Iannone, P. and Nardi, E. (2005), The pedagogical insights of mathematicians: effectiveness of interactive modes of teaching', in D. Hewitt and A. Noyes (Eds), *Proceedings of the sixth British Congress of Mathematics Education* held at the University of Warwick, pp. 73-80. Available from www.bsrlm.org.uk.

THE PEDAGOGICAL INSIGHTS OF MATHEMATICIANS: EFFECTIVENESS OF INTERACTIVE MODES OF TEACHING

Paola Iannone and Elena Nardi

University of East Anglia

Within the mathematical community a realisation has been growing recently that, alongside modifications of the syllabus to adjust to the background of new intakes of students, reflection on university-level pedagogical practice, is urgently necessary. We conducted a series of themed Focus Group interviews with mathematicians from six UK universities. Pre-distributed samples of mathematical problems, typical written student responses, observation protocols, interview transcripts and outlines of relevant bibliography were used to trigger an exploration of pedagogical issues. Here we elaborate the participants' reflections on the effectiveness of interactive modes of teaching in the context of lecturing, seminars / tutorials and student homework. We conclude with their evaluative comments on the project.

INTRODUCTION [1]

Numbers of students opting for exclusively mathematical studies are declining (Hillel in Holton, 2001). Recruitment of good graduates to mathematics teaching, at least in the UK, is low (LMS, 1995). As approaches to secondary mathematics teaching have been changing towards more *interaction* and *student participation*, the gap between these and university teaching approaches has increased - leading to even further student alienation from the *traditionalism of university-level teaching* (Thomas, in Holton, ibid.). Moreover, as *school curricula have been shifting away from mathematical abstraction and formalism*, new intakes of students have increasing difficulty in coping with traditional university curricula (Artigue, in Holton, ibid.). Reform has been focusing on adapting, content-wise, to the needs of these students (Kahn & Hoyles, 1997) but a realisation has also been growing recently within the mathematical community that reflection on and reform of university-level pedagogical practice, and, in particular, the engagement of mathematicians in self-reflective processes, is also urgently necessary (Jaworski, 2002).

Here we elaborate the issue of *effectiveness of interactive modes of teaching* in the context of university mathematics through presenting the views of five university mathematicians, Lecturers A-E. These views are characteristic of those expressed by the larger sample of twenty university mathematicians who participated in a study we recently completed at the University of East Anglia. Before presenting these views we outline briefly the methodology of the study.

METHODOLOGY

This 15-month, LTSN-funded study engaged groups of mathematicians (pure and applied, with teaching experience ranging from a few years to several decades, all but one male and of varying rank) from six institutions in the UK as educational co-researchers (Wagner 1997). There were 11 Cycles of data collection, six with five mathematicians from the University of East Anglia (Cycles 1-6), where the authors work, and five from elsewhere (Cycles 1X-5X).

Six Data Sets were produced for each of Cycles 1-6 on the themes *Formal Mathematical Reasoning I: Students' Perceptions of* ***Proof*** *and Its Necessity; Mathematical Objects I: the Concept of* ***Limit*** *Across Mathematical Contexts; Mediating Mathematical Meaning:* ***Symbols and Graphs****; Mathematical Objects II: the Concept of* ***Function*** *Across Mathematical Topics; Formal Mathematical Reasoning II: Students' Enactment of* ***Proving Techniques*** *and Construction of Mathematical Arguments;* and, *A* ***Meta-Cycle****: Collaborative Generation of Research Findings in Mathematics Education*. The Datasets for Cycles 1-5 were used also for Cycles 1X-5X.

Each Dataset consisted of: a short literature review and bibliography; samples of student data (e.g.: students' written work, interview transcripts, observation protocols) collected in the course of the authors' previous studies (http://www.uea.ac.uk/~m011); and, a short list of issues to consider. Participants were asked to study the Dataset in preparation for a Focus Group Interview – see in (Nardi & Iannone, 2003) the rationale for using this tool for data collection.

Interviews were digitally recorded. The interviews from Cycles 1-6 were fully transcribed (the data from Cycles 1X-5X were used as supportive material in the analytical process). Each interview was about 200 minutes long and generated a *Verbatim Transcript* of about 30,000 words.

