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NAMING A COLUMN ON A SPREADSHEET: IS IT MORE ALGEBRAIC?

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Spreadsheets use a meaningful algebra-like notation which, research suggests, can support pupils in developing an understanding of variables. In this paper we discuss the activity of Year 8 pupils who were taught to name a column on a spreadsheet, and who were asked to reflect upon their activity in a stimulated recall interview. More specifically, we discuss their understanding of notation, such as 'A2' and 'm', which they used when constructing spreadsheet formulae, together with the reflections of their teacher. We suggest that experience of naming columns may help pupils to develop a clearer sense of the notation as a variable, and to make links between their spreadsheet activity and use of standard algebraic notation.

BACKGROUND

Spreadsheets use a meaningful algebra-like notation. A cell reference refers to the particular number in a cell, and any number that may be entered into that cell, as well as describing its physical location. Spreadsheets also have the facility for filling down a formula through a range of cells; hence the column can also be seen as representing a variable. Research suggests that spreadsheets can support pupils in developing an understanding of variables. In a longitudinal study of two groups of 10-11 year old pupils working on traditional problems, Sutherland and Rojano (1993) conclude that 'a spreadsheet helps pupils explore, express and formalise their informal ideas' (p.380), moving from thinking with specific numbers to symbolising a general rule.

It is claimed that spreadsheet notation 'can ultimately be used as cognitive support for introducing and for sustaining the more traditional discourse of school algebra' (Kieran, 1996, p.275). Sutherland (1995) found that low achieving 14-15 year olds, who had worked on a unit which required them to write an algebraic version of a spreadsheet formula, were able to use their knowledge in a paper and pencil test. Sutherland suggests that the 'the spreadsheet symbol and the algebra symbol came to represent "any number" for the pupils' (p.285).

Noss (in press) recognises the shift of attention to pedagogy in research agendas, distinguishing between the 'impressive by-products of children's Logo experience,' and the lack of evidence that such Logo experience leads to 'understanding of pencil-and-paper algebra.' Noss calls for more research into this process, and points to the critical role of the teacher.

The Purposeful Algebraic Activity project [1] aimed to explore the potential of spreadsheets as tools in the introduction to algebra and algebraic thinking. To this end, we designed and implemented a spreadsheet-based teaching programme with five classes of Year 7 pupils (aged 11-12). The teaching programme, which consisted of six tasks, was taught by the pupils' usual teachers, with whom we worked throughout the project. We found that pupils from across the range of attainment could construct spreadsheet formulae, and we have evidence which suggests that some, but not all pupils interpreted the spreadsheet notation as a variable. We have also reported that some pupils tried to use a single letter rather than a cell reference when constructing a spreadsheet formula (Wilson, Ainley and Bills, 2004). These emerging findings informed the development of a related study, which focuses more closely on pedagogic strategies.

This related study seeks insight into the ways in which spreadsheet experience and teachers' pedagogic strategies shape the development of pupils' algebra. One of the classes who participated in the Year 7 teaching programme was traced into Year 8 (aged 12-13). During Year 8, the class were taught by Judith, an experienced mathematics teacher who was familiar with the content of the teaching programme (having taught it to other classes in Year 7). In their Year 8 algebra lessons, the class participated in follow up work which was driven by the Year 8 programme of study, the affordances of the spreadsheet and purposeful tasks. More specifically, Judith employed specific pedagogic strategies with the aim of optimising pupils' algebraic activity. This paper focuses on one of these strategies: naming a column.

A name for a column, or indeed a cell or row, can be defined on a spreadsheet. Letters or words can be used as a name and once defined the name can be used in a formula. When the formula is filled down through a range of cells, each of the new formulae includes the same name. This is unlike what happens when a formula with a cell reference is filled down:

	A	B
1	1	=2*A1+3
2	2	=2*A2+3
3	3	=2*A3+3
4	4	=2*A4+3

	A	B
1	1	=2*n+3
2	2	=2*n+3
3	3	=2*n+3
4	4	=2*n+3

The facility to name a column on the spreadsheet and use that name in a formula has not been widely researched in terms of pupils' algebraic thinking.

