EXPLAINING, QUESTIONING AND STATING

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This paper originated from an interest in issues relating to traditional formal notation and viewing a video two years after it was recorded of three boys working with a computer program manipulating arithmetic equations and being surprised about my role as I worked with them. So my interest is in two areas: the learning of formal written algebraic notation, and analysing my role as teacher within a particular framework. I consider issues relating to who or what might explain, question and tell, and when that might be done.

INTRODUCTION

Kirshner (1989) has pointed to the visual aspects of algebraic notation and argued that students sometimes use the visual spacing of symbols as a clue to the parsing of equations rather than knowing about order of operations as such. For example, the expression $3 + 2x$ has the '2' and 'x' closest together and so can be seen as a 'chunk' which is added onto the '3'. Thus the ordering of multiplication before addition might be carried out purely due to the visual aspects of the notation rather than any conscious acknowledgement of conventions. Neria and Amit (2004) commented that "The results indicate that only a few students, who are very high achievers, choose to communicate via algebraic representations, even after two extensive years of learning algebra" (p409). So formal written algebraic notation is not a natural vehicle through which students communicate. Malara and Navarra (2001) worked on a project where they created a fictitious person, *Brioshi*, who only understood formal written algebraic notation and through which primary students were asked to communicate with another group of students. This notion of a pedagogic activity which forces formal notation to be used is one which I was interested in pursuing through computer software where formal notation was the means through which students interacted with the computer. Radford (2000) called for a shift from viewing signs as representing internal cognitive process and instead considering them as tools to accomplish actions. So, the notation would not be a summative statement at the end of a piece of work, such as an investigation to find a rule, but would be the very objects which would be acted upon in order to carry out a task. Before saying more about the software I will consider the frameworks within which I analysed parts of the video.

THEORETICAL FRAMEWORK

I analysed the video using a framework of arbitrary and necessary (Hewitt, 1999). In this framework the mathematics curriculum is divided into those things which are (a) arbitrary: names and conventions – of which all students must be informed, and (b) necessary: properties and relationships – of which some students can find out through using their awareness. I have argued that a role for a teacher sensitive to this division is (i) to inform students of the arbitrary and assist students in memorising and adopting these names and conventions; and (ii) to provide activities, direct attention and use questioning to help students educate their awareness of what is necessary – properties and relationships. Thus I might imagine myself, as teacher, *stating* what is arbitrary, and using *questioning* in relation to what is necessary. *Explaining* is related to the question *why?* I argue that the question *why?* is something which may not be so relevant for the arbitrary since the choice of why something was given its particular name, or why a particular convention is how it is, is not so much a matter of mathematics but of historical interest. *Why?* is more relevant to the necessary – properties and relationships. Even so trying to address this question can lead to what I describe as the *explanation trap* (Hewitt, 1994) where a teacher does all the work and students tend to become increasingly passive.

THE SOFTWARE

Many studies report the significant role of software to develop algebraic thinking (e.g. Clark, G. and Redden, T., 2000) whilst others report on the role of software to develop the use of symbols to create and interpret expressions (e.g. Sutherland, R., 1993), but little work has been done on the role software can have in learning the traditional formal notation of algebra which is used on paper. Most computer algebra systems (such as Derive) involve students communicating with the computer in a way where algebraic expressions are entered in a linear format albeit that they then appear on the screen in traditional format. Spreadsheets not only have students entering expressions in a linear format but they also have their own idiosyncratic notation (such as 'A2') and do not translate the expression into formal notation. None of these pieces of software address the vertical spatial component of formal notation as part of the interface between student and computer.

The software I used was a trial version of *Working with Equations* (now published [1]) which is part of *Multimedia Mathematics School* and which has the following five features: (i) Equations can be created through spatial positioning of numbers, letters and signs; (ii) Equations can be created and manipulated (to a possible end goal or just explored); (iii) Equations can be manipulated through the metaphor of *doing the same to both sides* or through the *physical movement* of numbers and letters from one side of the equation to the other; (iv) When physically moving numbers, the spatial positioning of the numbers and letters is significant; (v) Notation is 'looked after' by the software in that numbers and letters can only be positioned in places which conform to formal notation conventions.