In the spirit of Data Grounded Theory (Glaser & Strauss, 1967) eighty *Episodes*, self-contained extracts of the conversation with a particular focus, emerged from a preliminary scrutiny of the transcripts and were transformed into *Stories*. These are narrative accounts in which we summarise content, occasionally quoting the interviewees verbatim, and highlight conceptual significance. The eighty *Stories* were grouped in terms of the following five *Categories*: *students' attempts to adopt the 'genre speech' of university mathematics* (Bakhtin 1986); ***pedagogical insight: lecturers as initiators in 'genre speech'***; *the impact of school mathematics on students' perceptions and attitudes*; *one's own mathematical thinking and the culture of professional mathematics*; and, *the relationship, and its potential, between mathematicians and mathematics educators* (25, 25, 4, 20 and 6 *Stories* respectively).

Eight subcategories emerged from the analysis of the data in the second *Category*, four under each of two overarching labels, 'interaction' and 'transition'. In this paper we elaborate the former.

THE EFFECTIVENESS OF INTERACTIVE MODES OF TEACHING

Our participants often find themselves largely disagreeing with the traditionalism of university-level mathematics teaching (Thomas, ibid.); in particular, they repeatedly expressed scepticism about the effectiveness of mass modes of teaching, such as lecturing. Nevertheless the majority of their comments is intensely focused on currently existing modes of teaching such as lectures, tutorials / seminars and student homework and on ways in which their practice can facilitate student learning from within these structures.

The effectiveness of learning through interaction (with a lecturer, a tutor, a peer, through verbal or written communication, through IT etc.) emerges strikingly as the overarching idea across their comments. Lecturer A suggests some reasons for this:

> A: Because I actually think that the element of difficulty in solutions is not something that you can pin down on a piece of paper. An argument, between the student and the teacher, ideally, why is this so and by the time I distil it in the model solutions the argument is dead, it becomes part of script. […] And we should have oral examination, and we should have lots and lots of oral interaction with the students to see that mathematics is about arguments.

Beyond this solid rhetoric on the benefits of interactive modes of learning, the participants were quick to list several forces that work against embracing the above proposals into a department's routine practice: student resistance and 'rumblings' (Lecturer E) from the university regarding the assessment of work submitted as the collective outcome of effort made by more than one students (issues of copying and plagiarism). 'We are actually supposed to discourage [*students from working collaboratively*]' he says. But 'we won't, because they learn a lot this way'. The participants' epistemology of mathematical learning, their preferred way of *coming to know mathematics* (Burton, 1999) appears to be clearly discursive. And, as many writers have emphasised (Bahktin, 1986; Sierpinska, 1994.; and, Mason, 2002, amongst them), this discourse consists of several levels of language at work as, for example, Lecturer B points out: students talking to each other, the language of a lecturer during a lecture, the language of a lecturer during a one-to-one interaction with a student, the language of textbooks. As he observes, difficulty for the students often emerges when these types of language appear in inappropriate places (e.g. a student query in a tutorial is dealt with a concise and compact textbook response, where a more informal, conversational style would fit better). In the following we elaborate these views for each of the three contexts of interaction with students that the participants focused on in their comments: lecturing, tutorials / seminars and student homework.

Lecturing. Lecturer C exemplifies the main shortcoming of lecturing as an effective way of learning mathematics as follows:

> C: I did that [*induction*] for the computer scientists. I thought it was all blindingly obvious and they all seemed to be nodding in the right points and I thought that

they found it blindingly obvious. And then, you know, since you have got work coming out …and there was gibberish written. So it was not blindingly obvious, it was actually quite hard to judge that students did found it difficult.

Hence, at least in the view of this lecturer, it is very hard to judge what students find difficult in the setting of lectures only. Below we exemplify the *tactics* (in Mason's (2002) sense of the term) that the participants employ in order to facilitate student learning through lecturing and we outline some of their statements on more long-term *desired outcomes* of lecturing.

Example of a *Tactic*: Using 'stepping stones' to facilitate the transition from less to more complex ideas.

E: I aspire to have […] moments in first year Analysis where [*the students*] meet with something they know.

