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THE ROLE OF SUBJECT KNOWLEDGE IN PRIMARY TRAINEE TEACHERS' APPROACHES TO TEACHING IN THE TOPIC OF AREA

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This paper presents the methodology for a study to investigate the relationship between trainee teachers' own subject knowledge in the topic of area and their approaches to teaching within this topic. By combining the methodologies of studies by Tierney et al (1990), Baturo and Nason (1996) and Berenson et al (1997) it examines connections between the kind of subject knowledge that a trainee teacher exhibits in mathematics (conceptual or procedural) and their perceptions of teaching. The topic of area was chosen as this was found to be successful in determining subject knowledge in previous studies. If effectiveness of teaching is determined by the type of knowledge and how this knowledge is used then the nature of the trainee teachers' knowledge would be seen as important as the content.

INTRODUCTION

Trainee teachers in England require at least a Grade C in GCSE mathematics before they are accepted on an initial teacher training course but this may not guarantee that the trainee teachers have sufficient subject knowledge for the teaching of primary mathematics. It is possible that the type of knowledge acquired in secondary education is not always sufficient for the knowledge required by primary mathematics teachers. Simmons (1993) proposed that "in order to teach well the teacher needs to know about the subject matter in both width and depth to a degree unlikely to be found amongst those beginning a teacher training course" (p. 9).

Baturo & Nason (1996) described four kinds of mathematical knowledge. The definition of the first, 'substantive knowledge', requires the knowledge of both concepts and procedures and the connections between the two. That is teachers "should also have a sense of the mathematical meanings underlying the concepts and procedures" (Baturo & Nason, 1996 p.236). The other three kinds: 'knowledge about the nature and discourse of mathematics', 'knowledge about mathematics in culture and society' and 'dispositions towards mathematics', provide a more complete picture of mathematical understanding. Without denying the importance of the other three kinds of knowledge, this study concentrates on 'substantive knowledge' and how it might influence teaching.

Berenson et al (1997) used the terms 'procedure-centred' and 'concept-centred' to classify extremes of knowledge in their study of trainee teachers' perceptions of teaching. Procedural knowledge is defined as the knowledge of algorithms, formulae and definitions and conceptual knowledge as the relationship of ideas. It is proposed

in this study that if trainee teachers exhibit good 'substantive knowledge' their understanding will tend to be conceptual. Conversely poor 'substantive knowledge' would be more likely to indicate procedural understanding.

Askew et al's (1997) research into effective teachers of numeracy suggested that primary teachers with a 'connectionist' approach to teaching were able to see connections in the mathematics they were teaching. 'Connectionist' teachers were considered to be more effective in teaching mathematics. Ma (1999) used the phrase 'Profound Understanding of Fundamental Mathematics' (PUFM) and reinforced the notion that teacher knowledge that connects concepts in mathematics is more effective. In this study it is considered that a teacher showing a 'connectionist' approach or demonstrating PUFM would have good 'substantive knowledge'. That is it will tend to be conceptual knowledge.

Research by Baturo and Nason (1996) and Berenson et al (1997) indicated that trainee teachers' knowledge in the topic of area tends towards the extremes of procedural or conceptual. If it can be shown that a trainee teacher with conceptual knowledge is more likely to become a teacher with a connectionist orientation then the promotion of conceptual understanding would be paramount in initial teacher training. There is the possibility that other factors are involved. A student teacher may be strongly influenced by supervising teachers or their own creativity and knowledge of how children learn may have an impact (Calderhead, 1988). Aspects such as the trainee teachers' subject specialism and their development of pedagogical knowledge could also be taken into account.

THE TOPIC OF AREA

The topic of area has been used effectively elsewhere to examine trainee teachers' substantive knowledge (Tierney et al, 1990, and Baturo & Nason, 1996) and their perceptions of teaching (Berenson et al, 1997). The intuitive notion of area can make it a difficult concept to define (Suggate et al, 1998) but it may be seen as "the measurement of the 'surface covered' by a certain region" is given by Deboys & Pitt (1995, p.73). The concept is more abstract than other measures such as linear.

