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GENDER DIFFERENCES IN KNOWLEDGE TRANSFER IN SCHOOL MATHEMATICS

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This paper draws on research in the areas of gender differences in mathematical cognition and transfer of mathematical knowledge. An attempted replication of Mevarech and Stern (1997) will be discussed, in which data indicates a difference in the ways that boys and girls transfer knowledge between isomorphic problem sets with varying context. Girls' improvement between trials is predicted by the strategy they use in the first trial, while boys' improvement is not. Boys' improvement seems to be predicted to some extent by the order in which abstract and realistic problem contexts are presented, although the mechanism for this is not clear. A future study will be outlined that aims to confirm and clarify some of the issues raised here.

TRANSFER OF KNOWLEDGE IN MATHEMATICS

For a student to be able to use and apply a mathematical principle, they must be able to transfer knowledge of concepts across contexts. This paper is concerned with differences in the ways that children transfer concepts. In a study of children working as street vendors in Brazil (Carraher, Carraher et al. 1985), it was reported that the children could solve some fairly complicated problems that related to their work but were unable to solve isomorphic problems outside of that particular context. It would seem that a child that can apply a mathematical concept across a range of contexts has a fuller understanding of that concept than a child who can apply the same concept only in one particular context.

The term 'procept' has been coined in order to describe a combination of process and concept. Gray and Tall (1994) argue that a successful student of mathematics does not distinguish between process and concept, instead accumulating understanding in term of procepts. Those students that think in this condensed way – understanding that one symbol or set of symbols can represent both a process and a concept – are doing a qualitatively different (and easier) kind of mathematics than those who cannot. An example they give of a procept is:

> The symbol ¾ stands for both the process of division and the concept of fraction
>
> (Gray and Tall 1994)

Understanding symbolism in this way requires abstraction in understanding. It allows children to derive new facts independently of experience – for example the fact that 13 – 11 = __ is the same as 11 + __ = 13. In the same way, children who think in terms of procepts are able to understand that '-5' represents both a process (either subtracting five or moving five steps to the left on a number-line) and a concept (the negative number -5). This will mean easier progress in solving problems involving the manipulation of negative numbers, compared to that made by a student who thinks of the process and the concept as two irreconcilable entities. The Brazilian children in Carraher, Carraher et al. (1985) had knowledge of the processes involved in the mathematical problems set in the context of their work, but did not have knowledge of the related concepts. Their lack of proceptual understanding meant that they were unable to solve the same problems in different contexts. Presumably, if a child can be said to have a proceptual understanding of a particular mathematical concept, then they will be able in many cases to use that concept in finding the solution to associated problems. Saxe (1991) has asserted that children with such context-bound understanding construct "different kinds of mathematics knowledge" to those with a traditional schooling in mathematics. Anderson et al. (1995) suggest that Carraher, Carraher at al. (1985) and others such as Lave (1988), "demonstrate *at most* that particular skills practiced in real-life situations do not generalize to school situations. They assuredly do not demonstrate that arithmetic procedures taught in the classroom cannot be applied to enable a shopper to make price comparisons or a street vendor to make change" (p. 4). Many questions regarding both effective conditions for conceptual understanding and individual differences between students in conceptual understanding remain unanswered.

Mevarech and Stern (1997) suggest that there is greater facilitation of understanding in sparse problem contexts and improved transfer of knowledge when moving from sparse context to realistic context. Two groups of children were given both the sparse- and realistic-context version of a set of graph problems, in opposite orders. Children's understanding of the concept of rate of change was facilitated to a greater extent by sparse context than by realistic context, also that transfer of knowledge was more effective from sparse to realistic context. There was an analysis of the number of correct answers given by the children and a restricted analysis of the explanations given. These findings complement the notion of the procept in that the experience gained from an abstract problem set is more likely to provide a student with recognisably transferable knowledge, whilst experience from a realistic problem set provides information to some extent tied to the context within which it is presented.
The aim of the present study was to replicate Mevarech and Stern (1997), and in addition to investigate the differences between students' answers and explanations. Analysis will be undertaken of the understanding of problems demonstrated by individuals and the relation between that understanding and improvement between trials. The data presented in Mevarech and Stern (1997) reported averages and left open questions regarding individual patterns of explanation type and improvement. It is clear that not all students responded in the same way. The data suggest that some students showed patterns of responses other than those discussed. One way in which it was predicted that students might differ in the present study was across genders. Reasons for this prediction are discussed in the following section.

