MEASURING LEARNING IN SITUATIONS WHICH ATTEMPT TO LINK SCHOOL MATHEMATICS TO OUT-OF-SCHOOL MATHEMATICAL ACTIVITIES

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This paper explores how learning, in situations which attempt to link school mathematics to out-of-school mathematical activities, may be measured. I present three possible measures which focus on students' motives, the mathematics employed and on the material resources. For each measure I consider its relevance, possible problems and how the measure may be constructed.

INTRODUCTION

I have just began a project[1] which will research ways that secondary school mathematics can be done in a manner similar to how it might be done in out-of-school activities. The aims are to understand: the problems in linking school mathematics to out-of-school activities; how learning activities can be designed so that links between school mathematics and out-of-school activities are made manifest; the role of the teacher and of resources in making these links; and how learning is affected in contexts which attempt to make such links. At the June 2004 BSRLM meeting I presented an overview of this project and a discussion resulted which focused on how we can measure learning in situations which attempt to link school mathematics to out-of-school mathematical activities. This is an important and a very complex matter and, after a brief review of literature, I take measuring learning in such contexts as my single focus in the discussion below.

MAKING LINKS TO OUT-OF-SCHOOL MATHEMATICAL ACTIVITIES

Most research on the use of mathematics in out-of-school activities shows a strong discontinuity between school and out-of-school mathematical practice. This discontinuity may be seen as a consequence of learning in and out of school being two different social practices, e.g. Lave (1988). Further to this, school mathematics is often ill-suited to out-of-school practices. Sometimes out-of-school problems are only apparently similar to school mathematics problems and in reality there is a range of explicit and implicit restrictions which makes school methods unsuitable, and other methods are used (Masingila, Davidenko & Prus-Wisniowska, 1996). In other cases (Scribner, 1984) work mathematics may appear to be simple, but there are no simple algorithms or methods to solve the problem and school-learnt procedures are of no use.

Despite the evident discontinuity between school mathematics and out-of-school practices some authors have observed an interplay between them. Saxe (1991) found evidence that school mathematics and the mathematics of street children's candy-selling practice in Brazil effect each other. Pozzi, Noss & Hoyles (1998) found cases

[1] See http://www.education.leeds.ac.uk/research/mathseducation/out_of_school_maths.htm

of nurses looking for a mathematical explanation for the conceptually simple mathematical procedures used in their daily practice. Magajna & Monaghan (2003), in a study of technicians designing moulds for bottles, found evidence that in making sense of their practice the technicians resorted to a form of school mathematics.

These studies suggest that school mathematics can be linked to out-of-school mathematical activities. I think it is highly unlikely that such links will arise without learning activities being specifically designed to engender these links. Further to this learning activities cannot be separated from the participants (teachers and students) and resources involved in using them. Tasks are central to making school mathematics relevant to out-of-school activities. Fitting a carpet to a room, for example, is not a simple area task, as it is in some school mathematics problems, and may involve metric/imperial conversions, considerations of matching the pattern and where to place the join. The resources used in a task effect the reasoning carried out in completing the task, see, for example, Magajna & Monaghan (2003). Carpet fitters use an electronic device for measuring distance, not a tape measure.

MEASURING LEARNING

A challenge for this research will be to measure learning. This is a challenge because intended learning in project classes is not learning mathematical skills and concepts but involves the learner developing an appreciation that the mathematics done in class is relevant to out-of-school activities. Activity theory (Leont'ev, 1978) provides a way that such learning may be measured. Activity theory differentiates between activity, actions and operations with regard to the objects to which these processes are oriented. Activities are oriented to motives. Actions are directed at specific conscious goals. Actions are realized through operations determined by the conditions of the activity. In the applications of mathematics in classrooms it is important to differentiate between motives, goals and conditions. If these are aligned, then the application of school mathematics is possible. Otherwise, then what the students are doing, though it may appear meaningful, may not have unity of purpose. Below I outline a set of measures, inspired by Stevenson[2] (2004), which may be used to evaluate this wider conception of learning. My starting point is three measures (which must be refined!)

Measure 1 Are students' motives consistent with the intended learning outcomes?

Measure 2 Does the mathematics employed by the students in realising specific goals assist them in realising the overall aim(s) of the activity?

Measure 3 Are the material resources available to the students consistent with the requirements of the activity?

