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SOME IMPACTS OF THE NATIONAL NUMERACY STRATEGY ON STUDENTS' WRITTEN CALCULATION METHODS FOR DIVISION AFTER FIVE YEARS IMPLEMENTATION

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This paper investigates some changes in year 5 pupils strategies for solving division problems since the introduction of the National Numeracy Strategy. Overall diversity in strategy use has changed little although there is some evidence of a new 'chunking' algorithm being used in some, but not all schools. Variations among the schools show there is little uniformity in the strategies taught despite the widespread availability of the Framework. Overall the boys were more successful than the girls and made more use of informal and mental strategies. The girls used more standard written algorithms and in the schools where girls did better they used mostly the chunking algorithm.

BACKGROUND

The final report of the Numeracy Task Force published in July 1998 gave a 'practical agenda for action to implement the National Numeracy Strategy' in England (DfEE 1999a: 3). This report proposed an emphasis on mental strategies with delayed introduction of standard algorithms and development of flexiblity in calculation methods (DfEE 1998). Students are expected 'to understand the four number operations' and 'the relationships between them' and to 'use mental methods if the calculations are suitable' (DfEE, 1998:69). Embedded in the advice for teachers is the principle that for each operation at least one standard written method of calculation should be taught in the later primary years (DfEE 1999a). The National Curriculum which specifies the statutory requirements for mathematics in school, specifies that to achieve level 4 (the target for 11 year olds) pupils must use an **efficient** method for calculating (DfEE, 1999b).

DIVISION IN THE NATIONAL NUMERACY STRATEGY

The Framework for Teaching Mathematics from Reception to Year 6 proposes that in addition to working mentally, for example using 'related facts and doubling and halving', in Years 4, 5 and 6 'pupils should be taught to develop and refine written methods for division' (DfEE 1999a:68 & 68). Examples are given of the outcomes expected for pupils in each year group and for 'Pencil and paper procedures (division)' for Year 5 these are separated into 'Informal written methods' and 'Standard written methods' for dividing a three-digit number by a one-digit number. Standard written methods have traditionally taken two forms: 'short division' in which the calculation is completed in a single line and 'long division', which involves written sub-procedures, recorded in a standardised format. In England the short division algorithm is used for division by a single digit number and long division may not be taught in the primary years, although it is given as an example in the Framework. Some schools have introduced a standardised procedure based on repeated subtraction of multiples (chunks) of the divisor and this will be referred to in this article as the 'chunking algorithm'. In Year 6 pupils 'continue to develop an efficient standard written method that can be applied generally' with progression indicated to the traditional long division algorithm (DfEE, 1999a:69).

THE RESEARCH STUDY

In 1998 a workbook with ten items involving division by one- and two-digit divisors was completed by pupils (n=275) in Year 5 in ten schools in and around one city (Anghileri 2000). At the time there was sensitivity about the poor results for English students in international studies of pupils' arithmetic (Harris, Keys and Fernandes 1997) and a criterion for the selection was that schools' performances in national tests was above average. In national test results in mathematics the ten schools had an average of 72.5% at level 4 or above, compared with the national average of 53.2%. The intention was for schools to be comfortable in having close scrutiny of their pupils' workings. In 2003 after five years of implementation of the National Numeracy Strategy the questions were given to year5 pupils using the same protocol. Nine of the ten schools were again involved, and one further school was selected to match as far as possible (school type, national test results, social and environmental characteristics) the one school that withdrew. In order to maximise comparability, the study was undertaken at the same time of year (June) and implemented in all schools by one researcher. The main research questions were:

- to what extent are the principles identified in the National Numeracy Strategy evident in pupils' solutions?;
- what changes from the 1998 study are evident?

