

## **LINKING MULTIPLE REPRESENTATIONS IN EXPLORING ITERATIONS: DOES CHANGE IN TECHNOLOGY CHANGE STUDENTS' CONJECTURES?**

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*This study investigates changes in conjectures of four typical students when they are using different kinds of technologies, particularly in relation to their preferences for representations and the way they express their conjectures in understanding the concept and properties of iteration. The first stage of the research was conducted using pen and paper (PP) with graphical calculator (GC) in a classroom while the second stage used PP with graphical software (GS) in a laboratory. The findings suggest, with important caveats, that different technologies significantly influence the students' preferences for representations. Also, this study shows that students' conjectures can be an effective unit of analysis in researching students' understanding of iteration and preferences for representations.*

### **BACKGROUND AND AIMS OF THE STUDY**

To be able to understand iteration, students need to successfully link the different representations involved. However, previous studies have revealed that this can be problematic (e.g. Dunham and Osborne, 1991). For example, Sierpinska (1994) has stated that 'students normally conceive that the fixed-point is the intersection of the graph and the iterative formula (p. 91).' But, many have believed that technology can influence students' understanding of mathematics and the way they link multiple representations (e.g. Elliot et al., 2000; Kaput, 1992; Kaput and Thompson, 1994; Hennessy et al., 2002). In fact, Weigand (1991) has found that students' understanding of iterations' properties is influenced by the considered representations generated by technology. Also, Keller and Hirsch (1998) have claimed that students' preference for representations is vital to an understanding of how they link multiple representations; that GC users preferred graphical representation; and that the technology lessened the difference in preferences when compared to non-GC users.

Aczel's (1998) study has provided evidence that investigating students' conjectures can be an effective approach to analysing their understanding of algebra concepts. Similarly, Villarreal's (2000) study has confirmed the use of this approach in investigating how students link representations or how students' preferences for representation change. She has categorised students' thinking processes as either preferring algebraic or visual approach which are neither exclusive nor disjoint.

This study aimed to investigate how, when they are using different kinds of technology, students' conjectures change in relation to— a) understanding the concept of iteration b) discovering the properties of iteration and c) their preferences for representation in understanding the concept of iteration. It is hoped that this may help

to illuminate the reasons behind the difficulties that students experience in relating multiple representations in learning iterations.

## THE METHODOLOGY AND THE ANALYSIS

Two A-level classes (having four and eight students) in a suburban school in England participated in the first stage of the research. The setting was conducted in a classroom environment and involved PP and the GC (TI-83)<sup>1</sup>. This was the students' first formal introduction to iteration at A-level, but they had some experience of rearrangement of equations. The students were asked to work in pairs in order to capture the 'students talk' relevant for analysis.

In the second stage of the research, four students from these classes, along with another student who had no experience of the materials in the first stage, used PP and the GS (*Autograph*)<sup>2</sup>. This second stage was conducted in a laboratory designed to capture the participants' activity by means of video and audio. Four video streams are recorded simultaneously and combined into a single stream: two video streams are of the students working on their worksheet, another video stream is of what they are doing with the mouse and keyboard, and the fourth stream is of the computer display.

Though, there are differences on how representations (i.e. graphical, numeric, and algebraic) can be represented by GC or GS, two parallel worksheets (figure 1) were carefully designed to take account of the differences of the representations that the technologies (i.e. PP, GC, and GS) can present in its interface (figure 2). The worksheets consisted of items 1) requiring procedural skills (required items to make inferences) 2) encouraging the making of inferences and 3) seeking reflection on their experience of using technology and the worksheet in the classroom. The items were also categorised for the purpose of analysis as follows: **I** – in understanding of the concept of iteration; **II** - in discovering the properties of iterations; and, **III** – students' combination of their inferences based on **I** and **II**.

These worksheets, similar in style to that of Weigand (1991) were designed to elicit students' solutions and inferences about their solutions. However, the worksheets from this study are focused more on conceptual skills and on an exploratory approach, where the items are to be connected in order to come up with meaningful conjectures.

The data collected from the worksheets are supported by techniques similar to those of Villarreal (2000) and Weigand (1991): audio transcripts based on think-aloud protocols, video data, interviews and fieldnotes. The data were validated through triangulation of the interview with the teachers and the students, and the researcher's fieldnotes. The main data collected were categorised using a coding scheme based on a number of previous empirical studies relating to how students approach graphing

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<sup>1</sup> Texas instrument model TI-83 is graphing tool capable of producing the graphs, equation and coordinates at the same time.

