'PRE-EIGHTEEN STUDENTS HAVE LOST SOMETHING MAJOR': MATHEMATICIANS ON THE IMPACT OF SCHOOL MATHEMATICS ON STUDENTS' SKILLS, PERCEPTIONS AND ATTITUDES

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It is not surprising to hear and read mathematicians' consternation about the form and content of school mathematics. The high dropout rate at the AS phase (age 17) of A'level mathematics, seems to support the argument that we need to rethink school mathematics. In doing so mathematicians' views may merit more notice. We conducted a series of themed Focus Group interviews with mathematicians from six UK universities. Pre-distributed samples of mathematical problems, typical written student responses, observation protocols, interview transcripts and outlines of relevant bibliography were used to trigger an exploration of pedagogical issues. Here we elaborate the theme "On the impact of school mathematics on students' skills, perceptions and attitudes" that emerged from the data analysis.

INTRODUCTION

International evidence suggests [Wood in (Holton 2001)] that increasingly smaller numbers of students undertake university level mathematical studies. Beyond reasons such as a problematic relationship between mathematics and society [Thomas in (Holton 2001)], attention has recently turned towards the interface between secondary and tertiary education; in particular, the often difficult transition from school to university mathematics both in terms of content (Schoenfeld 1994) and teaching style (Wood ibid.). In England and Wales an emphasis on school mathematics as an instrumental activity (Tall 1997) and, specifically, a number of largely market-driven changes in the A'level mathematics provision (Hoyles et al 2003) have impressed certain images of mathematics on newly arrived university students which often have a dramatic impact on their skills, perceptions and attitudes.

The evidence we present in this paper elaborates upon this issue through drawing on the views of university mathematicians whose pedagogical views and practices have had to adjust considerably to the rapidly changing needs of their students - as, for example, numerous chapters in (Holton 2001) suggest. The four STORIES that follow - for an outline of the aims and methodology of the study see ENDNOTE - originate in a study currently in progress at the University of East Anglia, UK. Tutors A-E are five mathematicians, pure (two analysts, one algebraist) and applied, with experience in teaching varying from several years to several decades, all male and of rank varying from Lecturer to Reader.

Story I: '2,500 years of thought crashed in one afternoon'

(Cycle 1, 5 MINS)

Following an exploration of students' responses to a question that required the use of Proof by Contradiction, discussion turns to students' perceptions of the nature of mathematics as fostered by their school mathematical experiences. 'I wonder whether the students feel that they should always be calculating and that they are the calculations that count' suggests Tutor B. Tutor C recalls a student's disillusionment about the fact that this was a *mathematical* question, not 'some sort of philosophical discussion'. Tutor A adds the example of a teenager family friend who equated pure mathematics with statistics, 'a skewed view of the field'. Tutor E asks whether it is possible to leave school without having seen a proof – the group agree it is likely - and recalls his 'Pythagoras' horror story', GCSE students proving the theorem by counting the number of squares on squared paper: 'two and a half thousand years of thought crashed in one afternoon', he concludes in despair. Tutor A and Elena concur and then outline proofs of Pythagoras' Theorem which require minimal use of language and logic and involve a vivid pictorial representation (e.g. the Indian Proof). Tutor E recalls a student's reaction to the Indian Proof: this proof applies to *these* triangles. But does it apply to *all* triangles? Elena mentions research on generic examples. Tutor C observes that Pythagoras' theorem is the one that most people would associate with the word theorem - perhaps Fermat's Last too, adds Paola. Tutor B laments the demise of Geometry in the school curriculum which, like in the case of Pythagoras, provided good opportunities for proving (e.g. the expansion of squares). The conversation concludes with speculation about what still remains in the school curriculum (e.g. Pascal's Triangle?) and Tutor C's observation that the proof of Pythagoras' Theorem is perceived by students not necessarily as a 'proof of a theorem' but as a 'demonstration of a rule'.

STORY II: There remain 'few doable proofs', 'if you have given up on Geometry completely'

(Cycle 5, 5 MINS)

