ACCELERATED LEARNING OF PROBLEM SOLVING SKILLS

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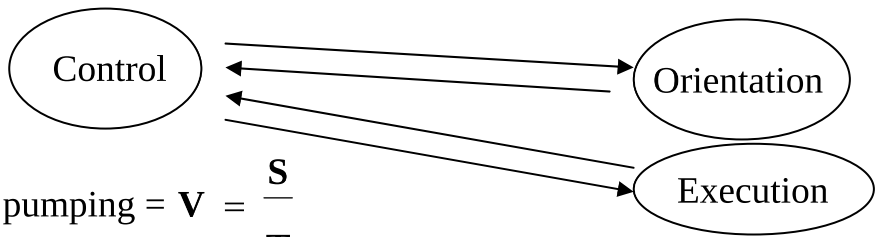
Two year 7 classes in a Manchester school were taught multiplication, division and fractions. An experimental group was taught these numerical skills, but their teaching program included practical problem solving, based upon activity theory principles, as an integral component. A control group practised their number skills in more traditional abstract contexts. As expected, the control group was not able to transfer number fluency to practical problem solving tasks. The experimental group, however, demonstrated a problem solving ability at higher GCSE level and achieved a significant improvement in mean scores over the dynamic assessment that followed the teaching program. The dynamic core of this assessment was computer based and there was a strong negative relationship between hints given by the computer and residual gains. Analyses of the computer records have provided important clues to guide a qualitative analysis of video records of the teaching program.

Engeström (1999) has suggested that a study of mediating artefacts (such as mathematical models) is centrally important in research into practical problem solving. Within this ‘Activity Theory’ perspective, theory is seen to be of greatest importance when it can be used to mediate a process of practical activity. In the teaching program summarised below there is no separation between presentation of theory and practice and the practical creative nature of mathematical tools became directly apparent to the students while they were solving problems. The teaching material was in turn prepared on the basis of detailed analyses of practical problems involving the notion of rate already carried out by the Gal’perin School (See Haenen 1996 for a more detailed account of this teaching method).

The experimental teaching programme focused on teaching basic number skills of multiplication, division and fractions in a meaningful context. I chose to develop these skills in the context of problems on rate of processes, for example the rate of movement, rate of production or rate of flow in water, because the students will encounter these problems regularly in later studies. I chose the notion of rate as a practical (substantive) generalization that would be widely applicable in practical activity. I looked at this learning in terms of developing practical creative abilities rather than simply of acquiring abstract knowledge of formal calculation rules. This diagram shows the key components of actions that were taught.



Rate of pumping = $\mathbf{V} = \dfrac{\mathbf{S}}{\mathbf{T}}$

Fig. 1

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Gal'perin's Activity Theory suggests (ibid) that the process of orientation, of knowing what to do next at any point, requires identifying the main operations to carry out and the order in which to do them. An appropriate control model must be taught and developed in the course of problem solving activity. A more or less developed form of this model can then be brought to mind when it is required during practical problem solving. Orienting activity will then appear as 'attention', which directs itself towards the model. This attention is thus an abbreviated and condensed control procedure for practical activity. An example of a problem solving action from the teaching program (fig 1) was: '*A pump produces 100 litres of oil in 5 hours. How much oil would be produced in one hour?*'

In this example, a formal mathematical notion was introduced as a model, which acted as a control for the action. The process of orientation, of knowing what to do next at any point, was also taught, in this case by means of two cards (shown below), which indicate the main operations to carry out in simple calculations, and the order in which to do them. These instructions were operations in verbal form and they were abbreviated during practice to a coded form '1,2,3,4' or '1 to 6'and then, with more practice to a simple awareness of what to do next, or 'attention'.

CARD 1

In each question you must find:

1) **Who is carrying out the action?**
2) **What is the person or thing carrying out the action getting through, producing or using up? (S =?)**
3) **How long do they take? (T=?)**
4) **How much do they get done in one unit of time? (V= ?)**

Card No 2

1) **How many actions are going on?**
2) **Do they begin and finish together?**

Do the actions work: a) together

b) against each other?

4) What is known in the task about overall production values (S_o, T_o, V_o)

5) What is known about component action values (S_1, T_1, and V_1)

6) What do you need to find in the task?