Of course resorting to familiarity is a tactic that is to be used with caution, he adds. He then offers the example of utilising the known-from-school fact of the geometric sum to facilitate the calculation of the limit of a series: very soon in the course, 'you part company with this sort of approach', he stresses. Students should not be misled into believing that a technique such as the one mentioned above can be generalised any further merely for the sake of building their confidence (affective role of a 'stepping stone'). In addition to this affective function, there ought to be also a purely cognitive one: for example, in our particular context, that of facilitating the transition from calculating a sum to determining a limit. Elsewhere (Iannone & Nardi, 2002) we elaborate a further example of a 'stepping stone', this time in the context of Group Theory.

In close association with the ad-hoc tactics exemplified above are certain, more long-term, desired outcomes of lecturing. Amongst those emphasised by the participants were: the **acquisition of synthesis skills** and **sense-making through a repertoire of examples**. With regard to the former Lecturer E proposes:

E: One very interesting technique that I have seen done is to get them [*the students*] to write down anything they like from the course, that fitted on two sides of A4, hand it in, in advance, and then it is on their desk in the exam. And they choose what to write down and […] it makes them think through the course and think what matters and think what they need to know.

With regard to the latter the participants frequently suggested that a lecture course should contain a 'robust set of examples' (Lecturer E) to help students make sense of what they hear. For example, in the context of Analysis, a 'toolbox of examples', such as basic analytic functions, to be retrieved when needed, is of paramount importance (see, also, (Nardi, Jaworski & Hegedus, in press 2005) for further elaboration of this central issue).

Tutorials and seminars. The participants recognise that these are often the only opportunity for interaction between lecturers and students. As the students become accustomed to the more passive role of attending lectures, the lecturer needs to invent ways in which the students can become more involved in the seminar/tutorial process. One such way is suggested by Lecturer E:

> E: Sometimes when […] we had tutorials in small groups, I get them [*the students*] to read out what they have written, and then, then they think it is nonsense. […] But if they try and read this they would immediately start to question what they had written.

Class size is a crucial factor in the success of this tactic, he elaborates: small tutorial classes are more appropriate. However, and perhaps surprisingly, the participants think that classes of 15-20 students might be more effective than very small ones: the former allow a reasonably intimate exploration of each individual's ideas without the potential intensity of a one-to-one or two-to-one session.

As Lecturer B emphasised, beyond comfortable class size there is a confidence-building function of these sessions. Ways to achieve that include handing out exercises which will not be assessed and which will be discussed in the session in a laid back, non-assessment-driven manner. The homework, which will be assessed, can then contain similar exercises. In his opinion this tactic helps the students not to feel intimidated by material they are not familiar with and enhances the likelihood of the student actually engaging constructively with the tutorial process. This is crucial since students, often unaware of the potential benefits lying within such participation, are less willing to engage in the process. A successful orchestration of the interactive process is then largely up to the lecturer who must avoid the session becoming a mini lecture and must ensure that the invitation to participate is absolutely clear to the students.

Part of this success is circumventing what Lecturer A calls 'a degree of detachment': between planning and conducting a seminar or a tutorial often lies a period of weeks or even months. Problem sheets are planned in advance, therefore without much tailoring to the needs of the particular students. This implies a potential gap between what one wishes to achieve theoretically in the seminar and what the students actually need in this particular moment of their learning. To this, Lecturer A adds the sometimes insufficient time between tutorials and lectures during which the students have not been able to grasp the importance of certain ideas and therefore do not come to the tutorial with the intention to pay analogous necessary attention. Finally, another degree of detachment is added by what Lecturer A refers to as 'other things', the pressing priorities that may distract the typically busy lecturer from engaging with preparing a seminar more elaborately and with more sensitivity towards the students' needs.

Student homework. The participants' comments revolved largely around: the choice of questions to appear in the homework (this is mostly problem sheets that students are required to submit on regular intervals - marking these is the major source of formative assessment of the students' progress); the use of homework for gaining insight into students' thinking, monitoring student progress and informing teaching; and, issues around marking and feedback.