A person's concept of area also develops with their study of mathematics. Gowar (1979) explained how the concept of area in a rectangle is quite straightforward but, in order to understand how the formula for a rectangle relates to the formula for a triangle or a circle, conceptual knowledge is required. For example how is the formula for a triangle from the formula length x width determined. Even further, the notions of limits and integration are needed to relate this to finding the area of a circle using the ratio of Π or to finding an area within curved boundaries. Tierney et al's (1990) study identified misconceptions in trainee teachers' knowledge of area such as an over reliance on the length x width formula and its incorrect use with irregular quadrilaterals. These misconceptions can point to a lack of underlying meanings in a trainee's substantive knowledge.

The topic also provides a good vehicle for examining approaches to teaching. There are procedures, such as counting squares or using formulae, which can be given (procedure-centred approach) or taught conceptually by explaining relationships. It is also a topic in which examples (some real-life) can be employed to help children generalise the concepts or provide problem solving situations to provoke conflicts, for example comparing shapes where visual inspection is difficult, or investigating perimeter and area. The use of these would again suggest a concept-centred approach to teaching.

THE STUDY

The Sample

Due to the time-consuming nature of data collection using semi-structured interviews (Robson, 1997) a small sample of twenty trainee teachers would be used. This would be insufficient to provide conclusive generalisations but may provide indications. To ensure other factors are taken into account the sample should ideally represent different subject specialisms, qualifications in mathematics or abilities as roughly indicated by subject knowledge audits. Such a range may not be possible in practice. The trainees' time spent on the course would also need to be acknowledged as this may affect their pedagogical knowledge. The use of volunteers may also introduce bias. Trainee teachers would also need to be reassured that the interviews would not influence their overall course assessment.

The use of lesson plans

Previous studies have shown that trainee teachers' views of planning reflect their conceptions of teaching (John, 1991 and Berenson et al, 1997). The trainee teachers will be set the task of planning a short scheme of work for top infant or lower junior children to introduce the topic of area. The trainee teachers would be asked for a brief outline of teaching objectives and activities for three or four lessons to provide insight into the intended progression. This could determine a trainee teacher's intended progression into the use of formulae. The content will not be left entirely open as the National Curriculum (DfEE, 1999) provides objectives. In order to provide a realistic planning situation, trainee teachers would have access to resources available to the establishment. The selection of activities would reflect the trainee teachers' substantive knowledge and subsequently their selection of teaching methods.

Semi-structured interviews would be used to ask the trainee teachers to explain their lesson plans and the tasks. Biographical information such as age, qualifications in mathematics and specialist subject areas would be asked, as well as how the plan was developed (for example: using own experiences as a pupil, previously taught on a teaching practice, use of books or original idea). The trainee teachers would be asked to go over areas again, present their personal views and explain their rationale for including activities. Examples of more structured questions would be:

How do you think these activities would facilitate the children's learning?

What instructions or explanations would you give?

What questions will you ask?

What difficulties might the children encounter?

How would you assess the children?

Subject Knowledge Tasks

Five tasks would be given to determine the trainee teachers tendencies towards procedural and conceptual knowledge. The tests would be carried out in a clinical interview using probes to encourage interviewees to expand on responses (Robson, 1998).

Task 1

Objectives: This task, taken directly from Baturo and Nason's (1996, p. 245) research, includes both open and closed shapes to test trainees teachers' perspectives of the notion of area. Two 3-dimensional shapes are included to test the ability to differentiate between area and volume.

Probes: Why did you select this shape? Why didn't you select this shape? For an open shape, colour the part you would measure for the area. What part of this shape (a solid shape) would you measure to find an area?

Task 2

Objectives: This task is adapted from Baturo and Nason's study and is designed to test the ability to compare areas, initially without the use of formula. Trainee teachers are presented with two pairs of cardboard shapes. Dimensions are not given. Comparison by visual inspection alone would be inconclusive so understanding of the area measuring process could be examined. A trainee teacher who can only use external measurements and a known formula would indicate a procedural tendency, as opposed to one who considers several methods, for example, cut and paste, use of grids, tiling and so on.

Probes: Can you think of another way? If you could measure the shapes, how would you find out? (Allow a ruler to demonstrate.)

Task 3

The trainee teachers are given three cardboard shapes - a rectangle 9 cm by 4 cm; a parallelogram where the area is the same but the perimeter has changed (base 9 cm and height 4 cm); a parallelogram where the perimeter has remained unchanged – and are asked to put the shapes in order of area. Dimensions are not given.

