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INCORPORATING DYNAMIC GEOMETRY INTO SECONDARY MATHEMATICS: TEACHER PERSPECTIVES AND PRACTICE

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Having identified the archetypical status of angle-property topics in the emergent practice of DGS use in English secondary schools, this study examined teacher perspectives and practice through lesson observation followed by teacher interview. Teachers saw DGS as making an important contribution to working efficiently with geometric figures. In developing viable approaches to classroom tool use they differed in the degree to which they expected students to make use of DGS and exposed them to apparent anomalies of operation. The prime purpose of DGS use was in evidencing geometric properties through dragging figures; most commonly, dragging to examine multiple examples or special cases; more occasionally, dragging to examine dynamic variation. The emphasis was on mediating geometric properties through numerical measures, with little direct geometrical analysis.

INTRODUCTION

This paper reports a multiple-case study of what preliminary research identified as being archetypical current practice in using dynamic geometry systems [DGS] in secondary mathematics education in England. Drawing on lesson observations, and most directly on post-lesson interviews with the teachers leading these lessons, the study sets out to illuminate the didactical thinking informing such use of DGS.

While some aspects of the educational use of DGS have been relatively widely researched, little attention has yet been given to their use in ordinary mathematics lessons (Gawlick, 2002; Sträßer, 2002). In particular –as more generally within research on ICT in mathematics education– few studies have examined integration of the technology into mainstream classroom practice, or the perspectives and practices of teachers regarding use of the technology (Lagrange et al., 2003).

A previous French study (Laborde, 2001), undertaken in a system taking a relatively formal approach to the geometry curriculum, reported that the types of DGS tasks favoured and devised by teachers evolved over the course of a three-year professional development project, from familiar paper-based tasks facilitated by the mediation of DGS, to quite new tasks inconceivable without such technology.

Recently, there has been significant advocacy of greater use of ICT –particularly DGS– in the teaching and learning of geometry in English schools (RS/JMC, 2001), as well as official endorsement of the use of DGS in a government-sponsored elaboration of the existing national curriculum at lower-secondary level (DfEE, 2001). It is reasonable to expect emergent use of DGS to be shaped by the longstanding orientation of English school mathematics towards ‘treat[ing] geometry almost entirely as an experimental science’ (Bell et al., 1983: p. 226).

DESIGN OF THE STUDY

This study forms part of a larger project examining ICT integration in secondary mathematics and science education in England, with a focus on the pedagogical thinking of successful practitioners. In the first phase of the project, a process of multiple recommendation was used to identify school departments regarded as successful, in terms both of the general quality of the education that they provide, and of integration of ICT into classroom practice. To identify what practitioners themselves regard as successful practice, we conducted focus group interviews with each subject department (during the latter part of the 2002/03 school year), in which teachers were invited to nominate and describe examples of successful practice.

In the second phase of the research (conducted during the earlier part of the 2003/04 school year), we invited teachers to help us to gain greater insight, through lesson observations and post-lesson interviews. The particular type of DGS use which had most frequently been cited as successful involved dragging a geometric figure so as to establish properties of its angles. To investigate this archetype further, we chose three teachers who had been particularly enthusiastic and expansive about it in the departmental interviews. They belonged to two departments with contrasting pedagogical orientations, both in state-maintained comprehensive schools. Within the period available for fieldwork, we were able to organise access to five lessons of this type involving these three teachers. As Table 1 indicates, these lessons did offer scope for comparison, not only between lessons taught by the same teacher (on which we also sought the teacher’s views), but between lessons on the same topic.

Table 1: Lessons observed

Lesson School_ Teacher/#	**Class** Year_ Set	**Mathematical topic**	**Organisation of DGS use**
N_F/1	9_low (3/3)	angle sums of polygons	teacher uses prepared figure projected onto ordinary whiteboard from laptop computer
N_F/2	10_low (3/3)	circle theorems	teacher uses prepared figures projected onto ordinary whiteboard from laptop computer
N_L/1	9_high (1/3)	circle theorems	student groups construct required figures on desktop computers, with each step demonstrated by teacher at interactive whiteboard
P_W/1	7_upper (1/2)	angle sums of polygons	student groups construct required figures on laptop computers, following opening demonstration by teacher at interactive whiteboard
P_W/2	8_upper (1/2)	corresponding angles	teacher uses prepared figure on interactive whiteboard, after it proves impossible to load copies onto laptop computers for use by students

Our evidence base consists of a detailed observation record for each lesson, with a transcript of the post-lesson interview conducted with the teacher. Transcripts were analysed through an iterative process of constant comparison, starting with open coding of a teacher's ideas about a particular lesson, proceeding to axial coding across lessons and teachers as a whole, resulting in thematic organisation of ideas. Teachers' accounts were also triangulated against observational records and departmental interviews. Material from these sources was used to refine some themes, particularly where it illuminated teachers' accounts, or extended them by identifying relevant aspects of teachers' practice not articulated in the post-lesson interviews.

