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GIVING EXAMPLES AND MAKING GENERAL STATEMENTS: 'TWO ODDS ALWAYS MAKE AN EVEN (IN MATHS)'

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The statement in the title is drawn from an answer given by an 11-year-old to part of a question to which many pupils responded with a general statement. In the second part of the same question children were required to provide examples in a way which implied they understood a generality. We explore answers to both parts of the question as well as comparing the two. Slightly fewer children were able to provide examples than make a general statement. Many children were successful on one part of the question but not the other.

INTRODUCTION

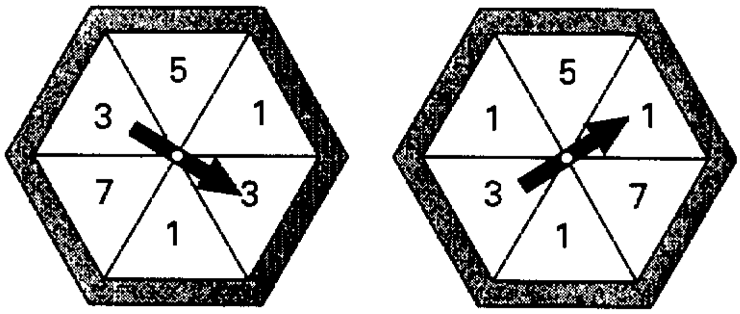
This paper considers pupils' written answers to a two part question (see Figure 1 on the next page). This question is drawn from the second phase of a project being carried out jointly by the Centre for Mathematics Education at the Open University and the Mathematics Test Development Team at the Qualifications and Curriculum Authority. An aim of the project is to establish how 11-year-olds respond to tasks designed to show evidence of early algebraic thinking. In particular we are interested in how different types of question may elicit different types of responses. The question considered here is set in the context of scores on spinners and both parts of the question are about total scores and what happens when odd and even numbers are added. The first part of the question requires pupils to write an explanation and the second part requires them to provide numbers so that a given statement is true.

BACKGROUND

The first part of the question used in our research requires children to give an explanation. There is substantial evidence that many primary children find this difficult. For example, Freudenthal (1978) documents the difficulty a 9-year-old child has in giving an oral explanation having successfully solved a problem. Asking for an explanation in writing presents another hurdle and a study carried out by Pekhonen (2000) found that many children were unable to give a mathematically valid argument in writing, even though many of them appeared to understand the task presented.

A clear way of correctly answering this question would be by making a general statement about what happens when two odd numbers are added, though it could also be answered by 'exhaustion' if all possible pairs of numbers were tested. Research suggests that younger primary pupils have difficulty in offering justification for

Nadeem has two spinners**.**



He spins the 2 spinners and adds the 2 scores to make a total.

Nadeem says, ***'The total will always be even'***

Is he correct?
Circle Yes or No.

Yes / No

Explain how you know.

....

....

....

Nadeem has 2 more spinners.
He spins them and adds the scores to make a total.

Write a number in every section so the total will **always** be **odd.**

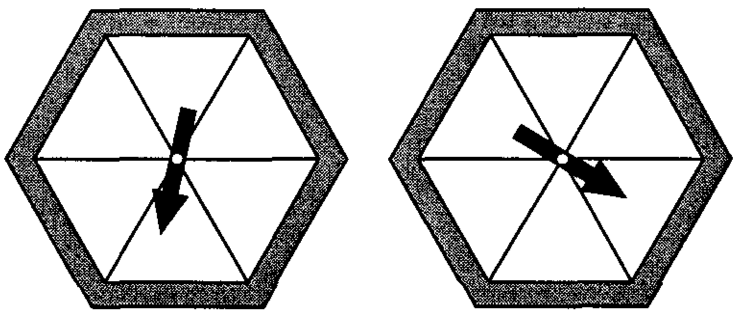


Figure 1: the question under consideration

mathematical statements, other than by providing examples, though this improves as children approach the end of primary schooling and is susceptible to intervention (Carpenter and Franke 2001, Carpenter and Levi 2000). For many students, a preference for using examples rather than more general forms of justification persists well into secondary school (Healey and Hoyles 2000, Lee and Wheeler 1989).

Work on justification and proof tends to assume that finding examples to match a general statement is a lower level activity than providing a mathematically valid proof. Curriculum documents in England also suggest that providing examples to match a general statement is easier than making a general statement in words. For example in the *Framework for Teaching Mathematics* (DfEE 1999) investigating general statements by use of examples is first mentioned for Year 1 children (aged 5 to 6), though this only becomes 'making and investigating' at Year 4 (aged 8 to 9). Generalising in words is mentioned for children aged 9 to 11. The same document contains many examples of what children should be able to do which relate to odd and even numbers. It is likely that by the end of primary school most pupils in England will have met ideas related to this several times.

