DISTRIBUTION AS EMERGENT PHENOMENON

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In this paper I present the background and intentions that have shaped the design of a microworld to study students' understanding of probability distributions. The outcome of this paper is the microworld design itself.

INTRODUCTION

My research is centered on understanding how people think about the emergent behaviour of distributions. I am fascinated by the often unexpected macrobehaviour that emerges from the interaction of thousands of autonomous agents. As a statistician, I make analogous personal connections with the emergent behaviour of distribution. As a researcher, I want to observe the reasons that lie behind people's struggling to attend to the emergent behaviour and the core notions of probability and trace their thinking. As a teacher and a researcher, I wish to help them to adopt a new way of viewing mathematics through simulation models, and see probability distribution and stochastics more generally through the lenses of emergent binoculars.

EPISTEMOLOGY OF PROBABILITY

The epistemology side of probability is marked by two easily distinguishable approaches, namely frequency-type and belief-type theories. The former place emphasis on how often events occur in order to measure chance. The latter regard chance as subjective.

This separation of probability into distinct epistemologies notoriously leads to a profound confusion and controversy which surrounds the frequency and subjective interpretations of probability. This confusion and controversy, in fact, persists miraculously in the mathematical treatment of probability for the past 300 years.

My study will build upon a new type of epistemology of probability that has emerged from the science of complexity and self-organization and is based on Wilensky's work (1997). Will emergent behaviour add to this catalogue and confuse further, or will it offer some sort of unification?

EMERGENT PHENOMENA

When numerous (micro) agents of a system dynamically interact in multiple ways, following local rules and utterly oblivious to any higher-level instructions, they can form higher - level patterns. These kinds of discernible macrobehaviour are called "emergent phenomena"- that is phenomena that emerge from parallel complex interactions between local agents. For example, prices emerge from the accumulated interactions of buyers and sellers, physical objects emerge from particles, and the transmission of a virus in a human population emerges from the interactions of individual human beings. In fact, as our appreciation of emergence advances, we see our world as dominated by emergent entities.

An understanding of complex systems is increasingly becoming a core part of scientific knowledge, the adoption of this new perspective is essential….so it is time for mathematics educators to make a crucial shift from the traditional paradigms.

Spurred by the necessity of adopting the perspective of emergence, I am seeing probability distribution as a complex personality that self organizes out of many individual decisions (data); a global order emerges out of uncoordinated local interactions over its duration or pattern; distribution emerges out of the anarchy of randomness: that is "probability distribution as emergent phenomenon".

The centralized mindset (Resnick, 1991; Resnick & Wilensky, 1993) is considered as one source of many people's deep-seated misunderstandings about the workings of patterns and emergent phenomena in the world. Resnick termed as "centralized mindset" the existing globalized tendency to have strong attachments to centralized ways of thinking that means that someone (the leader) or something explicitly creates and orchestrates the pattern. For example, people appreciate distributions in the same way and wonder how a patterned behaviour without apparent causal explanation emerges out of low level behaviour that does have explicit causal explanation.

To help people move beyond the centralized mindset, Resnick (1991) and Wilensky (1999) (also, Wilensky & Stroup, 2000) designed StarLogo and NetLogo respectively. Both are modeling complex, dynamic systems evolving over time. They allow modelers to give instructions to thousands of independent computational objects, all operating in parallel, and controlling their actions and the interactions among them as well.

Another major source of confusion lies in the failure to discriminate and move between levels (Resnick & Wilensky, 1998). Levels characterize the micro-behaviour and macrobehaviour of a complex system. According to Wilensky and Resnick(1998) the notion of levels is the a cornerstone to a new framework for developing better causal accounts of the relationships, and interactions among simple elements of various systems in the micro-level and understanding the mechanisms which underlie emergent phenomena and patterns.