THEMES ARTICULATED AND ENACTED BY TEACHERS

There were some broad pedagogical differences in the perspectives and practice of the three teachers, but space does not permit discussion of these here. We will focus solely on the three key didactical issues of DGS use which emerged. Again because of restrictions of space, we will summarise these themes, making sparing use of direct quotations from teachers. (A complete analysis of all themes, supported by teacher quotations, is available as a full paper from the authors).

Working efficiently with geometric figures

All three teachers were concerned with efficiency in constructing and measuring the geometric figures used in classwork. The benchmark to which they referred was the familiar situation in which these processes were carried out by hand. In particular, once a DGS figure had been constructed and the desired measurements specified, further examples could be created simply by dragging the figure, whereas to achieve this by hand required repetition of the whole drawing and measurement cycle. Teachers saw DGS as increasing the efficiency and accuracy with which figures could be created and measured, so expediting the pace and progress of lessons:

> Everything that we did there, I could have done by hand on the board, piece by piece by piece… It's very quick for me, I don't have to spend a long time drawing these things out. And then measuring the angles... We would make very little progress compared to what we've done already… It keeps the lesson moving at a good pace.

While use of DGS could remove difficulties that students experienced in drawing and measuring figures by hand, teachers also reported that some experienced (analogous) difficulties in physically manipulating DGS:

> If they're supposed to click on a point, the mouse isn't quite on it, so they'll click and create a new point, and then when they move the point they are supposed to move, the angle doesn't change with it because they've attached it to a different point. So there's all sorts of little things that you constantly have to [attend to].

Accordingly, N_L gave priority to instructing students in techniques for simplifying DGS figures through deleting points and lines. P_W reported that such difficulties were more frequent when manipulation was by means of a touchpad rather than a mouse. Although the observed classes were relatively inexperienced in making use of DGS, such difficulties appeared to be the norm in schools, accentuated by the very occasional use of this technology. In effect, then, both old and new technologies for creating geometric figures presented difficulties of physical manipulation for some students, with intermittent use playing some part in this.

Developing viable approaches to classroom tool use

Differences between teachers in giving students opportunities to use DGS were related to their views on the accessibility of DGS technique and its value to students, particularly regarding students learning how to carry out DGS constructions.

N_F (observed in both lessons with academically less successful classes) used DGS only for demonstration. He doubted that getting students to work with DGS would sufficiently repay the time and effort necessary to develop the necessary technique:

> If I wanted the students to do it, it would take a long time in order for them to master the package and I think the cost-benefit doesn't pay there… And there's a huge scope for them making mistakes and errors, especially at this level of student... And the content of geometry at foundation and intermediate level just doesn't require that degree of investigation.

His comments indicate that this judgement was influenced by the character of the curriculum prescribed for such students, and its assessment. While N_L was similarly sceptical as to whether working with DGS would directly benefit his students in examination terms, he saw it as having a potential to increase their enjoyment and understanding. Hence, (observed with an academically successful class), he regarded preparing students to undertake an element of construction as a worthwhile investment. His normal practice was for students to construct figures (following his step by step instructions) rather than using a prepared file (which he reported he would find difficulty in distributing to students' computers). P_W (observed with classes more academically successful than those of N_F, but younger than that of N_L) took an intermediate position, reporting that her use of DGS was carefully structured to minimise complexities, typically calling for students only to drag prepared figures, but occasionally involving them in simple construction.

Thus, although N_F and P_W shared concerns about the accessibility of DGS technique to their students, they made rather different decisions about its use. A crucial factor appears to have been their conception of DGS as a tool. Whereas N_F took a pragmatic view of DGS as 'just a drawing program', P_W sought to exploit the way in which DGS construction brought out mathematical relations:

> One of the main parts of this lesson was that they could… see that the software works geometrically... And so when they were trying to measure the angle, that really brought out the idea of what is an angle… Just the action of doing it... and they really understood that angles, these three points that are on two lines, and what it means.

