Thwaites, A., Huckstep, P. and Rowland, T. (2005), 'The knowledge quartet: Sonia's reflections', in D. Hewitt and A. Noyes (Eds), *Proceedings of the sixth British Congress of Mathematics Education* held at the University of Warwick, pp. 168-175. Available from www.bsrlm.org.uk.

THE KNOWLEDGE QUARTET: SONIA'S REFLECTIONS

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This paper is a continuation and extension of ongoing work, investigating the mathematics subject matter knowledge of prospective primary school teachers, which has recently become a high profile issue in the UK and elsewhere. In earlier work, a grounded approach to data analysis led to the identification of a knowledge 'quartet', with four broad dimensions which we are calling foundation, transformation, connection and contingency. Subject knowledge could be evidenced in practice through these dimensions. This most recent development builds upon the original work. It provides a case study of a trainee who has been invited to reflect upon her lesson a few hours after teaching it. In talking freely about her lesson and also by responding to an interviewer who had previously viewed and analysed a video recording of the lesson, further insights were gained.

INTRODUCTION

There is now a growing body of UK research on prospective primary teachers' mathematics subject knowledge. The process of audit and remediation within Initial Teacher Training became high profile with the introduction of government requirements. (D*f*EE, 1998) However, the balance of advice has gradually shifted away from assessment of this knowledge, by some form of 'testing', towards seeking evidence for trainees' subject knowledge in the act of teaching i.e. within the school-based placements. (e.g. Rowland, Martyn, Barber and Heal, 2000; Goulding and Suggate, 2001; Jones and Mooney, 2002; Sanders and Morris, 2001, Morris, 2001; Goulding, Rowland and Barber, 2002)

The focus of the case study reported in this paper is, in the first instance, on the ways a trainee's mathematics content knowledge may be seen to be cashed out in practical teaching during a school-based placement. In this respect, it is similar to previous work. (Rowland, Huckstep and Thwaites, 2003a) However, what is distinctive about this particular study is that some of the inferences made about the trainee's mathematics content knowledge are informed by the reflective responses that take place in an interview with the trainee during the same day that her lesson had been taught. This process of discussion and reflection is at heart of the learning relationship between trainee teacher and mentor (university and school-based) and the earlier work has been used as a basis of guidance to mentors. The new study provides another perspective.

METHOD

This study, as in the previous one, is of a trainee's final professional placement during the last of term of a one-year PGCE course. From earlier grounded research

(Rowland, Huckstep and Thwaites, 2003a), critical episodes in trainees' teaching were identified. In these, Subject Matter Knowledge and Pedagogical Content Knowledge could be articulated in terms of one or more of 18 coded descriptions such as *adherence to textbook* and *recognition of conceptual appropriateness*. But it soon became apparent that 18 codes were too unwieldy for classroom observation. The resolution we have termed the 'knowledge quartet'; the 18 categories are grouped into four broad, superordinate units – the dimensions or 'members' of the quartet. (Rowland, Huckstep, and Thwaites, 2003b) We have named these units foundation, transformation, connection and contingency.

For this new part of the study, a trainee was videotaped teaching a lesson by one member of the research team. Soon afterwards the team met to view the tape and to identify some key episodes in the lesson using the codes from the earlier work. Then, a team member met with the trainee to view the videotape and to discuss some of the episodes. An audio recording was made of this discussion, and later the recording was transcribed. In this stimulated recall, the trainees were able to reflect on the lesson and give further evidence about the role of their subject knowledge in their teaching. In practice, the trainees wanted to talk about a range of aspects and the researcher focused the discussion on the mathematical content of the lesson. One of the aims of this phase of the project was to complete the three stages (videotaping the lesson, team reviewing the lesson, discussion with the trainee) in a short time span. In the case considered in this paper, the whole process occurred within one day.

THE KNOWLEDGE QUARTET

The brief conceptualisation of the knowledge quartet that now follows draws on the extensive range of data (24 lessons) collected from the study so far. Some aspects of the characterisation below will emerge from our consideration of the lesson that we have singled out for attention later in this paper.