THE TEST ITEMS

Ten division calculations were devised with numbers to invite informal calculation methods. Five items were given within a context and five 'bare', some calculations were exact and others involved a remainder. Eight items were the same and two differed in 2003 and 1998. The reason for changing two questions was to reflect better to focuses of the National Numeracy Strategy and to match items being undertaken by pupils in a parallel study in the Netherlands. [The international comparison will not be considered in this paper.] Two items from 1998 involving division by ten [604÷10 and 802÷10] were replaced by a 3-digit number divided by a 1-digit number [424÷4 and 868÷4] because the 1998 study had included 2-digit and 4-digit, but not 3-digit numbers divided by a 1-digit number. Additionally, 'halving and doubling' are highlighted as strategies in the National Numeracy Strategy and the two new items (q4 and q9) involving division by 4 were accessible through these strategies. The 2003 test items were as follows:

1. 96 flowers are bundled in bunches of 6. How many bunches can be made?
2. 84 pencils have to be packed in boxes of 14. How many boxes will be needed?
3. 538 children are transported by 15 seater buses. How many buses will be needed?
4. The Taylor family want to buy a TV and DVD player that cost £424. They can pay for them over 4 months, paying the same amount each month. How much will they have to pay each month?
5. 1542 apples are divided among 5 shopkeepers. How many apples will each shopkeeper get? How many apples will be left?

6. 98÷7; 7. 64÷16; 8.432÷15; 9. 864÷4; 10. 1256÷6.

RESULTS

Perhaps the first consideration is any change in facility since implementation of the National Numeracy Strategy. In national tests there has been considerable improvement from 1997 (data used for original selection of schools), when an average of 53% of year 6 pupils gained a level 4 or above, to 73% in 2002.

For this study of division, there were ***8 directly comparable questions*** and pupils from ten schools in 1998 (n=275) and 2003 (n=308). For these questions the mean for 1998 was **3.45** (43%) with standard deviation 2.1. The mean for 2003 was **3.85** (48%) with standard deviation 2.2. This is heartening as there has been considerable investment in implementing the National Numeracy Strategy, but perhaps not as positive as results for mathematics generally. Other studies suggest that improvements in division calculations are not as positive as other areas (Brown et al. 2003: 15). The analysis which follows will compare strategy use across the schools and differences between those of boys and girls.

Classification of the strategies

As in 1998, the pupils' written methods ranged from inefficient strategies such as tallying or repeated addition to use of a standardised written procedure. It was not appropriate to use identical categories to 1998 but comparability was maintained as far as possible. The eight categories used for analysis:

Traditional algorithm, which was generally short division but also included long division in a structured written record;
Chunking algorithm, which was based on repeated subtraction in a schematised format as illustrated in the National Numeracy Strategy Framework;
Informal methods often using multiples of the divisor or the dividend in an unstructured written record. This category also included halving and doubling;
Low level strategies included tallying, repeated addition or subtraction of the divisor and sharing with some image of a distribution;
Mental strategies where a solution was given but no working shown;
Other including unclear and less frequently used strategies;
Wrong where the wrong operation was used, for example 98-7 instead of 98÷7;

Missing where there was no evidence of any attempt.

Features associated with different strategies

A notable difference between the ***traditional algorithms*** and the ***chunking algorithm*** is that the former involves calculating with digit values while the latter uses whole number values throughout. In the example 98÷7, the traditional algorithm will start by noting that seven 'goes into' nine 'once with a remainder of two'. This 'two' is then converted to 'twenty' and added to the eight in the units column. In the chunking algorithm, seventy is used as a multiple of seven and subtracted from ninety-eight to leave 28 which is also a multiple of seven.

Informal methods were those using known number facts in an unstructured written record. Halving and doubling were also included in this category.

Low level strategies generally showed pupils had understanding of division as sharing or repeated subtraction and these approaches, although inefficient, could lead to a solution.

Other included working that was unclear and some attempts involving place value partitioning of either the divisor or the dividend, or both. For example 64÷16 was tackled as 64÷10 and 64÷6 while 1256÷6 was attempted by finding separately 1000÷6, 200÷6, 50÷6 and 6÷6.

Strategies by school

Whereas the traditional algorithms dominated in 1998 being used in 49% of all items (Anghileri, 2002) there was more evidence in 2003 of informal strategies (27%) and the chunking algorithm (9%) with less use of traditional algorithms (19%).

