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WHAT DO TEACHERS OF MATHEMATICS TEACH? AN INITIAL EPISODIC ANALYSIS OF FOUR EUROPEAN TRADITIONS

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This paper derives from a small-scale comparative study of the teaching of mathematics in five European countries, Flemish Belgium, England, Finland, Hungary and Spain. Drawing on coded video recordings of sequences of lessons taught on standard topics, we examine what the project team has called the mathematical foci of the episodes that comprise a lesson; where an episode is that period of a lesson in which the teacher's observable didactic intention remains constant and a mathematical focus is an observable generic mathematical skill or understanding independent of the explicit mathematical content of a lesson. The analysis, based on the lessons taught in four of the project countries, shows considerable variation in teachers' observable objectives across countries.

INTRODUCTION

Several writers have argued that comparative educational studies tend to polarise into distinct philosophical camps. Some globalise mathematics in their assumptions that not only is a uniform curriculum and pedagogy possible but also desirable, while others internationalise mathematics by seeking to account for both the similarities and the differences between systems and then learning from them (Clarke 2003, Le Tendre et al, 2001). The project reported in this paper falls, we believe, into the latter category in its attempt to examine one system's adaptive potential for another (Clarke 2004).

It is widely accepted that teachers' professional perspectives are influenced by unarticulated and culturally-located assumptions which affect all aspects of their work (Stigler et al 2000). So strong are these influences that teachers' "pedagogical strategies and approaches" are "enacted repeatedly in a country's classrooms" to the extent that they appear almost "below the conscious level for most teachers" (Cogan and Schmidt 1999: 71/72). Comparative studies can offer "a powerful way to unveil unnoticed but ubiquitous practices" (Stigler et al, 2000: 88) and provide a means by which one's country's practices may be examined critically and enhanced.

THE PROJECT AND ITS METHODS

The mathematics education traditions of Europe (METE) project is a five-way, EU-funded, comparative study of mathematics teaching in Flemish Belgium, England, Finland, Hungary and Spain. These countries represent well the socio-economic diversity of the continent and diverse attainment on recent international comparisons of mathematical attainment like TIMSS, PISA and their repeats. However, since the wider cultures of all project countries are located in the Judeo-Christian tradition, it is the project team's belief that many problems of pedagogic transference (Hatano and Inagaki, 1998) would be alleviated.

The project collected data on the teaching of key mathematical topics by means of video recordings of sequences of four or five lessons taught by teachers perceived by their local team to be representative of the better practice in that country. The topics, which were thought to be representative of the breadth of school mathematics, concerned the teaching of

- percentages (a topic of arithmetic applicability) in grades 5 or 6
- polygons (a routine geometrical topic) in grades 5 or 6
- polygons (not only a routine geometrical topic but an opportunity to examine curriculum continuity) in grades 7 or 8
- linear equations (an early topic of formal algebra) in grades 7 or 8.

Videographers were instructed to capture all teachers' utterances and, as far as was practicable, as much of the work written on the board as possible. Teachers wore radio microphones while unobtrusive telescopic microphones captured student-talk. After filming, videotapes were compressed and each lesson transferred to CD ROM. Each lesson was coded by its home team against the project's schedule. Additionally, the first two lessons in each sequence were transcribed using TransTool® and then, where appropriate, translated into English. This allowed colleagues to code other countries' lessons in order to establish satisfactory levels of inter-coder reliability.

Data collection was preceded by a year of live observations in each country to develop, in a bottom-up, grounded manner, a framework for describing and analysing mathematics lesson activity. This followed a decision to circumvent the English linguistic imperialism of the educational research literature and not appropriate existing frameworks in order to allow all colleagues equal access to the project's processes. This process has been described elsewhere (Andrews and Sayers 2004). The coding schedule comprised three sections. The first, the mathematical focus, addressed the observable generic objectives or outcomes of a given episode. The second, the mathematical context, focused on the conception of mathematics underlying the tasks presented in an episode. The third, the didactics, considered the observable strategies employed by teachers in their classroom activity. In project terms, an episode was that part of a lesson in which the teacher's didactic or managerial intention remained constant. Thus, for example, a period of seatwork would have qualified as a single episode as would the taking of a register at the start of a lesson.