DATA COLLECTION

The data we present here relates to three lessons in which the Year 8 class worked on a task, *Myphone*. This task built upon and extended work from the Year 7 teaching programme. In each of the lessons, a range of data was collected. A pair of pupils, in this case Erin and Julian, were video taped whilst working on the task, and screen recording software was used to capture their spreadsheet activity. Their dialogue was semi-transcribed and the transcript later interwoven with details of the pupils' non-verbal behaviour and interaction with the computer. An audio recording was made of

Judith's interaction with the class using a radio microphone; this was semi-transcribed. Field notes were made by the first named author.

Following the lessons, a small group interview took place using the technique of stimulated recall (Calderhead, 1981). Five pupils were invited to watch short video clips from the lessons, some with and some without sound. Questions included: Can you remember what was happening here? What do you think Judith meant when she said …? Can you remember what you were thinking? Naming a column was one focus of discussion in this and in a subsequent stimulated recall interview with seven other pupils. The discussion was characterised by the friendly openness of the pupils, who had known the researcher for over a year and who represented the range of attainment within the set. Although the pupils were not always confident that they could recall what they were thinking at the time of the lesson, the interviews provided some valuable insights into their interpretations of notation. Judith, their teacher, was also interviewed at the end of Year 8. All interviews were transcribed.

THE *MYPHONE* LESSONS

The task involved solving problems related to four different mobile phones tariffs.

Myphone tariff	Monthly line rental	Cost of calls
Anytime A	£15.00	16p per minute
Anytime B	£22.00	12p per minute
Anytime C	£30.00	10p per minute
Anytime D	£50.00	8p per minute

In the first two lessons, the pupils were asked to set up a spreadsheet to show the cost under each of the tariffs, and then use the spreadsheet and scatter graph to solve various problems. The third lesson was classroom based and involved comparing spreadsheet approaches with paper and pencil approaches to solving the problems.

Erin and Julian entered the label 'minutes' in cell A1, and entered the numbers 10 down to 1 in column A. They knew that they needed to write a formula in cell B2.

	A	B
1	minutes	cost in pounds (using anytime A)
2	10	

Erin suggested that instead of thinking about a formula for the cost of *ten* minutes of calls, they could 'start at one.' They decided to write a formula in B11, since A11 contained '1'. Julian knew that the total monthly cost would be £15.16. Together, they generated the formula '=£15.00+a11*£00.16' and filled it up the column.

After some time, Judith stopped the class and invited two pupils to share the formulae which they had entered. The relative merits of each were discussed. At this point, Judith taught the class to name a column on the spreadsheet, which is something that they had not experienced in the teaching programme. The aim was twofold: to make clearer the links between spreadsheet notation and standard algebraic notation and hence to give meaning to paper and pencil activity; and to encourage pupils to see the

spreadsheet notation as representing a variable.

Judith What might be confusing about that formula [=A2*0.16+15] compared to doing it algebraically? If we were in the classroom doing this would you write it like that? … What would A2 normally refer to in algebra?

Pupil A number times by two …

Judith And are we actually doing a number times by two? … You can actually change the cells so that they can look more algebraic … We don't wanna call those cell A2, B2, er, A3, A4, A5 … We want to give them a letter like we would do in the classroom if we're gonna do that algebraically … Now instead of having to write down it's that cell times nought point one six plus fifteen, you can now say it's nought point one six times *x* if you've used *x* or nought point one six times *m* if you've used *m* and add fifteen

Judith taught the class how to name a column by highlighting it and entering a letter, such as *m*, directly in the name box. Erin and Julian named their minutes column as shown, and changed their formula from '=15+A2*0.16' to '=15+0.16*m'. They went on to write a formula for Anytime B and filled it down. When talking about the different numbers of minutes, Julian said 'we just use *m* don't we?' and both appeared to have a clear sense of what *m* denoted. In the second *Myphone* lesson, Judith showed the pupils how to name a column using 'Insert,' 'Name' and 'Define' which the pupils confidently explored.

PUPILS REFLECTING ON THEIR ACTIVITY

'It doesn't really feel like algebra on the spreadsheet,' explained Ruchira, when talking in general about her conception of relationship between spreadsheets and algebra. When introducing the naming of columns in the first lesson, Judith told the class that it was 'more algebraic,' and in the second lesson she reinforced this idea.