Foundation

Our first unit is rooted in the foundation of the trainees' theoretical background and beliefs. Both empirical and theoretical considerations suggest that the other three units flow from it. Our conceptualisation of this category includes trainees' beliefs, knowledge and understanding – gained both in their 'personal' education and in their learning *in the academy*, in preparation for their role in the classroom. The key, defining feature of the category under immediate consideration is its propositional form (Shulman, 1986). We take the view that the possession of such knowledge informs pedagogical choices and strategies in a fundamental way. The key components of this theoretical background are: knowledge and understanding of mathematics *per se*; knowledge of significant tracts of the literature and thinking which has resulted from systematic enquiry into the teaching and learning of mathematics; and espoused beliefs about mathematics, including beliefs about how and why it is learnt. (Bramall and White, 2000)

Transformation

The second of these four categories shifts attention from the acquisition of knowledge to knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself. At the heart of this category, and acknowledged in the particular way that we name it, is Shulman's observation that the knowledge base for teaching is distinguished by " … the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful". (1987, p. 15) Our second category, unlike the first, picks out behaviour that is directed towards a pupil (or a group of pupils), which follows from deliberation and judgement. The trainees' choice and use of examples has emerged as a rich vein for reflection and critique. This includes the use of examples to assist concept formation, to demonstrate procedures, and the selection of exercise examples for student activity.

Connection

The next category unites certain choices and decisions that are made for learning discrete parts of mathematical content. It concerns the *coherence* of the planning or teaching displayed across an episode, lesson or series of lessons. Mathematics is notable for its coherence as a body of knowledge and as a field of enquiry, and the cement that holds it together is reason. In recent, influential work (Askew, Brown, Rhodes, William and Johnson, 1997) five of the six case study teachers, found to be highly effective, gave evidence of a 'connectionist' orientation. Our conception of coherence includes the *sequencing* of topics of instruction within and between lessons, including the ordering of tasks and exercises. To a significant extent, these reflect deliberations and choices entailing not only knowledge of structural connections within mathematics itself, but also awareness of the relative cognitive demands of different topics and tasks.

Contingency

The fourth member of the quartet is distinguished both from the possession of a theoretical background, on the one hand, and from the planned deliberation and judgement involved in making learning meaningful and connected for pupils, on the other. Our final category concerns classroom events that are almost impossible to plan for. In commonplace language it is the ability to 'think on one's feet'; it is about *contingent action* and is thus knowledge-in-*inter*action. The two constituent components of this category that arise from the data are the readiness to *respond to children's ideas* and a consequent preparedness, when appropriate, to *deviate from an agenda* set out when the lesson was prepared. When a child articulates an idea, this points to the nature of *their* knowledge construction, which may or may not be quite what the teacher intended or anticipated.

THE LESSON

Sonia had completed a joint honours degree in Religious Studies and Education. Although she had passed the required GCSE mathematics examination she joined the

PGCE course with concerns about her own mathematical knowledge and confidence. Her lesson is with a Year 4 class. She begins with a short exploration of shape, introducing the learning outcome of the lesson - that pupils will be able to "... make and describe repeating patterns which involve translations and/or reflections". Then she moves onto a numerical task, as an oral and mental starter, before returning to the work on shape. We shall outline and discuss three episodes within the lesson.

Episode 1

Sonia's differentiated oral and mental starter concerns finding complements in 100 and 1000. The three pairs of examples she uses are:

82 + ? = 100	35 + ? = 100	63 + ? = 100
820 + ? = 1000	350 + ? = 1000	630 + ? = 1000

A valuable insight into Sonia's ability to undertake subject knowledge **transformation** comes from her response - firstly instantaneous, then reflective - to the interviewer's question about the *choice of examples* that she uses for this activity.

Teachers often structure their examples to achieve optimal learning uptake. Sometimes this might be simply a matter of making them progressively more demanding in some way as their pupils display success. But this raises the question of what counts as one complement in 100 being more demanding than another. More specifically, does Sonia have explicit (or implicit) decision criteria for her choices? In our earlier work we were obliged to *infer* from our observations, in the present case we were in a position to *defer* to Sonia in an interview. Nevertheless, her first response on the post-lesson audiotape was no more reliable than our own inference might have been:

Interviewer: You know when you did these … something add something equals…

Sonia: mm

Interviewer: … 100 and 1000 and so forth, and the examples that you chose were 82 …

Sonia: Completely random.

Sonia … there was whatever came into my head

It is tempting to suppose that since Sonia's 'choices' involved no apparent deliberation, they must have been arbitrary. Yet on further questioning, she was able to account for what simply seemed to have come straight to her:

Interviewer: … Sometimes there's a choice, when you're giving examples, sometimes … students or teachers have a particular reason for doing it. In your case these were just sort of…

Sonia: What were they? There might have been a reason

Interviewer: 82, 35

Sonia: 35 because it was a smaller … was an actually smaller number, I remember the reason for that one.

Interviewer: So you had a smaller number after the …

Sonia: Yeah, after the big number. And then I made sure that that the … the last digit of the 63 was a different last digit to the other two

Interviewer: Why did you have the smaller one … in the middle?

Sonia: Don't know. I just thought, this, I'd have a smaller number, like a substantially smaller number than 82

…

Sonia: The units … the ten was intentional but the unit was random, in that case. And in the last one, 63, the … ten was random but the unit was intentional.