Following an exploration of students' attempts at a Proof by Contradiction that $\sqrt{2}$ is irrational, the discussion turns to whether this could be the only proof the students have seen at A'level. If this is the case, wonders Tutor A, then what is the point of this unique exposure to proof? 'If you have given up on Geometry completely', observes Tutor E, 'naturally' there remain 'very few doable proofs'. There is some co-ordinate Geometry left, says Tutor B. But it is mostly trigonometry, adds Tutor E. Binomial formulae can be proved by Mathematical Induction, suggests Tutor A. However only some students arrive at university having done this, responds Tutor B. Elena proposes that there is some implicit experience of proving in algebraic manipulation, starting from one side of an equality and ending up on the other – 'manipulative proof' as Tutor A labels it. Tutor B observes that students do not see that there are sentences implicit in algebraic expressions and that 'getting them to write down additional words or conditions is hard'. As in STORY I, the group speculate about whether this proof would have been seen at A'level and return to the point that, when bereft of all Geometry, all that is possible is the proof implicit in algebraic manipulation, e.g. in proving a trigonometric identity. Tutor A then contemplates on the rationale underlying the presence of the proof that √2 is irrational in the school curriculum: to explore the fundamental issue of the existence of irrational numbers (for some mathematical cultures such as the Greeks)? Or, as Tutor E suggests, to expose the students to an example of Proof by Contradiction? Tutor B points out that students are probably 'left with the fact' and do not necessarily 'perceive it as an important idea of proof as a notion'. The conversation concludes with Elena's observation that a proof that √2 is irrational would counteract its calculator-generated image as a decimal number. According to Tutor D, at least ten years ago, students would leave school at ease with the idea that √2 is irrational. However, he adds, this is not necessarily the case any longer.

STORY III: The 'utilitarian', 'Charles Clarke' approach to education 'has been tested to destruction'*

(Cycle 5, 11 MINS)

*at the time of writing Charles Clarke is UK Minister of State for Education

Following a conversation on students' understanding of Proof by Mathematical Induction (MI), Tutor A points out that its simplicity should imply its presence in school mathematics; yet it does not. Tutor E reminisces about his fascination with MI in an early school encounter. The group agree with Tutor A that, along with other forms of reasoning, consistent student failure at this type of exam question has led to a removal first from exams and gradually from the classroom ('an ornament that is a waste of time' says Tutor E). Is there 'anywhere else, in any other subject, say in the sixth form, where one learns something as powerful as the notion of proof as A-level maths was done?' Tutor B wonders. 'If that is gone, then pre-eighteen students have lost something major' he concludes. Tutor A agrees and laments the fact that A'level students may 'have never seen what might be beautiful'. The 'bizarre' task to introduce things like MI in Year 1 at university implies less content knowledge for mathematics graduates, therefore less mathematical confidence for prospective mathematics teachers, he continues. The group share anecdotes on the above until Elena asks them to consider a hypothetical scenario: how would you argue the case for a reinforcement of mathematical reasoning in school mathematics? Tutor E responds that an answer would depend on the audience you are addressing: the 'utilitarian, Charles Clarke approach', he explains, 'has been tested to destruction, and no-one is doing mathematics, or engineering or physics or chemistry. So, if the initiating of this question comes from someone who is concerned about the fact that no one is doing mathematics then there is a reasonable answer. If this comes from someone who doesn't care that no one is doing mathematics then there is no point talking to them.'. He then wonders: 'why not let someone with some scholarly view have an input'? Tutor A concurs and points out the need for a 'cultural change' where university studies is no longer estimated merely in terms of salary increase for graduates. Tutor B recalls his first ‘moving’ mathematical experiences as a teenager and Tutor E sums up the task as ‘making mathematics intellectually attractive’. Given that only 40% of mathematics teachers hold a mathematics degree – and given that ‘not all mathematics degrees are the same’ according to Tutor E – richer mathematical content would have implications for teacher education, the group agrees. Tutor A adds that there is a need to attract better mathematics graduates to the profession through better payment and social status as is the case in some other countries. Returning to the issue of mathematics as an ‘intellectually attractive’ subject, Tutor E suggests ‘it is less attractive because the audience is much tougher’. ‘We grew up in a time where […] delayed rewards were more acceptable’, he explains. The ‘sell’ that it may take ‘ten minutes of thinking before you get that reward for something aesthetically pleasing’, he concludes, ‘is becoming ‘harder and harder’. Does this apply to mathematics only? Elena asks. Tutor E agrees it does not but explains that Mathematics by its nature is a subject that cannot easily accommodate these ‘declines in attention span’: ‘it just doesn’t come in this soft, soft chain’. Tutor A adds Grammar as a conceptual domain that has suffered similar losses in school for similar reasons. Tutor E expresses a wish that the National Numeracy Strategy entails a richer school mathematical experience and the conversation concludes with Tutor B’s suggestion that a benefit of exchanging mathematical ideas in grammatically sound, ‘full sentences’ can imply a student’s overall enhanced power of persuasion in speaking and writing.