Judith So now when I double click on that … you can see the formula actually has the letter *m* in it which makes it more algebraic, okay.

In the stimulated recall interview, the pupils were asked specific questions such as: How would you explain to someone what A2 (or *m*) means in the formula? What do you think Judith meant by 'more algebraic'? From our analysis of the pupils' responses and the ensuing discussion about the merits of naming a column, we report on the pupils' interpretations of 'more algebraic' under two broad themes: using letters; and letters as variables.

Using letters

When asked what they thought Judith had meant by 'more algebraic,' some pupils referred to the fact that having named a column, formulae could be written using letters. They felt that using letters was 'more algebraic' because they had used letters in their algebra lessons.

Julian It looks more like an algebraic formula, because instead of A2 it's like "a,

and you always use *a* on paper (open hand gesture)

Jane And it would be easier to write it down on paper, as well. Like, 'cause if you wrote A2, people would get confused. You just write *a*

Julian They know it's a number, it's representing a number

Researcher And what if you wrote A2 instead?

Julian It would look like A times two

In an earlier lesson, the class had worked on finding equivalent expressions, and were encouraged to multiply out brackets. They had used the spreadsheet to support their activity. In this lesson Erin had tried to multiply 4 by A2, and 2 by A2, but had written the products as 8A2 and 4A2 respectively. Erin commented on this possible confusion, and suggested that it was 'easier' to use a letter.

Erin Because in the classroom they might think it's A times 2 … normally in algebra, when you're doubling something, you should put (.) say if it was *x*, it would be 2*x*. So they might think A is just another *x*

Other pupils, however, felt that this wouldn't be a source of confusion because in standard algebra 'it's 2*a*, it would be the other way round' and 'algebra letters are never capitals.' Thus, there were mixed opinions about whether naming a column was a useful thing to do.

In the third classroom lesson, Judith projected the image of the spreadsheet onto the whiteboard and invited the class to think about writing formulae in standard algebraic notation and solving the problems using paper and pencil methods. The pupils were asked to 'set it up algebraically' but there was very little response from the class. Having watched the video of this part of the lesson, Ruth explained that because *numbers* were displayed on the spreadsheet, she didn't see it as algebraic.

Ruth It doesn't look like anything [meaning anything algebraic]. It's just like pounds and I didn't think of like what you'd put in, the formula for that, which is what she [Judith] meant.

Erin also commented that she did not always recognise the links between their spreadsheet activity and paper and pencil algebra.

Erin I see links between them when she [Judith] talks about links between them but when they're like separate then I think they're separate (laughs)

Overall, the pupils' comments suggest that using letters and then viewing the formulae (rather than the numbers) are useful bridges in helping them to make links between spreadsheet activity and standard algebraic notation. This, in turn, may help pupils to give meaning to paper and pencil activity such as solving equations.

Letters as variables

The pupils were also asked to reflect upon their understanding of A2, used in the spreadsheet formula, and upon their understanding of *m* or *a*, the name they had

defined for the column. Responses suggest that the pupils saw A2 as the location or name of the cell, or more specifically *the number* in cell A2. When constructing a spreadsheet formula, most pupils clicked on the number in a cell. Although they were aware of the potential of filling down the formula, the pupils did not really need to engage with the notation when they constructed the formula. More often, it appears that pupils thought about a particular arithmetic example to help them to construct a formula, such as when Erin and Julian knew that the cost should be £15.16. In the *Myphone* task, pupils filled down the formula and tried different numbers in the minutes column. They didn't actually *change* the number in A2.

When pupils engaged in naming a column, they actively defined the set of numbers that appeared in the minutes column using a letter. We were interested in how pupils understood this activity. When asked how he would explain what the letter *a* meant in the formula, Julian's response indicates a sense of variable.

> Julian Just like a column. Just the A column I think (moves hand up and down) … A2 is just that individual column (points to a single point), column, like cell thing (makes box shape with hands), the *a* is the actual whole thing column (moves hand up and down)

For Julian, it appears that a letter meant anything in the column, which is consistent with his comment in the lesson that 'we just use *m*' to write a formula for different numbers of minutes. Ruth seemed to have a similar understanding of the notation to Julian, again consistent with her activity in the lesson of trying different numbers of minutes in the column which she had defined using the letter *m*.