In summary, teachers differed considerably in the degree to which they involved students in carrying out DGS construction, manipulation and measurement. Decisions about involving students in these technical aspects of DGS use were shaped by teachers’ assessment of the immediate demands and eventual benefits of such investment. These assessments, in turn, were influenced by whether teachers saw educative potential in the mathematically disciplined character of DGS use.

All three teachers commented on how they managed particular types of apparently anomalous result, occurring when the operation of DGS measurement diverged from expectation. One of these types of result arose in measuring reflex angles:

> Sometimes it doesn't do quite what you expect. For example, if you mark an angle… it will always mark the one less than 180 and that’s not always what you want it to do. And when you move things round sometimes the angle that it’s displaying isn't quite what you expected.

In his circle-theorem lesson, N_L considered only situations where the angle at the centre of the circle was obtuse, finally dragging it to a value of 180 degrees, so that its arms formed a diameter, in order to establish his final target result about the angle in a semi-circle. The issue of what might happen when this angle was dragged beyond that position to become reflex was not considered. Likewise, in his lesson on this same topic N_F avoided reflex angles.

In his earlier polygon lesson, N_F did inadvertently create a reflex angle, quickly dragging it back once he realised what had happened. By contrast, in her lesson on this same polygon topic, P_W made no move to prevent students encountering reflex angles, and indeed used this as a springboard for more extended mathematical discussion. In effect, she treated this as a situation open to mathematisation:

> For me, success is when the kids produce something and then say, 'This can't be right because it's not what I expect.'… So that happened in slightly different ways around the room, but it was one of the key things that the kids learned. That you can't assume that what you've got in front of you is actually what you want. And you have to look at it… and question it.

Another type of anomalous result could arise as a result of rounding numeric values. For example, in both the lessons on polygons, episodes occurred where the sum of angles diverged from the expected value. P_W again used the anomaly to promote more extended discussion and mathematisation. N_F dismissed it briskly, later explaining that he was concerned to keep to his lesson agenda. A related example occurred in the circle-theorem lessons, where both N_F and N_L carefully set defaults and managed dragging so as to avoid students meeting situations which might obscure the underlying rule or impede them from finding it:

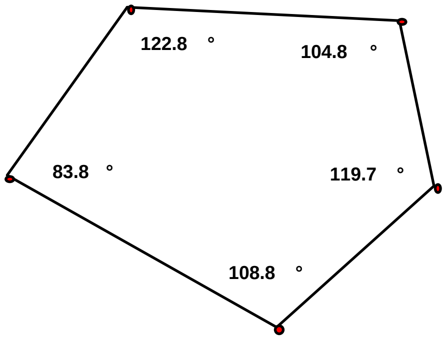
> I made sure the angles were always integer values... That way you don't have half angles to deal with. So if you noticed, the angle at the centre was always an even number of degrees because that way the angle at the outside can be halved quite successfully... So I did that to help make it a little bit easier for them to spot the rule.

In summary, teachers differed considerably in the degree to which they exposed students to apparent anomalies in DGS operation, rather than suppressing these. Such decisions were influenced by whether teachers saw such situations as providing opportunities for mathematisation, and for instilling a critical attitude to computer results.

Evidencing geometric properties through dragging figures

All three teachers identified the manipulation of figures through dragging as the central contribution of DGS use to their practice. To understand this aspect of practice more fully, it proved particularly useful to amplify the teachers' accounts by directly examining their use of dragging in action.

The logical development of both *polygon angle-sum lessons* followed an inductive sequence. N_F's lesson started with the familiar case of the triangle, which he used to introduce the dragging approach, then proceeded to quadrilateral and pentagon, with a view to establishing a table of angle sums, and formulating a pattern. Likewise, in her lesson on this topic, P_W reviewed the triangle case with the class, and then used the quadrilateral case to introduce drawing, measuring and dragging technique to the class. In both the lessons, dragging was treated as a means of generating different examples of each type of polygon. Identical types of DGS figure were used, on which measures of angles were marked, and any point was to be dragged (see adjacent figure).