This paper focuses on an analysis of the mathematical foci of project lessons. Shortened definitions of the seven mathematical foci can be seen in below.

Conceptual	The teacher emphasises or encourages the conceptual development of his or her students.
Derivational	The teacher emphasises or encourages the process of developing new mathematical entities from existing knowledge
Structural	The teacher emphasises or encourages the links or connections between different mathematical entities; concepts, properties etc.
Procedural	The teacher emphasises or encourages the acquisition of skills, procedures, techniques or algorithms.
Efficiency	The teacher emphasises or encourages learners' understanding or acquisition of processes or techniques that develop flexibility, elegance or critical comparison of working.
Problem solving	The teacher emphasises or encourages learners' engagement with the solution of non-trivial or non-routine tasks.
Reasoning	The teacher emphasises or encourages learners' development and articulation of justification and argumentation.

RESULTS

Due to unforeseen delays in the production of the Finnish data - the production company folded mid-way through the programme – this report is based on the data from Belgium, England, Hungary and Spain and draws on sequences of lessons taught by 16 teachers - four in each country, one for each of the four topics.

Each national team was given responsibility for the analysis of different aspects of the project's data. The English team focused on the analysis of the coding sheets. To confirm the project team's belief that shared vocabulary and conceptual understanding had been achieved, inter-coder reliabilities, in the form of Cohen's Kappa, were calculated. Assuming a kappa value of 0.75 as acceptable, appropriate levels of inter-coder reliability were established between the coders of England and Flemish Belgium ($\kappa = 0.877$), England and Hungary ($\kappa = 0.875$) and England and Spain ($\kappa = 0.793$).

Readers are reminded that since the codes were, essentially, categorical, non-parametric techniques have been used for comparative purposes. Also, since lessons varied in length it is it is acknowledged that the number of episodes may vary also. However, we argue that an episodic analysis will provide an indication as to the opportunities teachers offer their learners. Thus, while comparisons should be interpreted with caution, an episodic analysis should suffice in respect of highlighting teachers' mathematical emphases. Separate analyses, which are also reported below, examined the proportions of each lesson in which the different codes were observed.

The figures of Table 1 show some variation across project countries in respect of lesson lengths and number of episodes per lesson. The figures indicate that English lessons have significantly more and Hungarian lessons significantly fewer episodes per lesson than other project countries. The figures also show that Hungarian lessons are significantly shorter, and more consistent, in their duration while Spanish are significantly longer. Flemish lessons reflect most closely the project norm in both number of episodes and lesson duration.

	Flanders		England		Hungary		Spain		Composite	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Episodes	5.6	1.7	7.1	2.2	4.3	0.8	5.2	1.6	5.5	1.8
Duration	50.1	6.8	53.1	5.7	45.9	2.3	58.2	10.1	52.0	8.3

Table 1: The figures above show the mean (with standard deviation) lesson length (in minutes) and number of episodes for each project country. Italicised figures show where a country's mean is significantly different from that of all other countries at better than the 0.01 level.

THE PROJECT COMPOSITE LESSON

The figures of Table 2 reflect the different emphases found in project countries. Firstly, though, we discuss the project composite lesson, which is derived from means and provides a metric for comparison.

	Flanders	England	Hungary	Spain	Composite
Conceptual	3.9	5.4	2.9	3.6	3.9
Derivational	0.3	0.1	0.3	0.1	0.2
Structural	0.7	0.1	1.9	0.6	0.9
Procedural	3.0	3.7	2.4	3.3	3.0
Efficiency	0.6	0.7	1.7	0.7	0.9
Problem solving	0.4	1.4	1.3	1.8	1.2
Reasoning	1.8	2.1	2.1	1.1	1.8
Total	10.5	13.3	12.6	11.3	11.8

Table 2: The figures above show the mean number of episodes per lesson per country in which the different mathematical foci were observed. An episode was not restricted to a single code and may have received several. Italicised figures indicate a significant difference from all other lessons at better than the 0.01 level.