Thus the medium through which students engage with the activity of trying to manipulate equations ensures that students have to conform to formal notation conventions. As Ainley (1997) commented *it disciplines communication… by only accepting instructions which follow particular conventions* (p93). Since the conventions in this case are arbitrary then this is a matter of helping students to adopt these conventions rather than address questions such as *why do I have to write it this* *way?* The necessary issues, such as performing calculations and deciding equivalent expressions, is left to the student as this is a matter of awareness and therefore not something for the computer to 'perform' on a student's behalf. The only thing the software will do is to reject anything which is mathematically inconsistent with the original equation.

THE CONTEXT

I worked with a small group of three boys from a school in Birmingham. The school has just over 40% of students who get five or more grades A*-C at GCSE. The students are from a Year 9 class and in the seventh set out of nine. During the session they struggled with negative numbers and the manipulation of equations was not something with which they were confident. This was the first time the students had met the software. The whole session lasted for 37 minutes. The excerpt from a video of the session starts 27 minutes into the session. I intended to work with students using an arbitrary and necessary framework by informing what is arbitrary but not explaining why the arbitrary is so, and not telling what is necessary, only using questioning and tasks to help students educate their awareness. I was interested in what students, who are not confident about manipulating equations, would make of trying to manipulate equations using the software, and how they dealt with notation. Thus my original intention was to leave them to it and only help with issues relating to the operation of the software.

To avoid issues relating to the use of letters, I decided to ask students to work with purely arithmetic equations. Before using the software I gave the students the statement $2+3=5$ and asked them to tell me different statements using just the 2, 3 and 5 which would also be true. They came up with a list of statements such as $5=3-2$ and $3+2=5$. Their task using the software was to transform the statement $2+3=5$ into their other statements. After they had obtained a number of statements on the computer, I then entered a second starting equation of $2\times3+5=11$. Their task was to try to obtain other equations by manipulating this one. During the course of the video, students were manipulating an equation by physically picking up a number and dragging it across the other side of the equation.

THE VIDEO EXCERPT

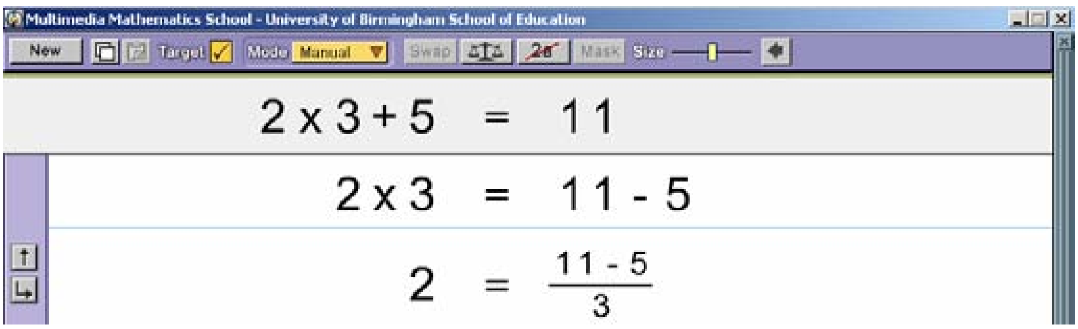

Figure 1: screen at start of excerpt.

I will give here only a brief extract from the session. Students had already manipulated the original equation to produce new equations and now had the screen as in Figure 1. They are working with the bottom equation and were trying to drag the number 11 across to the left-hand side (but on releasing the mouse the 11 did not stay there and automatically 'drifted' back to where it was on the right-hand side).

1	DH:	Why… shall I tell you why it won't let you drag the eleven there?
2	Student 2:	Why?
3	DH:	'Cos really, the eleven is being divided by three so the eleven is not… is not just on its own here. It's actually is being divided by three. So it's not going to let you just take eleven across because it's really eleven divi… eleven's being divided by three. It's not eleven on its own any more…
4	Student 1:	Yes… and one… times…

[The 3 in the equation $2=\frac{11-5}{3}$ was dragged over to the LHS next to the 2, which left a question mark under the division sign on the RHS: $2\ \ 3=\frac{11-5}{?}$. The student with the mouse was unsure about the box on the screen where the operation associated with the moved 3 was required to be entered. Also there was a space to enter a number which would replace the question mark. Until this was done, the OK button was not available.]