<sup>2</sup> Autograph version 2.10 capable of simultaneously presenting all representations and doing iteration dynamically.

and the linking of different representations (Even, 1998; Ruthven, 1990; Villarreal, 2000). In investigating the students' understanding of the concept and properties of iteration, the data were analysed by sequentially and reflectively looking at the questions, the video data and the data from the two worksheets. In finding the students' preferences for representation the same process was done this time including the coding scheme.

Items	Fieldwork (GC)	Experiment (GS)
Procedural	Find the solution of $x^2+x-6=0$ ; Sketch the graph using the graphical calculator; Find an iteration formula.	Find the solution of $x^2+2x-15=0$ ; Sketch the graph using the graphical calculator; Find an iteration formula.
Conjecturing	What can you infer from the graph of $x^2+x-6=0$ in relation to the graphs of $y=x$ and the iteration formula?; What can you conclude based on your inferences?	Write down your inferences for graphing $y=x$ , $x^2+2x-15$ , and the iteration formula; What can you conclude based on your inferences?
Difficulty/ Other	Please note down any difficulty encountered in the worksheet or in using the graphical calculator	How does your inference change when you use a computer compared to using a graphing calculator?

**Figure 1: Sample items on the two worksheets (extracts)**

Type	Fieldwork (GC)			Experiment (GS)		
	Item	paper	GC	Item	Paper	GS
I	A.5	A, N	V	A.3.c.1	A, N	A, N, V
II	D.1.a	A, N, V	A, N, V	A.6.c	A, N, V	A, N, V
III	A.9	A, N, V	A, N, V	A.4	A, N, V	A, N, V

Coding: V – purely visual; A – purely algebraic; N – purely numeric; AV – combined A and V; AN – A and N; VN – V and N; AVN – A, V and N; M – no answer or ambiguous

**Figure 2: Available representations offered by technology (extracts)**

General patterns of inferences were considered and revealed that the participants involved in stage two were typical of those involved in the classroom-based stage-one fieldwork. The worksheet data of the selected participants in the two stages were re-analysed to compare the changes in how their preferences for representations and their understanding of iteration in terms of aspects I, II, and III change when using different technologies. Figure 3 shows the general pattern of GC participants' inferences where the four typical participants were determined whilst; figure 4 shows the changes of preferences of the GS participants (See San Diego, 2003 for the description of the coding).

Type	Item	A	V	N	AV	AN	VN	AVN	M
I	A.5	3	1		4		1	1	4
II	D.1.a	2	5	2	2		1		2
III	A.9		4		3			2	5

**Figure 3: GC participants’ preferences for representation (n = 14) (Extracts)**

Fieldwork ( using GC)						Experiment ( using GS)					
Type	Item	S1	S2	S4	S5	Item	S1	S2	S3	S4	S5
I	A.5	AV	AV	M	AV	A.3.c.1	AVN	AV	V	N	AV
II	D.1.a	A	A	V	AV	A.6.c	VN	VN	N	N	VN
III	A.9	AV	AV	M	AV	A.4	AVN	AVN	AV	AV	AV

**Figure 4: Changes of selected participants’ preferences**

### LIMITATION, RESULTS AND DISCUSSION

The difficulty of identifying whether students have failed to notice something or failed to express something is recognised as a limitation of the findings of this study. The subtle differences between the two worksheets meant that comparison was problematic, but these differences could not be avoided. However, the worksheets were repeatedly trialled in order to minimise these effects.

Similarly to Aczel (1998), this has revealed that conjectures can be used as a unit of analysing students’ thought processes, particularly in this study, in investigating understanding of the concept and properties of iteration and preferences for representations.

Students in this study have difficulty in explicitly expressing the connection between the fixed-point and the solution of  $f(x)$ . However, this may be either associated with their difficulty in extracting information from a coordinate (Dunham and Osborne, 1991) or be influenced by the representation considered (Weigand, 1991; Keller and Hirsch, 1998; Elliott et al., 2000). The following are extracts of students’ written inferences using GC and GS when asked to relate the graph of  $f(x)$  to the graphs of  $y=x$  and  $g(x)$ . The corresponding related interview transcript is also presented.

- Stud1: (GC) The intersection of and the iterative formula represent the intersection of the x-axis.
- S1: (GS) Where the  $y=x$  and the iteration formula meet is where equation B meets the x axis.
- Stud2: (GC) The intersection between... and... will give you the value of the roots.
- S2: (GS) Where the  $y=x$  and the iteration formula intersect, we find the solution of the original equation, by looking at the x values of the intersects.

(Interview after using GS)

Researcher: Did your inference change?

S1: No. It didn't change! It's just faster... I've actually written down the x values.

S2: That's still the same innit?

S1: Yeah we knew that before...