The composite column of Table 2 indicates that the main objectives of project teachers, while concepts-dominated, are behaviourally rather than cognitively focused with mathematical procedures, reasoning and problem solving being privileged over the unique properties of mathematics – efficiency, structure and deductively derived knowledge. Moreover, in respect of the behavioural objectives of the composite lesson, the evidence indicates that procedural skills are viewed as more important than logical thinking.

A COMPARATIVE ANALYSIS BY EPISODE

One of the explicit intentions of the project was to examine how different educational systems conceptualise and teach mathematics. The figures of Table 2 show the mean number of episodes per country in which each of the different foci were observed. Mann-Whitney U-tests were performed to determine the significance of any differences between a country's mean and that of all other countries. Those which were significant at better than the 0.01 level are indicated by italics in the table. As indicated above, this analysis is based on episodes which represent, in project terms, the opportunities given learners.

The mean Flemish lesson, based on 20 lessons, resembled more closely the project composite than that of other countries. It differed significantly from all others only in respect of its lower number of episodes containing opportunities for problem solving. Indeed, with a mean of 0.4 compared with a composite mean of 1.2 episodes per lesson, the evidence suggests that the Flemish teachers of this sample place little importance on this form of mathematical activity.

The mean English lesson, based on 15 lessons, differed significantly from all others on two foci. There were significantly more episodes per lesson with a conceptual focus and significantly fewer with a structural. The latter, particularly when considered against the higher number of episodes per lesson found in that country, indicated that English project students were offered only very rare opportunities to examine the structural properties of mathematics. Indeed, a mean of 0.1 episodes per lesson, compared with a project mean of 0.9 suggests that English teachers place little importance on the structural properties of mathematics. The former, while demonstrating the importance placed by teachers on the conceptual development of their students, was no more than a commensurate number of opportunities.

The mean Hungarian lesson, based on 18 lessons, differed from all others on three foci. It comprised significantly fewer episodes with a conceptual focus, although, as will be shown below, this was proportionate to the mean number of episodes per lesson. The more interesting finding was that there were significantly more episodes per lesson with structural and efficiency emphases. The former, with a mean of 1.9 episodes per lesson compared with the composite mean of 0.9, tends to suggest that Hungarian teachers place substantial importance on the structural properties of mathematics. The latter, with a mean of 1.7 episodes per lesson compared with a composite mean of 0.9 suggests that Hungarian teachers regard notions of mathematical elegance as more important than their project colleagues.

The mean Spanish lesson, based on 16 lessons, differed from all others on just the one mathematical focus. There were significantly more episodes per lesson containing problem solving than in other lessons. The mean of 1.8 episodes per lesson, compared with 1.2, indicated that these project teachers place substantial importance on this type of activity.

A COMPARATIVE ANALYSIS BY PROPORTION

The episodic analysis above creates an image of some diversity across project countries. In order to assess its validity a secondary analysis was undertaken. This calculated the proportion, as a percentage for each country, of all episodes in which the various mathematical foci were observed. The figures for this can be seen in Table 3. It seems clear, that proportioning in this manner would exaggerate some national differences, diminish others and leave those that remain unchanged. The following examples highlight what we mean. On the one hand, the mean number of episodes in an English lesson was 7.1 while that of Hungary was 4.3. The lessons of both countries had comparable episodes with a reasoning focus (means of 2.07 and 2.11 respectively). When considered as proportions of all episodes these similarities become differences as their respective percentages of 31.5 and 47.2 show. On the other hand, the Hungarian mean of 2.94 for conceptual development was significantly lower than that of other countries and yet its percentage score of 65.7 approximated well the international average. Thus one can infer that analyses based on episodes may produce different results from those based on percentages. This is an interesting issue which raises important methodological questions. The production of percentages can provide a means of comparing like with like. Alternatively, an analysis based on episodes may offer a truer indication of the opportunities teachers present their learners. For example, it is probably more helpful to say that teachers in country so and so, on average, offer five distinct opportunities for learners to engage in problem solving than to say that 70% of all episodes contain problem solving. In short, unless presented and discussed with caution, both approaches may be prone to misuse.