One difficulty felt by people is that the relationship between micro and macro levels does not sit comfortably with the conventional view of relationships as being either hierarchical or inclusive. Another difficulty is that people's mind struggle to focus on the appropriate objects. Should they attend to the many individual interacting agents or the singular emergent pattern? Finally, there appear to be a natural or scheduled tendency to hold tightly to the deterministic mindset (Wilensky, 1997).

APPROACH

My research study adopts a Constructionist approach (Papert, 1991), which advocates the construction of knowledge in the context of constructing personally meaningful artifacts. The playful facet of constructionism is more likely to enhance learning because it is based on the fact that learners are more likely to grasp new "formal" knowledge when engaged in experiences (including verbal ones) and creating some kind of external artifact.

The major challenge for educators is to design powerful artificial environments or microworlds that would engage students in active experimentation and personal construction of knowledge. As an educator I respond to this challenge. Thus, I designed a NetLogo model for the purposes of my study. My study falls into the category of design experiments.

The purpose of design experiments is to develop specific theories about both the forms of learning and the means of supporting them, using an iterative design approach in which design, enactment, invention and revision cycle. The insights gained from each iteration feed into the next iteration.

In my research, I will use NetLogo as a platform for supporting student explorations (and studying student thinking) in high schools, in roughly two phases. In the first phase, I will present a "seed" model (a simple starting model) to the students and students play with the model in small groups and explore the parameter space of the model. I will engage them in discussion during their “game” as to what is going on, why they are observing that particular behaviour, how they can change the emergent macrobehaviour. Later I present the first iteration of the seed model. In the second phase, the group proposes an extension to the model and implements that extension in the NetLogo language.

In these activities, insights gained from the participants’ interactions with the model-setting and the “educational” intervention, both redefine my understanding of the learning issues involved.

The methodology is designed to explore whether and how: 1) students will make a link between causation and emergent distributions, and 2) the tools support students’ thinking as it moves from a micro only perspective to one which flexibly connects a macro emergent perspective to that prior micro view.

Due to the fact that we needed to bring the concept of levels into the mainstream of mathematics education, what is needed is more fine-grained research study that probe the conceptions which underlie the ways students understand (and misunderstand) emergent levels. My research will be devoted to: 1) a more fine-grained research study that probes the conceptions which underlie the ways students understand (and misunderstand) emergent levels in the case of distributions, 2) how learners can develop richer understandings of levels, and 3) how this understanding helped them to gain insight into the phenomena they were investigating.

In order to pursue these aims I realized that I needed to turn causality and distribution into something manipulable for students, a process that Pratt (1998) has called phenomenalising. My aspiration is to phenomenalise distributions in such a way that there is support for a Macro view of the features of distributions that I would like students to tune into. At the same time I would like to support the micro view of the messy randomness.

Furthermore, I intend to gain insight on students' tendency to hold tightly to the deterministic way of thinking by empowering students to view the relationship between them and perceive emergence as letting go of determinism.

The students will first experiment with the micro level where a deterministic world lies and get visual feedback for the macro level where an emergent world lies. The first stage of letting go of determinism will involve the introduction of error to the determining variables.

The students will then experiment at the Macro level where they must let go of determinism. Causal relations are now not determinable. Patterns must be explored as emergent phenomena.

BASKETBALL MODEL

I have now established the influential factor and set out the aims that underpin the first design of my microworld. Below, I describe this microworld in more detail.

Micro level project

At the micro level (Figure 1), the student is challenged to throw a ball into the basket. He can alter the basket design or the way he throws the ball.

This task directs attention of the student to causality (speed and angle of throw). Then the student can begin to let go of causality, by introducing an error factor in throws, perhaps perceived as allowing for skill level. The student is no longer completely in control and, consequently, the system is no longer completely determined.