GENDER DIFFERENCES IN MATHEMATICAL COGNITION

Fennema et al. (1998) investigated the difference between boys' and girls' strategy use and found that girls tended to use standard strategies while boys tended to use more abstract, invented strategies. In addition to these results, Fennema found that those students (both boys and girls) who did use more creative or inventive strategies in solving problems were more able to solve extension problems.

Boys will tend to use retrieval in order to solve simple arithmetic problems while girls will tend to use algorithmic procedures. Boys tend to rate social pressures to solve problems quickly and effortlessly very highly and high ratings for such pressures are a good predictor of the use of retrieval by the end of the first year of primary school (Carr and Jessup 1997). Characteristics such as impulsiveness are typically associated with boys and have been shown to be good predictors of the use of retrieval when solving problems (Davis and Carr 2001). However, strategy choice is not only a function of preference. When strategy choice was controlled so that children could use only retrieval, boys outperformed girls (Carr and Davis 2001). Strategy choice is determined by a function of social pressure, temperament and influence of parents and teachers (Carr, Jessup et al. 1999), related to, but not exclusively determined by, gender.

The work of Fennema and of Carr has involved children in the first few years of school education solving simple arithmetic problems. A study involving older students investigated the strategies used when solving some problems from the American SAT-math paper (Gallagher and De Lisi 1994). Problems were classified as either conventional or unconventional. The conventional problems could be solved using standard algorithmic procedures, while the unconventional problems were best solved using insight or intuition. The purpose of the study was to determine how boys and girls chose strategies according to problem-type. They found that boys were more likely to choose strategies appropriate to a problem than were girls, and were therefore more likely to answer problems correctly. In a later study, the results of Gallagher and de Lisi (1994) were replicated (Gallagher, de Lisi et al. 2000) and the authors concluded that, 'strategy flexibility is a source of gender differences in mathematical ability'.

The present study aims to analyse the differences in answers, explanations and patterns of improvement demonstrated by boys and girls in response to the abstract and realistic problem sets, with attention given to the consistency or flexibility of strategies used.

ATTEMPTED REPLICATION OF MEVARECH AND STERN (1997)

Design

A two-way factorial design was used, the independent variables being problem type and problem order, with repeated measures on problem type. The dependent variable was the number of correct answers given, out of a possible total of 9, for a set of problems. There were two conditions for each independent variable. Problem types were ‘sparse’ and ‘realistic’; Problem orders were ‘sparse to realistic’ or ‘realistic to sparse’.

Predictions were that students would perform better on problems set in sparse context than in realistic context (problem type), and that greater improvement would be achieved by students moving from sparse to realistic context. In addition, it was predicted that there would be differences in the type of explanation given for answers to problems depending on both problem type and problem order.

Tasks

Three isomorphic sets of problems were used, adapted from the study of Mevarech and Stern. Each task took the form of three printed A4 sheets stapled together. The top half of each sheet showed a graph – all of the questions in the set referred to the same graph. The instructions advised children that could do any workings out on the graphs if they thought it might help, and also that they should pay careful attention to their explanations when asked for.