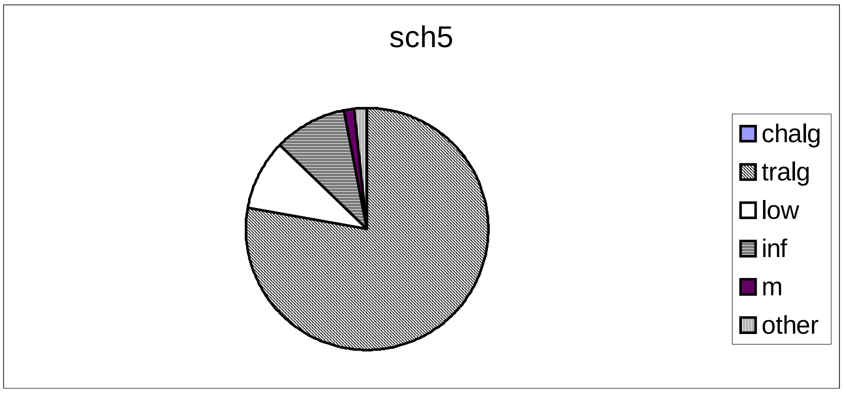
In six of the schools an informal method was most common whilst in the others the standardised written approaches predominated (Table 1).

There was variation across all the schools with no evidence of the chunking algorithm in some schools and up to 44% use in school 7. Informal strategies varied, from 51% in school 2 to 11% in school 7, and is notably lowest in the two most successful schools (5 and 7). Although the National Numeracy Strategy introduced emphasis on more flexibility in strategy choice, the number of different strategies used for the ten questions (mean 2.86, sd 1.15) varied little from the 1998 results (mean 2.98, sd 1.18). When considering the number of different strategies used by each pupils for the ten questions this varied from those who used a single strategy for all questions (11%) to a few who used 5 or more strategies (7%). The majority used 2, 3 or 4 strategies (29%, 31%, 22% respectively) but there was no correlation ($r = -0.1$) between the scores out of ten and the number of strategies used.

Success does not appear to reflect particular strategy choices as the two highest scoring schools (schools 5 and 7, see Figure 1) have significantly different patterns of strategy use. In one the only algorithms used were the traditional ones while in the other school pupils used both short division and the chunking algorithm according to the numbers and the context of the problems.

school		long-alg	short-alg	chunk-alg	low	inf	mental	other	wrong	missing	mean score/10	sd
1	**use**	**9%**	**20%**	**18%**	**8%**	**25%**	**7%**	**9%**	0%	4%	**5.37**	3.12
	correct	**7%**	**12%**	**10%**	**4%**	**14%**	**4%**	**2%**				
2	**use**	**4%**	**6%**	**0%**	**5%**	**51%**	**6%**	**20%**	2%	7%	**4.85**	2.75
	correct	**2%**	**2%**	**0%**	**3%**	**31%**	**2%**	**8%**				
3	**use**	**0%**	**9%**	**0%**	**12%**	**43%**	**13%**	**11%**	0%	12%	**4.42**	2.38
	correct	**0%**	**6%**	**0%**	**6%**	**24%**	**5%**	**3%**				
4	**use**	**4%**	**13%**	**1%**	**6%**	**39%**	**17%**	**11%**	2%	7%	**5.43**	2.79
	correct	**3%**	**5%**	**0%**	**5%**	**24%**	**14%**	**1%**				
5	**use**	**0%**	**66%**	**0%**	**9%**	**12%**	**3%**	**8%**	0%	1%	**6.5**	2.22
	correct	**0%**	**50%**	**0%**	**6%**	**6%**	**1%**	**1%**				
6	**use**	**1%**	**1%**	**9%**	**13%**	**40%**	**11%**	**7%**	1%	18%	**4.25**	2.23
	correct	**0%**	**1%**	**7%**	**9%**	**18%**	**6%**	**2%**				
7	**use**	**3%**	**29%**	**44%**	**3%**	**11%**	**0%**	**2%**	0%	8%	**6.1**	2.45
	correct	**3%**	**20%**	**29%**	**2%**	**7%**	**0%**	**0%**				
8	**use**	**2%**	**7%**	**2%**	**16%**	**30%**	**6%**	**15%**	5%	17%	**4.07**	2.96
	correct	**2%**	**4%**	**1%**	**8%**	**19%**	**3%**	**4%**				
9	**use**	**1%**	**5%**	**21%**	**16%**	**19%**	**7%**	**14%**	4%	13%	**4.10**	3.04
	correct	**0%**	**1%**	**13%**	**7%**	**11%**	**4%**	**4%**				
10	**use**	**0%**	**18%**	**0%**	**18%**	**32%**	**6%**	**11%**	6%	9%	**4.44**	2.92
	correct	**0%**	**10%**	**0%**	**8%**	**19%**	**4%**	**3%**				

Table 1: Strategy use and success in the ten schools.