The figures of Table 3 do not identify any significant differences not already discussed. They show that two of the differences identified above – conceptual development in both England and Hungary – were proportionally similar, while confirming the significance of the remaining four. Thus the mean Flemish lesson comprised a smaller proportion of problem solving episodes than found elsewhere. The mean English lesson contained a smaller proportion of episodes containing structural mathematics than elsewhere. The Hungarian mean lesson comprised a higher proportion of episodes involving both structural mathematics and mathematical efficiency than found elsewhere while the mean Spanish lesson contained a higher proportion of problem solving episodes.

An interesting perspective can be gleaned from the totals of Tables 2. The mean Hungarian lesson, despite comprising significantly fewer episodes, attracted more codes than the project mean. One explanation may be that Hungarian lessons have a more integrated quality than those found in other countries.

DISCUSSION

The reader is reminded that the analyses derived from only 69 lessons taught by 16 teachers in four countries. Consequently, the data has been interpreted with caution although the reader is reminded that since project teachers were considered representative of effective practice in their particular locations, some sense of generality should be possible. The data yielded both similarities and differences in respect of project teachers' observable objectives and that these differences were not unrelated to location. The similarities were that teachers' objectives were focused more on behavioural outcomes than cognitive; indeed, there was almost no evidence of mathematics as derived knowledge. In respect of the other project foci, observed lessons were dominated by conceptual development and procedural skills supported by not insubstantial amounts of reasoning. Thus, it seems that project teachers hold similar views as to the key objectives of mathematics teaching. The remaining foci discriminated one tradition from another. For example, Hungarian lessons emphasised mathematical structure and efficiency; a tradition identified earlier (Andrews 2003). Flemish and Spanish lessons showed lower and greater emphases respectively on problem solving while mathematical structure was rarely observed in English lessons. Such differences allude to nationally-located patterns of behaviour and which reflect long held and frequently unarticulated assumptions about the processes of education (Schmidt et al, 1996; Stigler et al 2000).

	Flanders	England	Hungary	Spain	Composite
Conceptual	70.4	79.5	65.7	67.9	71.1
Derivational	4.5	1.1	5.5	2.1	3.7
Structural	16.8	0.5	42.9	13.5	18.8
Procedural	56.0	52.4	58.1	62.5	57.4
Efficiency	10.8	11.1	39.0	11.8	18.0
Problem solving	5.7	21.7	29.5	35.0	22.7
Reasoning	32.8	31.5	47.2	27.6	34.6
Totals	197.0	197.8	287.9	221.9	226.3

Table 3: percentage of all episodes observed to emphasise the different mathematical foci per participating country. Differences significant at better than the 0.01 level, as determined by Mann-Whitney U-tests, have been italicised.

Of the four project countries, Flanders has been the most successful on all recent tests of mathematics attainment - the three TIMSS tests, on which Hungary has done well, have examined students' technical competence while the two PISA tests, on the first of which England did well, have assessed issues of mathematical applicability. It is worth noting that the Flemish lessons resembled most closely the project composite lesson. That is, with the exception of a lack of problem solving, Flemish lessons were the most balanced in respect of the opportunities presented to learners. Consequently, it is our view that one way of improving mathematics education in England would be to provide learners with experiences that resemble more closely those identified in the composite lesson. In particular, this would mean a dramatic increase in learners' exposure to structural mathematics - the cognitive links between and within topics. Such a shift will require of teachers the profound understanding of elementary mathematics described by Ma (1999) and which is substantially beyond the skills-based subject knowledge that successive UK governments have regarded as acceptable for initial teacher training.

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