Before addressing these measures is may be useful to attend to the word 'measure'. This is an emotive word for British mathematics educators. On the one hand, those of us who enjoy classical measure theory see measure as a beautiful mathematical

[2] I benefited from discussions with Ian Stevenson when he was first formulating his measures in 2003. Our distinct research needs have, however, resulted in distinct sets of measures.

abstraction. However, mathematics education in Britain has, for many years, been subjected to numerical measurement overload with regard to assessment[3]. An understandable reaction on the part of British mathematics educators, then, when I talk about measuring learning is 'oh no, not more measuring brain size!'. I am completely sympathetic to such sentiments but the need to measure (or assess or evaluate) learning (not ability) remains important, otherwise what is the point of such research?

I now consider each measure, attend to possible problems, examine if I have phrased them in an helpful manner and consider what their realisation may involve.

Measure 1 Are students' motives consistent with the intended learning outcomes?

'Motives' and 'learning outcomes' are key phrases here. Motive is crucial, it encapsulates why the student is doing what s/he is doing. In much of school mathematics the student is doing things because the teacher has instructed him/her to do it; the motive is to complete the work set. I am not necessarily critical of this in standard instruction. However, if the student is involved in a task linked to an out-of-school activity, then I feel the student needs to experience (at least something resembling) the motives a person in this out-of-school activity experiences. Indeed, I feel that if the motives of people in and out of school are different, then the activities they are doing, though they may be realised in similar actions, are distinct activities. Consider, for example, a student and a carpet layer engaged in planning to lay a plain carpet in the room. The carpet layer's motive is likely to be (i) to do it quickly, (ii) at a minimum cost, (iii) for it to be a perfect fit, (iv) for any join to be in an area of the room with minimum tread and (v) to avoid any mistake. 'Full marks' would be awarded on this measure if a student shared all five parts of the motive, 'no marks' would be awarded if a student was simply doing the task because s/he had to do it, but there are a number of problems. First I have assumed the carpet layer's motive. Different carpet layers may have different motives and carpet layer's may have motives without realising them or without seeing the need to say what they are. In my defence (i) – (v) came from interviews I have conducted with several carpet layers but interviews are both introspective and retrospective and present 'trustworthiness' problems. With regard to different carpet layers having different motives, this seems very likely. Consider, for example, a pair of carpet layers as compared to a single carpet layer. The motive of a carpet layer in a pair is likely to interact with his/her partner and this facet of the motive is likely to differ according to whether the pair are peers or whether one is senior to the other (in the latter case the motive of the junior partner is likely to include ensuring that the senior partner is happy with his/her work). Issues that I must attend to before refining this aspect of this measure is whether a common core of motives exists and can be used or whether a set of possible practitioner motives can be generated.

[3] See the Smith, http://www.mathsinquiry.org.uk/report/index.html, and the Tomlinson, http://www.14-19reform.gov.uk/, reports.

The phrase ‘intended learning outcome’ in measure 1 is, I now feel, problematic. A brief history of this phrase in this project may clarify my ideas. In conceptualising the project I used the phrase ‘intended learning’ to indicate that learning was not (just) learning ‘visible’ (Pozzi et al., 1998) mathematics but also learning about the context of the activity and how mathematics might ‘makes sense’ in this context. ‘Outcome’ is there partly because it is present in Stevenson’s (2004) measure 1 but also, I believe, because my own language has been influenced by British ‘education-speak’ where ‘learning outcome’ is a current ‘buzzword’[4]. In the BSRLM discussion of these issues the ‘education-speak’ connotations of this phrase clearly influenced people’s interpretation of measure 1 and this is a reason to omit the phrase. The use of ‘outcome’ in Stevenson’s report is legitimate because he takes “into account the preparations of the leader” (ibid., p.26)[5]. Although the preparations of the teacher will be important in my project I expect a complex (and far from smooth) interplay between students’ and teachers’ accounts of what (and why) students are doing. There is more to say on this but I have run out of space in this paper and I simply conclude that my first revision of measure 1 is: Are students’ motives consistent with the motives of practitioners?

A measure(s) of association between students’ and practitioner motives is required. The measure(s) may be quantitative and/or qualitative. I propose to first provide contextual descriptions of student and practitioner activities, ascribe motives and check the authenticity of my descriptions with participants. Further development will depend on what emerges from initial descriptions.

Measure 2 Does the mathematics employed by the students in realising specific goals assist them in realising the overall aim(s) of the activity?