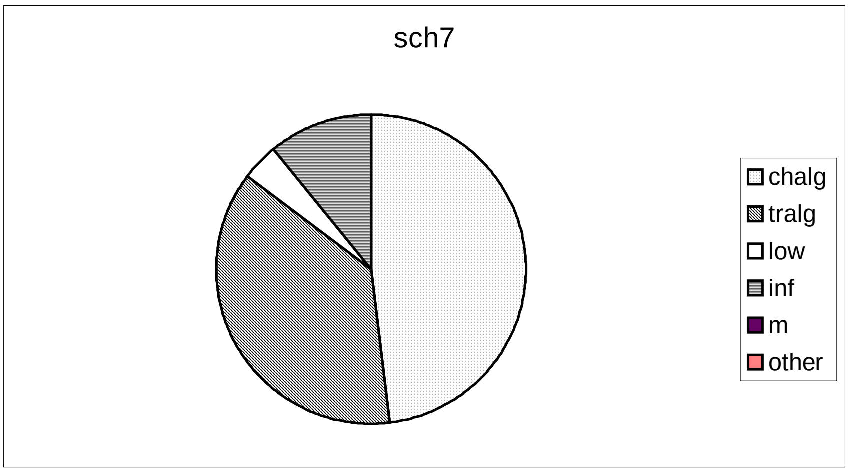


Figure 1: different pattern of strategy use in the highest scoring schools

In school 5 there was no use of the newer chunking algorithm. In school 7 the chunking algorithm was used most and, together with the traditional algorithm these accounted for over three-quarters of all attempts. Few pupils in these schools used a **mental** strategy (giving an answer but showing no working). Overall, comparing the eight identical questions, the mean for missing attempts was 9.4% in 2003 compared with 8.9% in 1998.

Boys and girls

In comparing the performance of boys and girls, in June 1998 there was no significant difference in mean scores for the ten items (girls 4.5, boys 4.2) but in June 2003 there was a significant difference (using a 2-tail t-test $p<0.005$) with mean score for the girls of 4.4 compared with the mean score for the boys of 5.3. The boys were more successful in every question.

	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10
girls	72%	51%	16%	49%	40%	64%	60%	14%	46%	30%
boys	80%	60%	38%	62%	42%	71%	67%	28%	52%	31%

Table 2: Success rates for boys and girls for each question

The biggest difference is in question 3, which involved the calculation 538 ÷15. In this question the only successful strategies for girls were algorithms which accounted for 12% of correct solutions and informal written calculations using chunking which accounted for the other 4%. Boys were successful with algorithms (10%), informal chunking (24%), repeated addition (1%) and mental (2%). Considering the strategies used overall in 2003, the girls used algorithms more frequently (35%) than the boys (23%), used more low level strategies and fewer items were solved mentally (Table 3). The boys used informal methods most (35%) with considerable success (22%).

	girls	n=144	boys	n=165
	attempts	correct	attempts	correct
chalg	11%	7%	7%	4%
lalg	2%	1%	3%	2%
alg	22%	14%	13%	9%
low	24%	9%	16%	8%
inf	24%	12%	35%	22%
m	2%	0%	12%	7%
u, wr, o	15%	0%	15%	0%
	total	**44%**		**53%**

Table 3: frequency of use and success of different strategies for boys and girls

The difference between boys and girls was not uniform across all the schools and in some schools the girls did better than the boys (Figure 2).