S2: We knew it.

Although the study was limited in various ways, it did provide some tentative results, as in Villarreal's (2000) suggestion on inter-media coordination. Students in this study tend to get confused when solving for the iteration formula expressed in terms of  $x=g(x)$  on paper. Instead of recognising the concept of  $x$  being a variable that can be changed, students' tendency is to change the set-up in the two technologies where the default setups of the variable are  $y$ ,  $\Delta x$ , or  $f(x)$ . The computer is found to be better in this sense, since it can provide immediate feedback for any non-logical operation that students may input (audio transcripts below).

(Using GC)

GroupA,S2: How do you change  $y$ ?... in the calculator?

Teacher1: You have to change the  $x$  into  $y$ . What it's asking you to do is to get the iteration formula and graph it but you have to change  $x$  to  $y$ . Got it?...

GroupB,S4: How do you change  $y$  to  $x$ ?

S9: Basically  $y$  equals  $x$ ... It's  $y$  equals  $x$ , you change  $y$  to  $x$  so it's  $y$  equals  $x$ .

(Using GS)

PairA,S1:  $y$  equals  $x$ . do it you're faster...  $x$  squared plus  $2x$  minus fifteen equals zero. (The computer gave a feedback saying invalid equation entry)

S1: Oh yeah! Equals  $y$  innit?

S2: Equals zero.

S1: Are you sure? (typed). . . It's not there!

S2: What is it doing?  $f$  equals  $y$  innit? No! uhh (laughing a bit, S2 typed in  $y$ ). Is that right? (The graph appeared.) Cool! (Both laughed)

It appears that in finding the solution of  $f(x)$ , students prefer to solve it algebraically rather than graphically, supporting Knuth's (2000) findings. The evidence provided by the video data shows that students in this study normally prefer to check their conceived algebraic solution using the graphical calculator or the computer.

Given the limitations of this study, the implications drawn from it are deemed to be tentative. This study does not attempt to find the exact nature of the links between representations; moreover, it attempts to show the value of using conjectures as a way of researching issues concerning the understanding of maths and suggests more empirical studies are needed focusing on students' conjectures or thought processes.

## REFERENCES

- Aczel, J. C. (1998). *Learning equations using a computerised balance model: a Popperian approach to learning symbolic algebra*. Unpublished DPhil Thesis, University of Oxford.
- Dunham, P. and Osborne, A. (1991). Learning how to see: students' graphing difficulties, *Focus on Learning Problems in Mathematics*, **13**, 35-49.
- Elliott, S., Hudson, B. and O' Reilly, D. (2000). Visualisation and the influence of technology in A-level mathematics: a classroom investigation. In Rowland, T. and Morgan, C. (Ed), *Research in Mathematics Education*, Vol. 2 British Society for Research into Learning Mathematics, London, pp. 151-168.
- Even, R. (1998). Factors involved in linking representations of functions, *Journal of Mathematical Behavior*, **17**, 105-121.
- Hennessy, S., Fung, P. and Scanlon, E. (2001). The role of the graphic calculator in mediating graphing activity, *International Journal of Mathematical Education in Science and Technology*, **32**, 267-290.
- Kaput, J. (1991). Technology and mathematics education. In A., G. D. (Ed), *Handbook of research on mathematics teaching and learning* Macmillan, New York, pp. 515-556.
- Kaput, J. and Thompson, P. (1994). Technology in mathematics education research: the first 25 years in the JRME, *Journal for Research in Mathematics Education*, **25**, 676-684.
- Keller, B. and Hirsch, C. (1998). Students' preferences for representations of functions, *International Journal of Mathematical Education in Science and Technology*, **29**, 1-17.
- Knuth, E. (2000). Student understanding of the Cartesian connection: an exploratory study, *Journal for Research in Mathematics Education*, **31**, 500-508.
- Ruthven, K. (1990). The influence of graphic calculator use on translation from graphic to symbolic forms, *Educational Studies in Mathematics*, **21**, 431-450.
- San Diego, J. (2003). *The effects of technology on making conjectures: linking multiple representations in learning iterations*. Unpublished MSc Dissertation, The Open University.
- Sierpinska, A. (1994). Processes of understanding. In Sierpinska, A. (Ed), *Understanding in mathematics*, Ernest, P., Studies in Mathematics Education Series: 2, Vol. The Falmer Press, London, pp. 72-111.
- Villarreal, M. (2000). Mathematical thinking and intellectual technologies: the visual and the algebraic, *For the learning of mathematics*, **20**, 2-7.
- Weigand, H. (1991). Iteration sequences and their representations, *Educational Studies in Mathematics*, **22**, 411-437.