Objectives: This task, developed from Tierney et al (1990) is intended to determine the trainee teachers' ability to consider changes in area and perimeter.

Probes: Can you think of another way? If you could measure the shapes, how would you find out? (Allow trainees a ruler to demonstrate).

Task 4

Objectives: Adapted from Baturo and Nason's study, this task aims to test the correct use of formulae, the use of 2 dimensional units and an understanding of the relationship with non-rectangular shapes, including the notion of limits and the ratio Π.

Probes: Does your answer seem reasonable? Query units if incorrect or not given in answer. What does base or height mean? Why is it over 2? (Refer to formula for the area of a triangle). What activities do you remember doing in school to derive the formula for the area of a triangle? What does Π mean? What activities do you remember doing to establish the meaning of Π?

Task 5

The following problem is given:

A gardener has a rectangular plot of land. She wishes to build a rectangular flower bed roughly in the middle. Both the length and width of the flower bed will be half the length and width of the plot of land. The rest of the land will be seeded with grass. She knows the area of the plot of land is $300m^2$. She needs to know the area to be grassed in order to buy the correct amount of seed. How can she find this area?

Objectives: The purpose of this task is to determine a trainee teacher's reliance on formulae or procedures. If totally reliant on a known formula, the trainee teacher may find this impossible to answer and ask for more information, for example, the actual length and width. If trainee teachers are able to find the area without applying the formula, sound connections may be apparent in their understanding. An attempt to check using a general formula may also indicate this.

Probes: How do you know your answer is correct? Could you use another method? What would the answer be if both the plot of land and the flower bed were triangular instead? If unable to answer - Why not? What would help you?

ANALYSIS OF DATA

Qualitative analysis of the data would be combined with quantitative analysis by ranking the degree of procedural/conceptual knowledge as assessed from the tasks, ranking the teaching approach as assessed from the lesson plans and relating the two. Using the five tasks described above a score of 1 to 3 would be given to each. A trainee teacher's understanding and the knowledge of relationships could be assessed against the objectives given for each task. A tendency towards procedural thinking would be concluded with a score of 1. A score of 3 could be given for trainees which show sound conceptual knowledge. A score of 2 would be given where there is no clear tendency. This would give a score out of a total of 15.

In analysing the lesson plans, five criteria could be used:

1. The language used in presenting the activities to the children.

   procedural - tell, give;

indeterminate - explain, demonstrate;

conceptual - discuss, question, find.

2. The types of activities used to help children develop a concept of area as the amount of surface

   procedural - very limited or formula given;

   indeterminate - demonstrations or verbal explanations;

   conceptual - covering with arbitrary units and tiles; use of different sized measures/grids.

3. Gradual introduction to help children generalise with planned opportunities for assessment.

   procedural - examples limited in number or variety; rush to give procedures;

   indeterminate - some use of different shapes;

   conceptual - a variety of shapes and contexts, including some real-life; use of different resources.

4. Opportunities for children to develop their own procedures.

   procedural - very limited or given;

   indeterminate - told how use of tiles, squares or formula relates to a notion of area;

   conceptual - appropriate resources to determine the best units; restricting number of tiles or using larger shapes where counting is undesirable to encourage the discovery of a formula.

5. Use of challenges or conflict provoking problems.

   procedural - examples limited or comparisons easily determined;

   indeterminate - some examples promote challenge;

   conceptual - comparison through visual inspection difficult; investigating changes in perimeter and area.

These criteria are given as an initial guide. Some adjustments may be required as the data is analysed.

The trainee teachers would then be assessed against each criterion, with a score of 1 for a procedural approach, 3 for a conceptual approach and 2 where there is no clear tendency. Again it is scored out of 15. These two scores for each trainee teacher could be plotted on a scattergram. In order to test reliability, the lesson plans, trainee teachers' workings and transcripts could be made available for subsequent analysis.

CONCLUSIONS

The analysis of the results would be used to test the hypothesis that the nature of the trainee teachers' subject knowledge influences their approach to teaching. Even though the sample size would be small, further insight into this relationship would be gained. It is possible that no extremes of conceptual or procedural are discovered or there is little or no correlation. In this case other factors such as creativity or pedagogical knowledge could be considered further. Any of these results would help provide more detailed knowledge of the nature of the mathematical subject knowledge of trainee teachers and the role this might play in their classroom practice.

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