> Ruth Um instead of like putting A2, just as a cell, if you put *m* it sort of means like "any cell and "any number. So it will change "everything, not just one thing
>
> Researcher … Okay. But for you A2 means
>
> Ruth Like just that one cell

If we place this in the context of the pupils' activity in these lessons, where they did not change the number in A2, we suggest that when she talked about *m*, she had a clear understanding of variable as 'any number.' It is less clear whether she understood A2 as a variable.

In contrast, Ruchira felt that there was not much difference between the notation A2 and *m*, and she didn't feel that using *m* was different.

> Ruchira It's like *m* stands for like a number, one thing, and (.) A2 stands for one thing as well so it doesn't really make much difference to me … it's just too much (.) like, you have to "change it all and everything when you can just click it. I mean if it was me I would just click A2. I think it's easier

Ruchira's conception sees A2 and *m* as standing for 'a number, one thing,' rather than any number in a column. Perhaps at the time of constructing the formula, Ruchira focused on the specific number in A2.

TEACHER REFLECTIONS ON THE PUPILS' ACTIVITY

Having discussed two ways in which pupils saw naming a column as more algebraic, we outline Judith's reflections on this aspect of the pupils' activity. Judith was positive about naming a column, seeing it as a valuable strategy in making links to the paper and pencil algebra from the Year 8 programme of study.

> Judith The naming of the column I think is successful in terms of them linking the algebra, certainly from my point of view … I think it was a good step to make and I think it's probably a step, if you're going to make the links to algebra, it's a step that they perhaps need to do fairly early

As reported earlier, the pupils had mixed opinions about whether it was useful to use letters in this way. Initial analysis suggests that the strategy was seen more positively by those pupils, such as Erin and Ruth who were unsure about the links between spreadsheets and paper and pencil algebra.

Following the second theme identified from the pupils' reflections, Judith also felt that naming the column helped pupils to develop a sense of variable, so that they saw the letter as 'anything in that column.' Further, she pointed to the distinction between listing a series of calculations and changing a number in a cell.

> Judith One of the things I think there's a slight confusion on their part is the "difference between having a "list of numbers in column *n* and having a "single number that you change. Um, I don't think that they see the subtle difference between them … I can't put my finger on "exactly what, but a few times I've got the impression from what the children have said. They don't "quite see that this calculation here is the same as the next one above but with a different number in your variable position, if you see what I mean. They think of it as a different position almost and therefore not the same, not quite the same variable

We have reported elsewhere on two different constructions of meaning for variable (Ainley, Bills and Wilson, 2004). Following on from Judith's comments, it would be valuable to study naming a cell as well as naming a column. In tasks such as *Myphone*, however, where pupils do make a list, naming the column may offer a useful image of a single letter as a variable. Judith also perceived that naming a column helped some pupils to identify the variable and to set out their work.

DISCUSSION

The picture that we have presented is complex. When pupils were asked what they had understood by 'more algebraic,' two broad themes emerged: using letters and letters as variables. Judith and her pupils recognised that formulae constructed with letters *look* more algebraic. Bridging the notation in this way seemed to help some pupils, such as Ruth and Erin, make links between spreadsheets and algebra which they may not have made otherwise. Such links may help pupils to develop a clearer sense of what it means to solve an equation, for example.

Whilst for some pupils the spreadsheet symbol and the algebra symbol do represent 'any number' as Sutherland (1995) reported, we recognise the complexity of pupils' interpretations, and the complexity of researching such interpretations. Nonetheless, we suggest that naming a column can potentially support pupils in developing a clearer sense of the notation as a variable, and that engaging in the process of defining a name fosters that sense of variable. We recognise, as some pupils did, that there is no real reason to name a column on the spreadsheet. These high achieving pupils understood that the value of *a* used in a calculation was the number in the same row. For other pupils, however, greater transparency, through the use of cell references, may be more beneficial.

Overall, we are encouraged by the pupils' and teacher's reflections on the value of naming a column on a spreadsheet. Showing pupils how to name a column may be a helpful strategy for teachers who want to ensure that the algebra doesn't get lost amidst rich spreadsheet activity. In our future analysis, we will explore further the relationships between pedagogic strategies and pupils' development.

NOTES

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