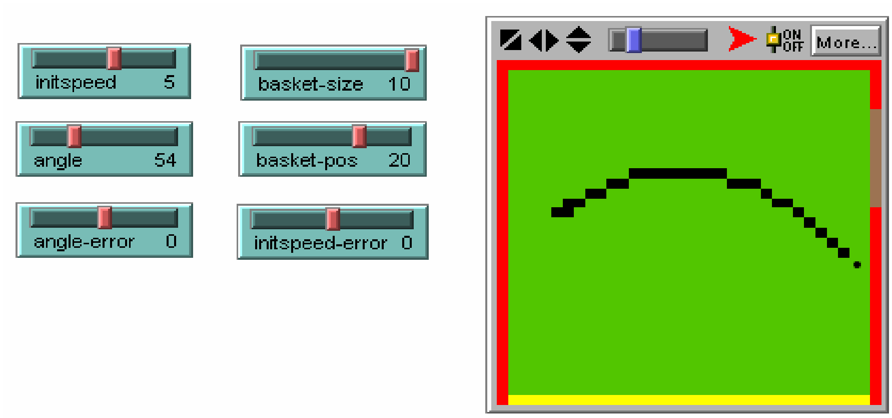

Figure 1: In the graphics window black colored balls move under gravity towards the basket. The top sliders allow the student to change the size or position of the basket. The bottom sliders enable the student to throw the ball at different speeds and angles and introduce error into those throws.

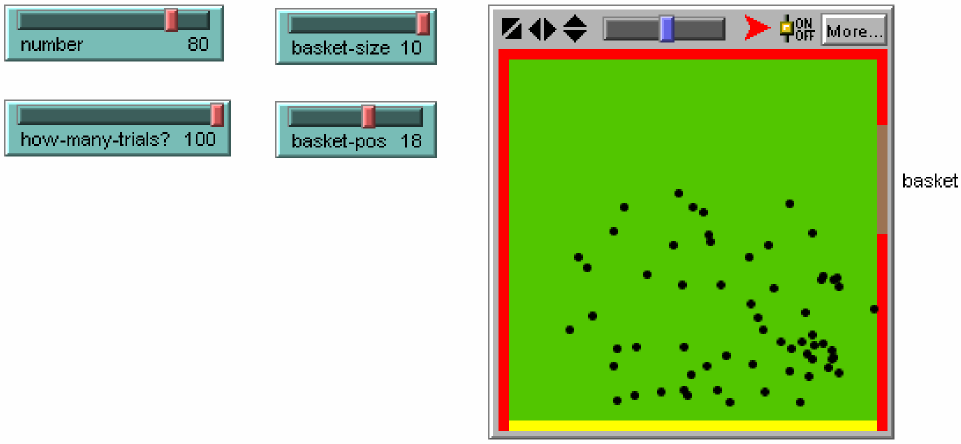

Figure 2: The top sliders again allow the student to change the size or position of the basket. The bottom sliders enable the student to decide how many balls should be thrown and how many repetitions are carried out.

Macro level project

At the Macro level (Figure 2), the student is asked to position the basket in such a way that scoring is neither too easy nor too difficult for a whole class of unknown children.

This task directs the attention of the student towards emergent distribution. The student has to let go of causality. Now, they have no control over the throws and they have to consider the distribution of the throws.

At both levels, feedback is an essential component of a simulation, because it provides the form of an output. At both micro and macro levels, the students can base their decision on various types of feedback, such as counters of goals, and three different types of graphs, namely average rate of goals per trial, a histogram of goals against position of basketball throw each, and successful-shoots against trials.

CONCLUSION

I have described the literature of emergent phenomena and the broadly constructionist approach that has let me to this particular microworld design. I shall be using this software with 16/17 year old students to probe into how students relate to the micro and macro levels and the role of causation and distribution in those two contexts.

In a future iteration I hope to find a way of integrating the two levels, which at present are presented quite separate, onto one project and, in the process, open up the software to enable students to change the model in more fundamental ways. As stated above, this is the first phase of an iterative process, part of a design experiment.

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