One of the reasons for including a question in a problem sheet, especially for Year 1 students, the cohort that most of our participants had teaching responsibilities for, is: in what ways does this question foster the skill of mathematical writing, both in terms of using mathematical symbols as well as fully-fledged explanatory prose? Below Lecturer E explains this rationale by offering the example of a question which asked students to translate the symbolic statement of convergence of a series into a verbal statement and then write its negation, both symbolically and verbally:

> E**:** You have to be fluent in both facets of the language […], and that is just hard. And the idea that one could do this entirely symbolically or entirely not symbolically I think is just mistaken. The meaning has to be in their heads and they have to have the ability to write it in words and in symbols and jump between them. And I don't see it very widely, I wish there was, to be honest. It would be marvelous if they all did it.

In other words the aim is to foster 'literacy in the language of mathematics' (Bullock, 1994), a theme frequently returned to by the participants in several contexts – see *Lecturing* above. Significantly, formal mathematical language, for these participants, does not consist merely of strings of mathematical symbols: it incorporates rich, fluent and precise prose.

Poring over student homework has multiple functions: gaining insight into students' thinking, monitoring student progress and informing teaching. If, as Lecturer A observed earlier – see *Lecturing* – teaching needs to be tailored to student needs, understanding these needs is a priority. Scrutiny of students' regularly submitted written work is a major source towards this understanding and far more than a conventional assessment exercise. In the words of Lecturer E:

> E: I never understand why they used them [homework sheets] to … attain marks where in fact we want to use them to teach mathematics. These are two completely different things. We use them to obtain marks, to test knowledge and to teach mathematics. And I can… I can imagine maximise how to use them to obtain marks or maximise how to use them to test knowledge or maximise how to use them to teach mathematics, but not all three simultaneously. […] The three out of ten person has done a fantastic job, they observe that they don't understand this so we have an opportunity to teach the maths, and they should get the seven.

Hence for this lecturer, as in the relevant literature (e.g. William & Black, 1996), there is an in-built contradiction in the function of homework: it is simultaneously a tool for formative (identifying students' needs and modifying one's teaching accordingly) and normative (assessing students' progress) assessment.

Written feedback is then a major opportunity for establishing a dialogue with students (Mason, 2002). As Lecturer A puts it, detailed written feedback is 'second best' only to face-to-face interaction with the student. As he explains below students also seem to appreciate an elaborate response in the context of tutorials and seminars.

A: I found out that the types of errors are often rather different from what I think they would be. So I haven't written [the suggested solutions distributed to the students once marking is done], while in the past I would have done this before I look at [their work]. Now I do it afterwards. And now I see that the students […] realise that […] I am responding to their collective errors, and they rather like this.

'It worked quite well [and it] is not a lot more work', he concludes reassuringly. 'And students learn that other students have had the same difficulties', adds Lecturer B.

CONCLUSION

Many of the views regarding the teaching of mathematics at university level offered by the participants do not necessarily sound new to researchers in mathematics education, who are typically well versed in issues such as the benefits and difficulties of interactive modes of teaching. However we believe that the novelty and the value of the enterprise reported here resides in the fact that these observations originated totally from the practitioners of university teaching themselves.

The participants often characterised the study as a forum in which to articulate pedagogical reflection, as an opportunity to explore the relationship between mathematicians and researchers in mathematics education (Nardi & Iannone, 2004) and as an opportunity to acquire a better understanding of the research paradigms of mathematics education Here is an example of the last of these three: faced with transcripts of audio-recorded conversations between lecturers and students in the Datasets (and, towards the end of the study with transcripts of their own interviews), the participants experienced a change of attitude towards the 'spoken word'. As Lecturer B observes in the world of mathematics the written word (such as published books, papers, or what is written on the blackboard during a lecture) has the highest status, whereas the spoken word is 'kind of transitory'. The experience of being confronted with a transcript of his own words has made him more sensitive towards the role that the 'spoken word' can have in his teaching:

B: I have become much more conscious about the spoken word. What I can say can have an impact, saying the right thing at the right time when you get one opportunity to introduce the students for the first time to how mathematics works and not fluff the line. That [*having seen his own words transcribed and having to articulate pedagogical thinking during the interviews*] I think has made a big influence on the way I lecture.

NOTES

1. A longer and more elaborate version of this paper (Iannone & Nardi, submitted) has been submitted to *The Journal of Mathematical Behaviour*.

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