On the second page there were 6 questions (1.a-c and 2.a-c) that asked children to read values from the graph given a value on one axis. The part c questions asked children to work out how much the value on the y-axis increased as the value of the x-axis increased. On the third page there were 3 questions (3. 4. and 5.) that were taken directly from Mevarech and Stern (1997), with only the wording changed in order to improve children’s understanding of the questions. These asked children about the rates of change of the two lines on the graph and also asked children to explain how they decided on their answer.

The only difference between the three sets of problems was the context. The sparse context problems involved a graph with axes labelled ‘x’ and ‘y’, and lines labelled ‘line A’ and ‘line B’. There were two sets of realistic context problems; one involved a graph with axes labelled ‘income’ and ‘year’ and lines labelled ‘company A’ and ‘company B’, while the other involved a graph with axes labelled ‘amount of water’ and ‘time’ with lines labelled ‘tank A’ and ‘tank B’.

Participants

Participants were 45 year 9 students (aged between 13 and 14) studying at a school in Nottinghamshire. Students in the school were divided into two populations, or streams (X and Y). Within each stream, students were taught mathematics in sets from 1 to 5 according to achievement, where students in set 1 were the highest achieving. The participants in this study were all of the students present for both sessions in the two set 3 classes in the school (22 in class 9Y3, 23 in class 9X3).

The tasks were administered in the students’ usual mathematics classrooms. Students had studied line graphs in their classes, but had not formally studied the topics of gradient, compound measures or rates of change.

Procedure

Each complete class of students was assigned to one of two conditions. Class 9Y3 were asked to complete the set of problems with sparse context in the first session, then asked to complete a set of problems with realistic context in the second session. In the second session, one week later, students were divided into two groups; half were administered the changing level of water with time graph, half were administered the changing level of income with time graph. Class 9X3 completed the same tasks, in reverse order.

In order to determine group differences, mathematics SAT scores were recorded for the children taking part in the study. The Key Stage 3 SAT tests had been taken by the children 1 month prior to the beginning of this study.

Results

Before looking at students' performance on the problems, mathematics SAT scores were compared. The means for the two groups of students were 91.18 (Realistic to Sparse group) and 58.18 (Sparse to Realistic group, 1 absentee). A t-test showed a significant difference between the SAT scores of the two groups ($t=8.2$, $p<0.001$). An analysis of variance with repeated measures on problem type indicated no significant main effect of problem type ($MSe=1.91$, $F=1.24$, $p>0.05$) or for order ($MSe=8.198$, $F=2.29$, $p>0.05$) (see figure 1). The findings of Mevarech and Stern were not replicated in this study, but this could be due to the groups' widely differing SAT scores, and the possibility of a ceiling effect for the group with the higher level of SAT achievement.

Predictors of Improvement Between Trials

It became apparent that boys showed a greater level of improvement between trials than did girls ($F=4.099$, $p=0.042$). This led to the conclusion that there was something different about the ways in which boys and girls approached these problems that caused the difference in level of improvement. The original aim of the study had been to demonstrate that the order of problem set (context) predicted improvement. Now that it was clear that some children were improving while others were not, it was important to find out what (if not order) was the cause of that improvement.

The data were analysed again, this time looking at boys and girls separately. The first possibility considered was that improvement might depend on the type of explanation given (representing the strategy used) for solutions. Correlations were calculated between the incidence of the use of steepness in the first trial, incidence of the use of steepness in the second trial and improvement between trials (Table 1 shows correlations for girls, Table 2 shows correlations for boys).

The correlations showed a marked difference between boys and girls, in terms of the effect of the explanation given on improvement between trials.

Table 1

Correlations between the incidence of the use of steepness in the first trial (STEEP1), incidence of the use of steepness in the second trial (STEEP2) and improvement between trials for girls (n=22)

		Improvement	STEEP1	STEEP2
Improvement	Pearson Correlation	**1**	**.492***	**.501***
	Sig. (2-tailed)	.	.020	.017
STEEP1	Pearson Correlation	**.492***	**1**	**.910****
	Sig. (2-tailed)	.020	.	.000
STEEP2	Pearson Correlation	**.501***	**.910****	**1**
	Sig. (2-tailed)	.017	.000	.