[…]

10	Student 2:	'Cos there's six is not dividing with anything so… what's the question mark?

[…]

15	DH:	So what would I divide by to just keep six to be six?
16	Student 1:	Nought.
17	Student 3:	Divide by…
18	Student 2:	One.
19	Student 3:	Divide by nothing.
20	DH:	Well…
21	Student 2:	Oh, it's divide by one, yes.
22	DH:	Is it divide by one?
23	Student 2:	Yes, because…
24	DH:	OK, so what I suggest is,… you see… choose the sign. What sign is it?
25	Student 1:	Times.
26	DH:	OK, then when you click on here and you enter what number you think should be there. It's alright, you carry on. [They entered one and clicked on OK and the equation become $2\times 3=11-5$]. There you go. And.. and usually when it's divide… when you are dividing by one it

doesn't bother showing you normally because dividing by one doesn't change anything.

DISCUSSION

Within the design of any software there are decisions made about what will and what will not be allowed. I will briefly say here that a pedagogic decision was taken not to allow students to move parts of an expression which forms a 'chunk' which is then operated on in this case by dividing by 3. This is so as to help force an awareness of order of operations and to work on the last operation first when carrying out inverse operations. It is possible with the software to swap the expression $\frac{11-5}{3}$ for $\frac{11}{3}-\frac{5}{3}$ and then the $\frac{11}{3}$ can be dragged across to the other side. This insistence that the single fraction is first swapped for two fractions helps force an awareness of what $\frac{11-5}{3}$ means. If the software had automatically shown $\frac{11}{3}$ when the 11 was dragged, then the software would be telling something about properties and relationships (the 'necessary'), which goes against a general principle of only tell what is arbitrary. So there is an issue about *telling* with respect to software design.

On viewing the video I was surprised to find myself immediately offering to explain something which is about number properties and relationships. I would have expected to use questioning to get them to think about why this may have happened. Instead I found myself entering into the *explanation trap* (Hewitt, 1994).

Within lines 1 and 3 I used "it" in a way as to attribute authority to the computer: *shall I tell you why it won't let you…* and *So it's not going to let you…* Giving authority to the computer for notational matters fits in with the fact that notation is arbitrary and students need to be informed of what is arbitrary and learn to adopt and work with such conventions. I have noticed on a number of occasions, within my own classroom in schools, how students often accept the way they have to conform to communicate with a computer but challenge me when I ask them to write something in a particular way. The question arises for me as to what extent I might exploit this phenomenon. When do I want to pass on the authority to a computer for pedagogic reasons? Who do I want to do the *telling*? Does this change for the things which are arbitrary and the things which are necessary? The case above concerns some mathematics which is not arbitrary and so it concerns educating awareness. So this is another example of students either accepting this as an example of the strange things a computer does, as if it is at the level of something which is arbitrary, or whether they become involved in trying to account for why it is not letting them do this and that there is a mathematical/pedagogic reason behind the phenomenon. In line 26 I am not clear whether I am attributing power to the software or to the notation itself. Maybe it is the notation which decides what it will or will not show.

From line 15 to line 24, I am using questioning, which does fit in with the arbitrary/necessary divide as what is being discussed is something under my necessary category. However, in line 26 I begin to get into explaining again, this time it is a mixture of a mathematical issue (that something divided by one is unchanged) and a notational matter (if it is unchanged then why bother showing it?). My explanation seems to be a mixture of *stating* what is arbitrary (*when you are dividing by one it doesn't bother showing you normally*) and *explaining* what is necessary (*because dividing by one doesn't change anything*).

1 *Working with Equations* is part of *Multimedia Mathematics School* and published by Plato Learning (www.platolearning.co.uk).

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