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TRANSFORMATION OF FUNCTIONS: LEARNING PROCESSES

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The topic 'transformation of functions' is commonly introduced, at least in the context of secondary Greek education where the study reported here is being conducted, in terms of the effects that the changes of parameters of functions have on their graphs. However, in my experience as a teacher, students, even at the later stage of upper secondary and further education, have substantial difficulties with the subject, especially in the lab courses. The focus of this paper is on students' construction of meanings concerning the structure of mathematical concepts, such as invariancy, while working in an IT-based environment of multiple representations. Two groups of students, engaged with a mathematical activity concerning the concept of transformation of functions and using a newly introduced piece of software were interviewed. Qualitative analysis of the interviews is currently in progress. The research reported here is part of a larger doctoral study.

THEORETICAL FRAMEWORK

Most research studies concerning the transformation of functions – mapping from R^2 into R^2 - offer variations of the standard school approach to the subject. Belonging to this category are the studies of Dreyfus and Eisenberg (1987), Guttenberger (1991) which discuss the altering of coefficients in a computer environment. These authors suggest a longer and more systematic establishment of connection between representations. A new approach to teaching transformations, by creating families of transformations, is offered by Borba and Confrey (1992). In his PhD Borba proposes a teaching experiment in which it is possible to knowledge constructed in different representations.

Conceptual development, as well as that of mathematical thought, is an interplay between concrete and abstract, between grounded activity and 'systematic inquiry' (Confrey 1993). The difficulties students face regarding the concept of the function, and transformation, seem to have their roots in the transition from the procedural (the notion is viewed as a process such as assigning values) to the structural conception (object on which operations may be performed) (Sfard 1991). Technology can be used to assist either or both of these aspects.

The present study was designed to examine, through a computer-based pedagogy, the potential of the computer to structure new concepts such as invariancy – points mapped onto themselves - and function of functions, as well as to bridge the gap between operational and structural conception. It aims to support the idea that the computer, as a tool, can contribute to understanding through assisting the dialogue between grounded activity and 'systematic inquiry' as well as through supporting the students in finding ways to develop structural conceptions (Kieran 1992).

METHODOLOGY

The research activity presented here was conducted as a pilot to the main study of my ongoing PhD. It is located within Action Research, ‘an approach to improving education by changing it and learning from the consequences of change’ as I was given the opportunity to introduce the teaching of Mathematics in a computer lab environment. The present research activity is ‘a close examination of the effects of such an intervention’ (Cohen and Manion 1994).

The setting

The institute where the research was conducted, is the newly founded, Higher School of Pedagogic and Technological Education (ASPETE) which is actually the only one operating in Greece. It offers a four year course and its graduates are given the opportunity to teach subjects of technological orientation in Technical Secondary Education (TEE). Technological subjects with two semesters of Mathematics cover 70% of the curriculum while pedagogic subjects cover the remaining 30%. As Head of the Department of General Subjects I introduced the teaching of Mathematics in a computer lab environment, for the second semester with mandatory participation as a prerequisite. Since during the month of October exams were in progress and the computer lab was not available, the present research was conducted in my office.

The sample

Four volunteer students, two female and two male, from the Electrical Department, who had successfully completed the second semester, formed two teams for the purpose of the present study. Their selection was based on their computer literacy and mathematical performance. The criteria used were: their grades in Laboratories of Mathematics and Information Technology, their Mathematics grades in the National Entry Exams and their performance in Mathematics during the two semesters at the school. The students were placed in two teams, ‘good’ and ‘average’.

The tool

The exploratory software was Function Probe, a multirepresentational software for exploring functions. It sustains the dialectic between grounded activity, for example direct act on transformations, and systematic inquiry that ‘stabilizes-extends the mathematics use’, for example calculator or table (Confrey 1993).

The activity

The research activity lasted four hours for the ‘good’ team and six hours for the ‘average’ team. It involved the Wheel in an amusement park, its cyclical movement and the distance of the passengers from their various positions to the platform. So that the students gain personal experience of the subject, I suggested visiting the amusement park near by, but the girls were frightened and the boys had already been there. Although the pre-test and post-test, focusing on the function and some of its transformations, were given to detect previous knowledge and possible changes, the time period allocated to the research activity was rather limited to observe alteration in perceptions. Both teams spent four hours training by studying a specific activity as well as exercises on function transformations.

Data collection and analysis method

The data were collected as follows: recording of the conversation in three cassette recorders, diary keeping with notes on my part, computer print outs of student work, saving files on disc, students' worksheets, personal semi-structured interviews (life story) of students (Goodson and Sikes 2001), meta-analysis (a discussion of the concept and the experience with the students).

After each session summaries and verbatim transcript were constructed with support from the materials mentioned above. Following a phenomenographic approach (Marton 1993) I attempted formulating categories that described the students' conceptions.

The data were looked at in terms of the following three phases: prior, during and after the activity.

RESULTS

The instances that will be discussed below are taken from the discussion that took place after the activity during the meta-analysis.

At the beginning of this session the question posed to the students concerned the title of the lesson.