* Correlation is significant at the 0.05 level (2-tailed).

** Correlation is significant at the 0.01 level (2-tailed).

Table 2

Correlations between the incidence of the use of steepness in the first trial (STEEP1), incidence of the use of steepness in the second trial (STEEP2) and improvement between trials for boys (n=23)

		Improvement	STEEP1	STEEP2
Improvement	Pearson Correlation	**1**	**.071**	**-.164**
	Sig. (2-tailed)	.	.747	.454
STEEP1	Pearson Correlation	**.071**	**1**	**.439***
	Sig. (2-tailed)	.747	.	.036
STEEP2	Pearson Correlation	**-.164**	**.439***	**1**
	Sig. (2-tailed)	.454	.036	.

* Correlation is significant at the 0.05 level (2-tailed).

For girls, the incidence of an explanation involving steepness in the first trial predicts the incidence of an explanation involving steepness in the second trial ($F=40.05$, $p<0.0005$). Also, the incidence of an explanation involving steepness in the first trial predicts improvement between trials ($F=4.603$, $p=0.044$). 12 of 22 girls gave explanations involving steepness in the first trial.

For boys, an explanation involving steepness in the first trial does not predict an explanation involving steepness in the second trial ($F=2.123$, $p=0.160$). Also, an explanation involving steepness in the first trial does not predict improvement

(F=0.102, p=0.753). 11 of 23 boys gave explanations involving steepness in the first trial.

Fisher's z transformation shows that the difference between correlations observed for boys and girls, between use of steepness in an explanation in the first trial and in the second trial, is significant (z=3.3, p<0.001). Neither boys' nor girls' explanations were affected by the order of the problem contexts. As many explanations involving steepness were observed in first trials for sparse-context problems as for realistic-context problems.

The data were analysed for the original hypothesis again, that improvement would be predicted by order of problem contexts, but this time analysing data from boys and girls separately. For the girls, the order of problem contexts does not predict improvement between trials (F=0.269, p=0.609). For the boys, there is some evidence to suggest that order of problem contexts does predict improvement between trials (F=3.064, p=0.095).

FUTURE PLANS

A study will be planned with the aim of confirming and clarifying the findings of the present study. The lack of detail in terms of the strategies used by children to solve problems will be the major impetus for change. In order to collect more detailed information on the strategies used by the children, they will be asked to solve problems individually, with session recorded using either audio- or video-tape. This will solve a number of problems encountered in the previous study. In the next study, children will be asked to explain what they are thinking as they solve each problem. After thirty seconds of silence, they will be interrupted with a prompt, asking what they are thinking. This method has been used in similar studies with good results (Gallagher, de Lisi et al. 2000). Records of children's answers and explanations will be used to classify children's strategy for problems.

The main areas that require clarification are in determining the mechanisms by which girls and boys improve between trials. Girls will be predicted to use similar strategies in both trials. Improvement made by girls would be predicted by the appropriateness of the strategy used on the first trial. In the present study, it was apparent that if girls used the concept of gradient (an appropriate strategy) for the first problem set then they were highly likely to use it in the second. What was not clear was the likelihood of repeating the use of inappropriate strategy. The increased level of detail hoped for in the planned study would help to clarify this point.

Boys will be predicted to be more flexible in their strategy use. Boys' strategy use in solving problems in realistic context will be predicted by the order of problem contexts. That is, if boys see the abstract problem set first, it will influence their strategy choice for the realistic problem set. The mechanism by which boys improve between trials is far from clear. The effect of the order of context was much greater for boys than for girls, but this was not explained by differences in patterns of strategy use between groups. The categorisation of strategies other than the use of gradients is hoped to provide some insight into the changes in solution procedure made by boys in order to achieve the level of improvement noted in the present study.

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