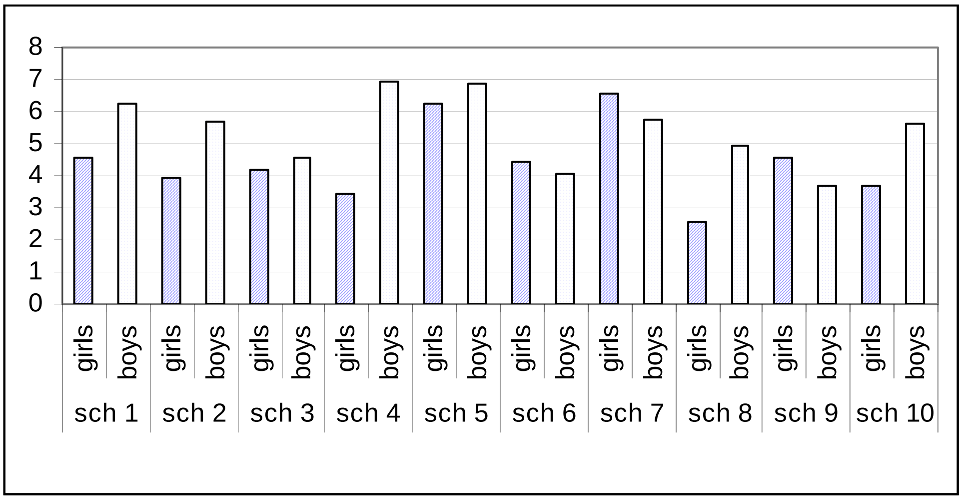


Figure 2: Mean scores out of ten for girls and boys in each school.

Small sample size for each school made it difficult to show significance in these results but in schools 4 and 8 the results were highly significant ($p<0.001$ for both). In school 4 boys used informal strategies (54%) and mental strategies (29%) for more than three quarters of the items. Only 2 boys used a chunking algorithm and one of those was the only user of the long division algorithm, suggesting that these were not learned at school. The girls in school 4 used short division as their only algorithm and used it more than any other single strategy (30% of all items) but fewer than half (41%) of these calculations were correct. None of the girls used a mental strategy. In school 8 one girl only used the chunking algorithm and one boy only used the long division algorithm, again suggesting that these were not learned at school.

In schools 6, 7 and 9, where the girls did better, more than half the girls used the chunking algorithm at least once and overall they used this strategy most frequently (32% of all items) and more effectively (71% correct) than any other strategy. In these schools fewer than half the boys used the chunking algorithm and they used more informal and mental strategies overall than the girls.

DISCUSSION

The implementation of a National Numeracy Strategy and the extensive availability of a Framework for Teaching (DfEE 1999a) might suggest some uniformity in the methods taught to a single age group across all schools. The data collected reveals different patterns of strategy use suggesting that teachers are interpreting the Framework in different ways. Indicative of this is the extensive use of the new chunking algorithm in some schools and its total absence in others. A few pupils using the long division algorithm were evident in small numbers across all schools suggesting pupils may have met this method outside the classroom. In some schools there is extensive use of informal working but little evidence of structured written recording. Overall the shift from extensive use of the traditional algorithm n 1998 to more use of informal strategies in 2003 has led to biggest improvements for division by a 2-digit divisor. In all schools the range of strategies used showed flexibility in addressing the numbers and the context of the question but the range of strategies used was not very different from 1998.

Those schools where pupils had mastered some form of standardised written method were more successful than those where informal and low level strategies dominated. It appears that pupils in year 5 are more successful with division calculations if they are taught to record their strategies in some structured way. This may be in terms of the traditional short division algorithm or the new chunking algorithm. This appears to be more crucial for girls who were less successful than boys in using informal working.

Overall boys appear to be doing better than girls and some have suggested that this is 'likely to be related to the predominance of mental over written work (in year 4) and more emphasis on public performance' (Brown in Thompson p204). This study has shown that boys use a mental strategy more frequently and more competently than girls. They are also more successful using informal working. In those schools where girls were more successful than boys they used the chunking algorithm successfully and this appears to help them with their need to organise their working. Although advice to teachers is to adopt one standardised format for written recording, pupils in one of the most successful schools used both the traditional algorithm and the newer chunking algorithm effectively for different question types.

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