Although I deal with measures 2 and 3 separately they are intricately related because the mathematics one does is mathematics-with-a-tool and using a different tool in a mathematical activity transforms the activity (Trouche, 2004). Two things to keep in mind with regard to measure 2 are: (i) that people in out-of-school activities (other than professional mathematicians) employ mathematics when mathematics is needed to realise actions, they do not use mathematics for the sake of doing mathematics; (ii) I expect teachers to experience a ‘curriculum pull’ to encorporate specific mathematical topics so that their students can gain experience of using that specific mathematical topic in an application. To return to carpet laying, although carpet layers do calculate areas to determine cost (which is a fixed cost per square yard/metre), they do not calculate areas in their decisions on how much carpet is required for a room; carpets come in rolls of a fixed width and the carpet layer deals with lengths, not areas.

[4] See, for example, http://www.qca.org.uk/14-19/11-16-schools/index_1194.htm

[5] ‘Learning outcomes’ is also a focus in attempts to improve out-of-school learning elsewhere. See, for example, *The Learning Impact Research Project* of the Council for Museums, Archives and Libraries, www.mla.gov.uk/documents/insplearn_lirp_rep.pdf

So, what if a teacher takes carpet laying as an activity and encourages her/his students to engage in area calculations for laying? Well, if dialogue does not persuade then otherwise, then I will have to accept it; teachers are subjected to a lot of pressures and it would be inappropriate, in this collaborative research, for me to direct what teachers do. But if a digression into unnecessary mathematics (for the activity) is neither significantly time consuming or distracting, then it is possible that this will not be a problem.

This measure is expected to raise the question ‘what is mathematics?’. A complication in reporting research in this area is that this question is highly individual. Mathematics educators will be influenced by their own mathematical studies. Research into work place mathematical practices and the use of technology in mathematics classrooms have, in my experience, resulted in me returning from these studies with an enlarged vision of what mathematics is; but this is very much my experience. I think that all I can do with the current project is to keep an open mind on what counts as mathematics.

I do not propose to change the wording at this stage. With regard to the nature of the measure I will explore two possibilities, one qualitative, the other quantitative. These measures will not necessarily be mutually exclusive. Details have yet to be decided but I would expect a qualitative measure to be informed by evidence-based accounts of students’ actions and descriptions of their work and relating these to general patterns of practitioner actions and descriptions of their work. With regard to a quantitative measure I will direct my initial efforts to constructing a statistic (rank correlation may be appropriate) from student and practitioner actions. A problem of whether a common core of practitioner actions exists, similar to the problem of whether a common core of practitioner motives exists mentioned in relation to measure 1, appears likely to arise in determining this measure.

Measure 3 Are the material resources available to the students consistent with the requirements of the activity?

For brevity I restrict discussion in this paper to physical tools. The ideas I held when I framed this measure were: all school and out-of-school mathematical activities involve tool use; tool use transforms mathematical actions; different institutions may sanction different tools but these different tools may realise similar aims. The last statement requires clarification. In engineering design in the developed world it is an expectation to use *AutoCAD*[6]. Indeed, a client may require, for subsequent use with numerically controlled machinery, that a specific version of *AutoCAD* be used. On the other hand school mathematics often stipulates that specific pencil and paper algorithms be followed or that calculators are not to be used. I am not aware of any out-of-school mathematical practices that make such stipulations. Now I would not like to assume, prior to experimental work, that a student involved in a school-based

[6] *AutoCAD* is software which produces engineering drawings.

task related to engineering design who did not use *AutoCAD* was not engaged in engineering design, but I would question whether s/he was.

I used *AutoCAD* as an example as it is an standard professional tool and illustrates my point well. I do not expect project classes to engage in engineering design but questions on school/out-of-school tasks and tools used are important and it is not clear, at least to me, what the issues and answers are. In the discussion of measure 1 I suggested that if the motives of people in and out of school were different, then their activities, though they may appear similar, are distinct. I do not assume *a priori* that the same applies with 'motives' replaced by 'tools' but there may be situations where this is the case. To appreciate why I do not make this assumption I return to carpet laying; I do not see that a student using a tape measure instead of an electronic device for measuring distance has fundamentally changed the activity.

In a school mathematics context this is a relatively unexplored area. How can a measure be developed? I do not see scope for a quantitative measure simply because I cannot envisage how numeric coding could be ascribed, but this may be possible. Rich contextual accounts of tool use in school and related out-of-school activities appear to be necessary. The principles of activity theory are a likely source of descriptors for this measure. For example, how does the use of different tools in ostensibly similar activities interrelate with the motives of the activities? Is it possible to graduate differences here? Different tool use will certainly result in different actions and operations but, again, is it possible to graduate differences here?

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