(R=Researcher / good team: D=Dimitra, J=John / average team: C=Christina, L=Lysimahos)

R: What would you write on the table?

L: A function.

R: Function?

L: Equation.

R: Equation?

L: An equation, function.

Lysimahos immediately replies that the title of the lesson would be a 'function'. My question however makes him reconsider and give an additional different answer 'equation'. My second question seems to prompt him towards including both.

It is quite possible that what Lysimahos had in mind is that function and equation, which he had been taught at lower levels in school, are identical concepts. According to the literature on students' concept images of function (Tall 1992) this is a common occurrence often attributed to the way the concept is taught.

The next instance refers to how the students perceived the mathematical content of the activity and the suggested title.

D: What can I say now. Function is not …The first questions are not questions. No connection to function. The function begins from the moment you get it out and then start doing what we call transformations.

R: From which question and on did you say?

D: From the point that we get it out.

R: Which question is that?

D: From the point that the function is altered and on, that is question 4 and on.

R: So what you are saying is that from 4 and on it is a function.

D: Yes! In the beginning it's … nothing . It's nothing. It is simply the rule of three, a very simple thing.

On the occasion of the title for the activity Dimitra clarifies how she personally understands the function. She defines that ' it starts from the moment you get it out and then start making what we call the transformations'. What she possibly means by 'get it out' is that we 'construct the equation' and that the function involves 'transformations'. Then she matches the questions of the activity to the mathematical concepts: the first three (construction of function-table and -graph) are rule of three, and from the fourth and on the function 'starts'. At this point Dimitra does not seem to consider this process as a function since it is something so 'simple'. Just 'doing' is to her 'nothing'. On the contrary question number 4 (writing-graphing the equation of the height from the platform and checking it with the graph-table values), which requires typing of the equation of the function, is fundamental to her.

What I observe here is that Dimitra's prevailing perception of function is the algebraic equation as an object and not the mathematical manipulation, the process within the 'rule of three'. In the discussions conducted during the activity the students revealed how influential their primary and secondary school experiences of the concept had been on their current images. Specifically the image of the equation, taught at the end of primary education (6-12 years of age), after the arithmetic operations, is supplemented with the function at the beginning of Gymnasium (12-15 years of age). So the algebraic equation is one of the representations that carries special weight in the school environment.

The next instance refers to question function of functions (change of graph-table-function while altering the speed of the wheel, the diameter etc). The question was exploratory: how would you teach this topic?

J: I believe as a title, something like that, transformations, basically, function transformations, now for the specific exercise it's best transformations-graph, that is, the form we did to the Function and saw when it went ------ and when ----- when it ---- forward-backward, transformation is that of the graph.

And

C: We have a table, we have its graph and we also have an equation, the equation resulting from the two and then we make some changes…

L: That the exercise suggests here.

C: To the height, the speed and whatever it says … and to the diameter and see the changes in some that already exist on the table, the graph, the equation.

John and Christina describe their perceptions of the exercise. Both students see only one function and the changes this goes through. They cannot yet see that one can create new functions through transforming an initial function, for example through a translation – e.g. via altering the height of the Wheel from the ground.

Lastly regarding phase III the concept of transformation as function of functions was approached only as a 'process'.

Finally I refer below to some instances from the discussion of the question about invariancy (installation of extra wheels with the same diameter, and meeting point at t=2sec at the same height from the platform). In these a student's perceptions of the invariable point of transformation are revealed.

D: At first it was zero (the anchor) and as we were moving it (the hand) the graph… Then, when we took the anchor and moved where we wanted in 2 started to move around 2.

And

D: when you read it, the picture that comes to your mind is this: it's where the three wheels meet at the same point and its like them moving together, but than one will move on, the other will stay, more behind.

Dimitra, the top student who since the first year of the Lyceum (15-18 years of age) enjoyed working on mathematical problems and who had never been taught the transformations and invariancy, describes to the other three students what happened when she moved the 'anchor' (part of the Function Probe software used in the activity) to point t=2 and what image the instructions of this exercise created in her mind. According to her description point t=2 is a meeting point. Prior and after that each of the functions has its own 'course'.

These instances describe very distinctively the concept of invariancy: immobility in mobility.

As far as phase II is concerned, in my view the fact that out of four students one managed intuitively to 'see' the invariable point and combine in one image the static and dynamic nature of the concept of transformation, is quite encouraging.

CONCLUSION

The above initial analysis of the data of this brief study seems to support the following:

a) the strong relationship between the variety of students' concept images and the concept definition (as described by previous studies such as Tall (1992)

b) a probable contribution of the computer, as a tool, in forming a path to understand the new concept of invariancy (Confrey 1993)

c) the bibliography on students' perceiving the newly introduced concept function of functions operationally (Sfard 1991)

As this pilot study was small in scale (in terms of time devoted to the activity and in terms of the number of students involved), further phases of the research will examine the validity and potential refinement of these findings. Particular attention will be given in terms of length and numbers of student involvement, a more thorough linking between the enactive and symbolic aspects of the students' understanding of functions, the diversity of activities to be looked at in different settings and the more detailed character of the clinical observations**.**

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