THE RESEARCH

The question being considered here (see Figure 1) was trialled informally then adapted and put together with other questions to form a short test paper which was completed by 364 Year 6 children (11-year-olds) drawn from 7 schools. In selecting the sample, the intention was to try to match the national profile, particularly as far as levels of attainment were concerned. Although children were aware this was not their 'real' test, it was administered in a similar way. Analysis of questions was carried out in order to gain a detailed picture of how children responded and the range of written answers they gave.

FINDINGS

Giving explanations

Like all questions requiring explanations, the first part of this question raises the issue of what is considered to be a correct answer. Because the question was written for research purposes, we concentrated on categorising answers by type rather than starting by saying whether they are right or wrong. The categories we used were based on those developed for analysis of another question requiring explanations (Evens and Houssart 2004), but adapted slightly to suit this question. The number of answers in each category is shown in table 1. Our categories were based on the explanations given rather than whether yes or no was circled, though it worth noting that nearly all children giving explanations involving restatement, testing all possibilities, or use of a general rule circled 'yes'.

Only 1% of children did not attempt to answer this part of the question, though 17% gave answers classified as wrong or irrelevant. These included a few children who seemed to have misread the word 'even' as either 'seven' or 'eleven'. Some children giving answers in this category either said or implied that the sum of two odd numbers is odd. Less than half the children giving answers in this category circled 'yes'.

Summary analysis for first part of question n = 364		
Category for first part (a)	Number	Percentage of total to 1%
N: No response to part (a)	4	1%
W: Wrong / irrelevant	61	17%
R: Restatement	51	14%
O: Noticed 'all numbers are odd' only	16	4%
ES: Specific numerical examples tested only	43	12%
EA: Claimed to have tested all possibilities	38	10%
G: Justification in terms of the general rule	151	41%

Table 1: Categories of responses for giving explanations

Some examples of wrong or irrelevant answers are given below. Answers used in this article are reproduced using the original spelling and punctuation.

I just added some up and they didn't make 11

On the spinner all the numbers are odd and when you add 2 odd numbers togeer it is odd

We have used the term 'restatement' for answers which just restate Nadeem's statement, either as it is, or slightly re-worded. All of these children circled 'yes' agreeing that Nadeem's statement was true, though they did not give a reason. In some cases answers in this category were written in a way that suggested the child may have tested examples, though this was not stated explicitly. In a real test it is unlikely that answers like this would be given a mark, because nothing has been added to the information given in the question. However it is possible that many of the children did understand the ideas behind the question and some may have had reasons for their answers, but failed to put them in writing.

because all the numbers add up to even numbers

I know because there all even

Another group of children noticed and stated that all the numbers on the spinners were odd, but did not say any more. Half of this group circled 'yes'. It is likely that some of this group, especially those who circled yes, also understood why the statement is true, but had failed to finish their explanations. As with the previous category we are limited as to what we can conclude from what is written.

because all the numbers on the spinner are odd

Because there are no even numbers on the spinners

The next category consisted of answers which were supported by numerical examples. These varied from use of just one example (23 children) through to eight examples, the latter being only two short of the ten distinct pairs of numbers possible. Some answers in this category came close to giving or implying the general rule.

> I no because 3+1=4 and that is even

> because 3+1=4 and 5+3=8 and 7+3=10. All of the numbers are odd so if you add each one up you get an even number.

A slightly different category was based on answers claiming to have tested all or most of the cases, but examples were not given so it was not possible to tell whether they had really identified and tried all the possibilities.

> I went through sistermalickly adding up 2 numbers at a time.

> I added each number two gether I did the same with every number.

The final and largest category, 'justification', consisted of statements of the general rule, with 41% of children giving answers of this type. All 151 children giving answers of this type stated the general rule 'odd+odd=even' in some form and 66 of them also explicitly stated that all the numbers on the spinners were odd.

> Odd + odd = even

> two odds make an even (in maths)

> because when two odd numbers are added together it always equals an even number

> there is only odd numbers on the spinner and everyone knows if you add two odd numbers together you will get an even number

Completing the spinners

The second part of the question differs from the first in that it is easy to categorise answers as correct or incorrect. This part of the question also offers the children a considerable amount of freedom, both in the answers they give and how they arrive at them. Some answers suggested systematic methods had been used, while others appeared to be the result of trial and improvement. A summary of responses for this part is shown in table 2.