Within this teaching method practical actions were converted to words and then to mental actions. Actions were first presented in materialised form as diagrams. These coded actions changed to a verbal form as they were spoken aloud. During practice, silent speech 'to oneself' was abbreviated and condensed and was eventually no longer accessible to introspection. In this process, the actions changed in their level of generalisation as a deeper understanding of rate formed from notions of speed, wages, flow etc. Abbreviation and fluency of all three aspects of the actions (orientation, execution and control) was developed in order to establish a sound long-term memory of the problem solving skills (see Talyzina 1981). A variety of techniques were employed to develop these skills and the problem solving tasks were gradually increased in difficulty until they eventually reached a level of difficulty presented in higher-level GCSE courses. For example:

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'Two bulls charge each other with a combined speed of [40 3/10 mps] and meet after [16 seconds]. The speed of the first bull is [12 1/5 mps]. [How far would the second bull travel] in [this time] if he went [2 1/10 mps slower]?'

Results: The diagram below illustrates the experimental design:

	Initial base line test	Teaching -------- program	First post test	Computer practice	Second post test
Experimental gp.	(IT)	------------------	(T1)	(P1)	(T2)
Control gp.	(IT)	-----------------	(T1)	(P1)	(T2)

Initial baseline measures compared the children from the two classes (the experimental and control groups) in terms of ability to complete questions on multiplication, division and fractions. These were questions taken from the normal school end of unit test on this topic. Average scores on these tests did not vary significantly between the classes. The mean overall scores in these initial tests were 61% and 63% (table 1). These scores indicated that children were generally competent to begin work on the elementary introductory examples in the teaching program. Initial scores on further practical problem solving questions about rates of processes were lower, as would be expected, but again showed no significant differences between the two groups:

	Experimental group		Control group				
Variable	**mean**	**sd**	**mean**	**sd**	**n**	**t**	**p**
Pre-test (number - %)	**61**	**16.5**	**63**	**16.5**	**23**	**0.6**	**0.57**
Pre-test (rate - %)	**18.6**	**18.6**	**22.9**	**21.4**	**26**	**-1**	**0.42**

Table 1

The control group followed the normal school program, which involved practice of the number skills in more abstract contexts. I expected that the control group would not be able to transfer their number skills to practical problem solving tasks. This was in accord with the principles of activity theory, which explicitly propose that mastery depends on the quality of particular orienting bases (cards one and two) employed, and is not an ingredient that is added separately from the material that is taught.

At the end of the program I looked both at what the children could do in formal numerical questions from the school end of unit test and at how easily they could apply this knowledge to problem solving questions on rate, with help from an hour of computer based teaching. In this three-part dynamic assessment, a computer-based assisted practice session separated two isometrically similar formal unassisted tests (see Day 2001 pp. 176-207 for a discussion of this procedure). No significant differences emerged between the groups in their measured abilities at numerical questions on these topics. A two-way Repeated-Measures ANOVA analysis of the number test scores showed no significant differences between classes (see Chart 1).

An analysis of rate test scores showed a significant difference between experimental group post-tests ($p<0.01$) and a significant difference between the two groups overall

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($p<0.05$) (see Chart 2). The experimental group scores on rate improved from a mean of 26% to 41% over the dynamic assessment and were more than twice as high as those of the control group in the final post test.