P_W talked in terms of students continuing to select different examples until the invariance of the angle sum became persuasive. N_F suggested that arbitrarily halting the dragging conveyed a sense of selection from amongst many figures:

> The fact they can see it changing as you're dragging and dropping it, makes the difference. It's a bit more convincing for them. And then also at one stage I got one of them to actually tell me where to stop… so it wasn't always me that was choosing it.

Indeed, in the lesson, N_F explicitly introduced the idea of choosing at random:

> We've just picked four triangles at random and shown that that's true. And there's no way that could have happened by accident.

As well as calculating the angle sums of polygons, both teachers envisaged showing that they could be decomposed into triangles. During his lesson, N_F drew by hand onto each projected DGS figure to show a decomposition of this type. However, he did not identify the key issue as one of decomposing the angles of a polygon. Rather, splitting the polygon into triangles was treated as a matter of getting an accepted form of diagram. P_W commented that, had time permitted in her lesson, she would have asked students to add segments to triangulate their DGS polygons.

At the start of their interviews both teachers described their *circle theorem lessons* in terms which organised results deductively. However, the way in which the angle-at-the-circumference property was expressed in dynamic terms was notable:

> The objectives were... to learn that the angle at the outside was twice the angle in the middle, and also that it therefore didn't change as it moved around the circumference.

In fact, this more dynamically striking result was actually presented first in both lessons, and in both teachers' (subsequent) statements of the lesson agenda. By contrast, treatment of the remaining results appealed to dragging more as a means of generating different examples, or of examining a special case.

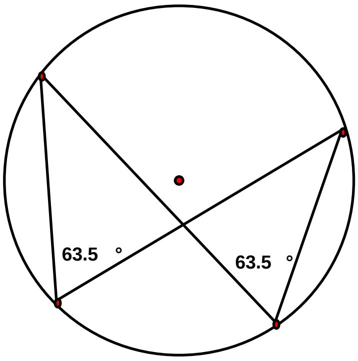
For the first result, N_F used dragging to convey a sense of the unchanging measure of the moving angle-at-circumference:

> The technology helps because they can actually see it getting dragged round, they see the angle doesn't change and they are much more convinced.

N_L's comments also appealed to dynamic variation, and illustrated its significance for students:

> They can actually drag it round and see that the angles change or don't change depending on what they are doing… in a way that you can't do without that dynamic… I heard one of the boys, for example, saying 'There's something wrong with this, it's always the same angle wherever I move it to'. And then it dawned on him that that was the whole point!

Although neither teacher commented on this aspect of their lessons, both incorporated episodes in which the 'dynamic' image of the moving angle-at-circumference was (tacitly) related to the more customary 'static' image of two fixed angles-at-the-circumference (see adjacent figure). Intentionally or not, these episodes can be seen as serving to establish an important relationship between the dynamic figures employed in these lessons, and the static figures which students would encounter once they moved on to tackle exercises on the page.



Dragging of figures was employed, then, to evidence properties in two ways. Most commonly, it was used to *examine multiple examples or special cases* of a geometric figure, without particular attention to variation during the dragging process itself, other than in evoking the multiplicity of possibilities. More occasionally, dragging was used to *examine dynamic variation* (notably non-variation) in a geometric figure during the dragging process, and this could extend to demarcating the domain over which a property held. In effect, then, this distinction rests on the degree of explicit attention to dynamic variation in the DGS figure as it is dragged.

Most strikingly, regardless of the type of dragging employed, *consideration of geometric properties was almost always mediated by the effects of dragging on numeric measures*, the brief exception being the overscribing of figures to show the triangulation of polygons (for which a geometrical rationale was not made explicit). While there were some allusions in passing to shape, space and movement, these played no part in the public analysis of geometric situations. This characteristic was common to the lessons of all three teachers, despite other important pedagogical differences.

DISCUSSION

Within the systemic subject culture (Ruthven et al., 2004) in which these teachers are working, mediating geometric properties through numeric measures is a well-established didactical norm, long predating the introduction of DGS, and presumably well adapted to current expectations of teaching approach and student learning. However, a more critical examination of these institutional circumstances would ask whether this represents an unacceptable degree of curricular narrowing. Accordingly, we have redesigned the central didactical artefacts –the dynamic geometry figures– used by these teachers, to make greater use of the dynamic potential of DGS, and to support classroom activity involving a broader range of mathematical thinking, extending –in particular– its visuo-spatial and logico-deductive aspects. We now hope to gain support to develop lesson designs exploiting and refining these new ideas, by working in collaboration with interested teachers.

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