Summary analysis for second part of question n = 364		
Category for part (b)	Number	Percentage of total to 1%
Correct	159	44%
Incorrect	162	45%
Incomplete	28	8%
Nothing	15	4%

Table 2: categories of responses for completing spinners

Of those children correctly answering this part of the question, the most common response was to use the numbers 1-12, with 1, 3, 5, 7, 9, 11 on one spinner and 2, 4, 6, 8, 10, 12 on the other. A further four children used the numbers 0-11 in a similar way. Usually numbers less than about twenty were used, though a notable exception, shown below, suggests confident understanding of the generality involved. The answers are reproduced in a slightly contracted format with the left hand block representing the left hand spinner and the right hand block representing the right hand spinner.

24	16	12		81	21	7
12	8	22		93	69	45

Many children showed an awareness that they could use a number more than once as shown in the first example below. Some took this further by using the same number six times on one spinner, or even on both spinners as shown in the other example. These two answers could be interpreted as confident understanding of the situation, or as playing safe by reducing the number of additions involved.

6	4	2		3	1	1
2	4	6		5	5	3

1	1	1		0	0	0
1	1	1		0	0	0

Some children who eventually gave correct answers showed evidence of trial and improvement, having amended answers which did not work. This usually meant crossing out a few numbers which did not give the correct total, though in a few cases the whole spinner was altered.

3	7	5		6	~~5~~ 4	8
5	7	~~6~~ 9		8	4	~~3~~ 2

12	2	4		~~12~~ 11	1 ~~2~~	~~4~~ 3
10	8	6		~~10~~ 9	~~8~~ 9	~~6~~ 5

Although a small number of children not getting this part of the question correct did not provide an answer or did not complete both spinners, the majority of wrong answers were from children who completed both spinners in a way that did not give the required totals. 162 children gave answers of this type and just over half of these showed evidence of using the properties ‘odd’ and ‘even’ but in an incorrect way.

- 27 used odd numbers throughout on both spinners
- 24 used even numbers throughout on both spinners
- 33 used 3 odd numbers and 3 even numbers on each spinner

Some of the answers in the last category above consisted of spinners filled in, like the one below, so that if numbers in corresponding positions were added then the total would be odd. It is possible that children giving such answers did not understand that they needed to consider every possible pair, perhaps through lack of experience with spinners, though they may have understood the mathematics involved.

14	9	10		21	16	17
13	12	11		20	19	18

Comparison

Comparison of correct responses to the two parts of the questions depends on what is considered to be a correct response to the first part. For the purpose of comparison we have counted only those giving the general rule. This means that slightly more children were successful on the second part of the question than the first, though many children were successful on one part only, as shown in table 3.

	Spinners correctly completed (part b)	Spinners not correctly completed (part b)	Totals
General rule used (part a)	102	49	151
General rule not used (part a)	57	156	213
Totals	159	205	364

Table 3: comparison of correct answers to the two parts of the question

49 children gave an explanation for the first part of the question in terms of a general rule, but were not successful in the second part of the question. This might be because they can recite the rules for odd and even numbers but cannot apply them. In some cases, where there was a clear but incorrect system for filling in the spinners it may be that the child did not know all the rules for adding odd and even numbers, or did not understand about spinners. Another group (57 children) were successful in filling in the spinners, but did not give a general rule for the first part of the question. It is possible that these children did understand the general rule but were not able to express it in writing. It is also possible that they felt that use of examples was an adequate explanation, especially in cases where they claimed to have tested all possible pairs of numbers.

COMMENTS AND CONCLUSION

Answers to the first part of the question are consistent with the findings of Freudenthal (1978) and Pekhonen (2000) in suggesting many children have difficulty in giving written explanations although they may understand the situation. Common inadequate answers involved the testing of a few examples or restatement.

Answers to the second part of the question suggest that the children did not find providing examples that much easier than expressing a general statement in words. Of particular interest were those children who got one part of the question right and not the other, including 49 children who were able to provide a general statement for the first part of the question but were not successful in providing examples for the second part. Although there are many possible explanations for this, it does raise the question of whether providing examples is necessarily easier than providing a general statement. This is interesting in the context of existing work which suggests there is a progression from providing examples to offering justification in words (Carpenter and Franke 2001, Carpenter and Levi 2000). It also suggests that although some children prefer use of examples to providing more general forms of justification, as suggested by existing research (Healey and Hoyles 2000, Lee and Wheeler 1989), there may be some children who prefer to work with general statements.

As usual with written response there are many unanswered questions about what the children did and why. The next step in our research will be to interview children about this question in the hope of finding out more about why they give particular answers. In particular we hope to focus on those children able to answer one part of the question but not the other.

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