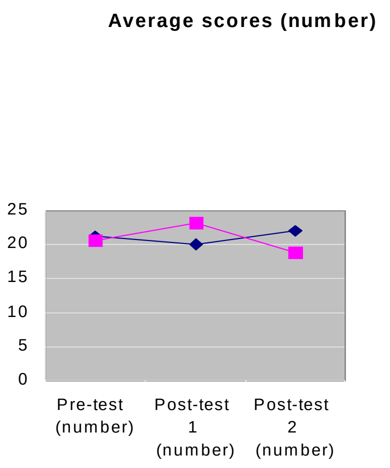

Chart 1

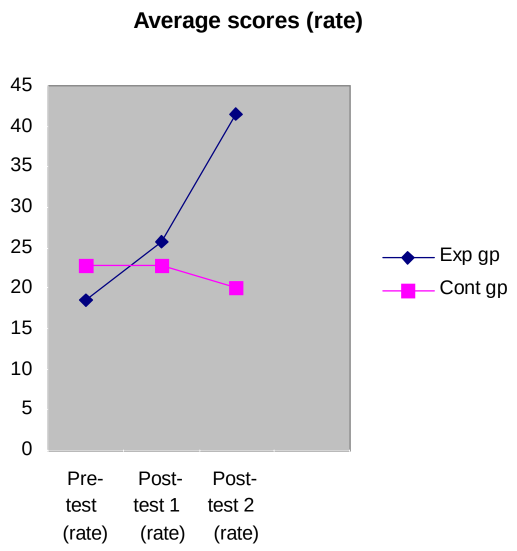

Chart 2

The next table summarises the comparison between mean scores on the pre-test and mean scores on the post-tests (T1, T2) for the two groups.

Descriptive statistics and t values for experimental and control groups

	Experimental group		Control group				
Variable	**mean**	**sd**	**mean**	**sd**	**n**	**t**	**p**
Pre-test (number)	**21.3**	**5.5**	**20.6**	**5.2**	**23**	**0.6**	**0.57**
Post-test 1 (number)	**20.1**	**6.6**	**23.3**	**4.8**	**26**	**-2**	**0.05**
Post-test 2 (number)	**22**	**5.9**	**18.8**	**7.4**	**12**	**1.5**	**0.16**
Pre-test (rate)%	**18.6**	**18.6**	**22.9**	**21.4**	**26**	**-1**	**0.42**
Post-test 1 (rate)%	**25.7**	**25.7**	**22.9**	**25.7**	**26**	**0.3**	**0.8**
Post-test 2 (rate)%	**41.4**	**31.4**	**20.0**	**24.3**	**26**	**2.6**	***0.014**

table 2

I have already shown (Day 2001) that the rate of adaptation to similar but more complex problems can provide important information about a child's 'Zone of Proximal Development'. In this work I demonstrated that the number of hints given during interaction with a tutor can provide a useful (inverse) index of intellectual maturity and readiness for the more difficult problems. Because of time constraints within the busy school teaching program, theoretically based hints, generated from the teaching program, were given during a computer-based practice until all the problems were solved. Amount of help required was recorded, categorized and compared with the mathematical gains made in unassisted performance over the two tests carried out before and after the computer-assisted session.

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[In order to minimise validity problems due to effects of the distribution of results in test T1 on results in test T2, mathematical gains over the practice session were defined by residual gains in test two, above or below what was predicted by the overall trend of results. (See Elliot and Lauchlan (1997), Embretson (1990) for a discussion of the reliability of gain scores measured by test, train, retest procedures). Unreliability due to unequal scaling effects for level of difficulty between test items remains, but scaling defects will be the same in both experimental and control groups and changes of scale that occur because items that become easier in the second test will also be generally replicated over the two groups. Use of the second post-test will therefore not affect the reliability of my comparison between the two classes].

Mean residual gains and hints given for the two groups are shown in the diagrams below. The negative mean residual gains for the control group (fig.2) suggest that the control group children were unable to gain as much from the practice sessions as children from the experimental group. They could not transfer their formal mathematical knowledge to their practice session on the computer. This was, of course, what was expected. These results largely replicated the results of earlier studies and confirm the number of hints needed in practice to be a useful indicator of proximal development zones. Control group children received almost 50% more assistance on average (fig. 3) than children from the experimental group.