Interviewer: Right, so there's some … thinking behind it.

Sonia: Yeah.

Through this discussion, the rationale for Sonia's choice of examples has been teased out. This type of discussion can be helpful in reflecting on practice and making explicit those decisions in planning that can have a marked effect on the children's learning.

Episode 2

Perhaps the most interesting episode arises when Sonia dwells on the pupils' solutions to 63 + ? = 100. Since she is presented with three answers 37, 27 and 47 the way is open for **contingent action** on her part.

It is to Sonia's credit that she does not immediately announce the correct answer but decides to invite a volunteer to discuss his method publicly. Matt firstly finds the complement in 10 of the 3 of 63, saying "if you do 3 add 7 that makes 10". It is at this point that Sonia prompts him by asking "where have we got to?". There is something ambivalent in this utterance. In saying it, Sonia could simply be drawing the pupil back onto the task by asking him how much of the problem had been solved. On the other hand, the "where" could be a tacit way of suggesting a place in a specific mathematical sequence suggesting that she is guiding Matt into a sequential method. Either way he takes the cue and increasing the 63 by 7 writes 70 + 30 = 100. Sonia then ties things together asking "what have we added on?". She rings the 7 and the 30, asks the class what 30 and 7 make and finally draws out the answer to her original question: 63 + 37 = 100.

Hitherto, the team would have had to conjecture about the intention of the question "where have we got to?". The interview, however, is illuminating:

Interviewer: Yes, and he said, … 3 add 7 that's 10, so, …you … referring back to what you said earlier on, you make it up to a nice number

Sonia: Yes

Interviewer: … and you said, "where have we got to?"

Sonia: Yes

Interviewer:… and he said 70

Sonia: Yeah … I think, was it 63, was the number?

Interviewer: Yes, but he said *three* add 7 is 10 … were you trying to get him to do it in sequence, then?

…

Sonia: I thought that was what he was going to do, so I was just hoping he was, and tried to push him in that direction.

Interviewer: Yes, so "where have you got to?" is just the right sort of prompt there …

In this interchange we confirm with Sonia that she was hoping to draw out a specific process from Matt. This may also have helped to clarify the thinking of those children who gave answers of 27 and 47.

Episode 3

The choices of shapes Sonia selects to transform in the next stage of her lesson reveals some shortcomings in her **foundation** knowledge. In particular, she does not appear to realise that the internal properties of a transformed figure can mask certain effects of a transformation, particularly reflection.

With the learning objective of pattern-making in place, she tries to establish that when a shape is transformed, a second shape is generated which is "the same" as the first. This is slightly confusing, because if her notion of a transformation is a movement, when using *objects*, there will not be two shapes. The moved object will become the image of the transformation but the domain shape will no longer exist.

The shapes (including a circle and a rectangle) that she is using are special ones and do not reveal a change of orientation under the transformations of reflection and translation. An astute pupil presents her with an opportunity for **contingent action**. He chips in with "if you reflect it with an L shape it wouldn't turn out the same". This time, whilst Sonia endorses the pupil's response she makes no attempt to enact his suggestion publicly. However, when questioned later the same day Sonia readily saw this as a missed opportunity:

Interviewer: … it's about the boy who did the … who asked for the L-shape

Sonia: Yes

Interviewer: The shapes that *you* chose were a rectangle

Sonia: Yes

Interviewer: … and a circle, which … have got a certain amount of regularity

Sonia: Yeah

…

Interviewer:… if you flip the … the rectangle, the same … but the L-shape … hasn't got any symmetry in it, if you like

Sonia: mm

Interviewer: Emm, so did you, were you aware of that, or just…

Sonia: [laughs] I took random shapes off a pile [laughs] um, yes.

Interviewer: Well, you can see the boy's point…

Sonia: Oh yes, definitely

Interviewer: It's quite a good reply

Sonia: If I were to do it again, I would …

Interviewer: It would be striking what has happened to the shape if itself it didn't have any symmetry

Sonia: It would be much easier for them to see

So here we hope that the discussion has helped Sonia extend her understanding of transformation and how particular example shapes can demonstrate the essence of a specific transformation. Through the act of teaching, Sonia is showing some of the areas of her subject knowledge which will benefit from further development.

CONCLUSION

The knowledge quartet has given us a framework to be able to view trainees' lessons from a subject knowledge perspective. This enables a focus on the mathematics rather than more generic issues such as organisation and management of the learning. The new layer to our work has added the trainee teachers' reflection on their lesson, assisted by viewing a recording of the lesson. We hope that this will help to develop further our understanding of the trainees' own subject knowledge but also the ways in which we (as mentors as well as researchers) can engage them in reflective dialogue which supports and builds upon their existing subject knowledge.

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