STORY IV: ‘If you are aiming for the B/C or C/D threshold you just omit chunks of the syllabus’

(Cycle 6, 3 MINS)

Following speculation on students’ previous knowledge in Linear Algebra, Tutor E opines that this knowledge consists of mostly instrumental elements. Tutor A, who says he recently scrutinised school textbooks, agrees there is little effort in these books to ‘try to create … concept images. It is just: you do this and then you do this and that’s it’ - with ‘no concern whatsoever’ for a student’s more ‘personalised attitude’, he adds. Furthermore Tutor E observes: ‘In the old days a bad teacher or a bad student did all the syllabus badly, and had in fact seen all of it’. ‘Whereas nowadays’, he adds there is ‘pressure’ from the ‘system’ to ‘just omit sections of the syllabus’. This applies to A’level and at GCSE it is ‘even worse’ especially ‘if you are aiming for the B/C or C/D threshold’. This is more significant than thinning the syllabus, he claims. Is this an immediate consequence of teaching in sets in primary school? Paola asks. But ‘the lower sets in primary school mathematics don’t end up doing A’level mathematics’ Tutor E conjectures. He is more concerned about people who ‘will come out with an A-level mathematics at grade B with a chunk of the syllabus that they never saw’.

CONCLUSION

According to the participants in this study, students arrive at university with a perception of mathematical thinking as primarily involving calculation. Their school mathematical experiences are seen as significantly responsible for this: textbooks foster an instrumental image and parts of the syllabus are often omitted to adjust to the perceived ability of weaker students; students rarely, if at all, see a mathematical proof (e.g. the irrationality of $\sqrt{2}$); and, with the demise of Geometry, a domain that excels in its capacity for accessible, short proofs, only an implicit experience of proving remains, e.g. in the context of Trigonometry and Co-ordinate Geometry, in the form of algebraic manipulation where, however, the logic and the overall valuable linguistic structures underlying mathematical reasoning are overwhelmed by the act of calculation. Simple proving techniques, such as Proof by Mathematical Induction, can be used as a demonstration of how intellectually attractive mathematical thinking can be. However in a cultural climate that favours instant gratification and where university studies are appreciated merely in terms of salary increase for graduates, convincing young people of the appeal of mathematics, an activity that has sometimes a slower turnover in terms of intellectual and emotional gratification, is a particularly difficult task. Failing to do so has resulted in a regrettable depletion in the numbers undertaking mathematical studies and opting for the profession of mathematics teaching. Improved social recognition as well as better earnings are vital if we are to address the now urgent need to attract better mathematics graduates to the profession.

ENDNOTE

This 15-month, LTSN-funded (http://www.ltsn.ac.uk) study engages groups of mathematicians from six institutions in the UK as educational co-researchers (Wagner 1997). There were 11 Cycles of data collection, six with five mathematicians from the University of East Anglia (Cycles 1-6), where the authors work, and five from elsewhere (Cycles 1X-5X). Six Data Sets were produced for each of Cycles 1-6 on the themes *Formal Mathematical Reasoning I: Students' Perceptions of **Proof** and Its Necessity; Mathematical Objects I: the Concept of **Limit** Across Mathematical Contexts; Mediating Mathematical Meaning: **Symbols and Graphs**; Mathematical Objects II: the Concept of **Function** Across Mathematical Topics; Formal Mathematical Reasoning II: Students' Enactment of **Proving Techniques** and Construction of Mathematical Arguments;* and, *A **Meta-Cycle**: Collaborative Generation of Research Findings in Mathematics Education*. The Datasets for Cycles 1-5 were also used for Cycles 1X-5X. Each Dataset consisted of: a short literature review and bibliography; samples of student data (e.g.: students' written work, interview transcripts, observation protocols) collected in the course of the authors' previous studies (http://www.uea.ac.uk/~m011); and, a short list of issues to consider. Participants were asked to study the Dataset in preparation for a Focus Group Interview - see Madriz (2001) and Nardi & Iannone (2003) for a rationale for using this tool for data collection. Interviews were digitally recorded.

The interviews from Cycles 1-6 were fully transcribed (the data from Cycles 1X-5X were used as supportive material in the analytical process). Each interview was about 200 minutes long and generated a *Verbatim Transcript* of about 30,000 words. In the spirit of Data Grounded Theory (Glaser & Strauss 1967) eighty *Episodes*, self-contained extracts of the conversation with a particular focus, emerged from a preliminary scrutiny of the transcripts and were transformed into *Stories*. These are narrative accounts in which we summarise content, occasionally quoting the interviewees verbatim, and highlight conceptual significance. The eighty *Stories* were grouped in terms of the following five *Categories*: *students' attempts to adopt the 'genre speech' of mathematics* (Bakhtin 1986); *pedagogical insight: tutors as initiators in 'genre speech'*; ***the impact of school mathematics on students' skills, perceptions and attitudes***; *one's own mathematical thinking and the culture of professional mathematics*; and, *the relationship, and its potential, between mathematicians and mathematics educators* (25, 25, 4, 20 and 6 *Stories* respectively). Here we focus on the third *Category*.

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