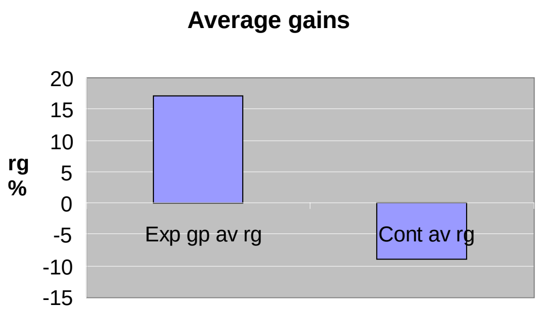

Fig 2

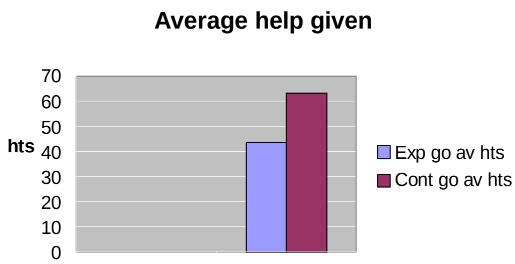

Fig 3

It can be seen from table 3 that the number of hints provided was negatively correlated with gains that were made and with scores on the first post-test. I found, as expected, that lower scores on the post-test meant that, generally, more help would be needed in completing the practice papers and lower gains would be made during practice. The amount of help needed was clearly an important factor in predicting these gains. In a multiple regression (table 4) the number of hints (hts) accounted for 13% of variation in residual gain scores over and above the general mathematics ability measured in the pre-test and a specific test of the work (t1, rate).

	Rg	**t1 num**	**t1 rate**
t1 num	**0.1**		
t1 rate	**-0.030**	***0.55**	
Hts	***-0.31**	***-0.46**	***-0.52**

*P < 0.05

table 3

Model	STEPS TABLE					
0 (no predictors)	STEP	df	deviation reduced	df2	F	rsq chg%
1 (pre)	1(0-1)	1	0.97	27	0.25 ns0.010	1
2 (pre,t1)	2(1-2)	1	0.95	26	0.24 ns0.009	1
3 (pre,t1,hts)	3(2-3)	1	13.18	25	3.64 P<.05	13

table 4

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Computer-based hints have thus been shown to have some validity as an index of a students progress and can help to guide a more descriptive account of the program.

Qualitative results: This final diagram shows hints plotted against residual gains for two children who completed the entire program. These children (Louise and Lisa) sat together in class, achieved equal scores in the second post-test and made similar mathematical gains. Lisa, however, needed far more help in completing the computer-based practice than Louise. Video records of these two children confirm that Louise was far more influential in interactions between the two children. She seemed to have a greater mastery of the topic than Lisa and because of this would be expected to progress more quickly in future. The imbalance in their working relationship was only indicated quantitatively by amount of help required during practice. An inductive analysis of the data, beginning with video transcript records of Lisa and Laura and then looking at other related events that were observed has modified my view of the teaching activity. As I review the data looking for events that contradict my original idea, I hope to arrive at set of ideas developed within an integrated and well-defined theory that could describe aspects of the teaching program in a way that will provide suggestions for future improvements. A model that accounts for the qualitative observations will be presented later.

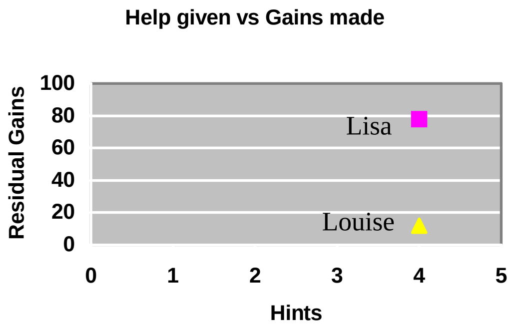

Chart 3

BIBLIOGRAPHY

Day.C. 2001. An activity theory approach to the teaching and dynamic assessment of mathematics, with particular reference to the use of algebra and functions in problem solving. *PhD Thesis, South Bank University*.

Elliott, J.G., & Lauchlan, F. (1997). Assessing potential: The search for the philosopher's stone? *Educational and Child Psychology*. 14(4), 6-16.

Embretson, S. (1990). Diagnostic testing by measuring learning processes: Psychometric considerations for dynamic testing. In N. Fredericksen, R. Glaser, A. Lesgold, & M.G. 1 Shafto (Eds.), *Diagnostic monitoring of skill and language acquisition* (pp. 407-432). Hillsdale, NJ: Erlbaum.

Engelström, Y.: 1999, 'Activity Theory and individual and social transformation.' In Engelström, Y. Meittinen, R. and Punamäki, R. *Perspectives on Activity Theory* Cambridge University Press.

Haenen, J. 1996 *Piotr Galperin: Psychologist in Vygotsky's Footsteps*. Commack NY: Nova Science.

TALYZINA, N. F.: 1981, 'The Psychology of Learning: Theories of Learning and Programmed Instruction.' Moscow, Progress Publishers.

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