ERRORS AND MISCONCEPTIONS IN KS3 'NUMBER'

Chris Bills

University of Central England

In June 2003 Y7 pupils in five schools completed a test based on questions first used in the CSMS and APU studies. The aim was to collect data in order to inform new teachers about pupils' common errors and misconceptions. The data may also be used for purposes of comparison with results collected by teachers in other schools. The study indicates that in this sample a high proportion of pupils gave correct answers but that there were significant numbers of pupils with misconceptions. In a separate study Y7 pupils in one school were asked to perform written calculations in order to identify the strategies used. Accuracy levels were low for multiplication and division and there were a wide variety of non-traditional strategies.

INTRODUCTION

The National Numeracy Strategy (NNS) has influenced teaching and learning in primary schools in England since September 1997. The KS3 Strategy has encouraged secondary school teachers in England to adopt an approach to 'number' which complements this. In both strategies the emphasis is in on 'teaching for understanding' where mental calculation is a first resort. This is in contrast to the previously more common approach of emphasising written algorithms as a foundation for calculation.

The CSMS tests (Hart, 1982 and 1980) and APU surveys (e.g. Foxman, 1981) gave indications of facility levels for items but gave little analysis of errors. The purpose of the test devised for this study was to help identify common errors and misconceptions to inform the teaching in the schools involved and to provide a database of the prevalence of mistakes and misconceptions. A total of 879 Y7 pupils were tested

The results suggest that there are widespread common errors and misconceptions but also that the schools varied widely in the distribution of these mistakes. For one question the most common error was given by 60% of the pupils varying between 40% and 80% for the five schools. In many other questions between 20% and 40% of pupils gave the common error.

In a separate study 220 Y7 pupils in a high achieving school were asked to perform four calculations on paper (376 + 248, 376 – 248, 46 × 57, 1000 ÷ 7) to identify the common strategies that might be prevalent in the post-NNS era. Nearly 80% of the pupils used a compact algorithm for addition, 76% for subtraction, 30% for multiplication and 39% for division. The accuracy levels were 90%, 72%, 18% and 27% respectively. Formal and informal methods seemed equally prone to common errors but pupils who used algorithms more frequently gave correct answers. There were some differences between pupils in different sets (each half-year grouped by achievement level) in both accuracy and methods chosen.

METHOD

Five schools were chosen at random for the 'errors and misconceptions' test but proved not to be a representative sample in that they were collectively above national average in terms of KS3 test results and GCSE performance. The schools varied in size: (172, 269, 144, 167, 127 pupils involved in test) and achievement levels:

KS3 level 5 and above (79%, 75%, 65%, 76%, 80% - overall 75%
national average 67%)

GCSE A*-C (62%, 51%, 36%, 55%, 67% - overall 54%
national average 52%)

Pupils were allowed 20 minutes for the 'errors and misconceptions' test consisting of 27 questions. Answers were recorded for each pupil and proportions of each answer calculated. If more than 3% of pupils in any school gave a wrong answer it was deemed a potential 'common' error. The results reported here are for those answers that were given by at least 7% of pupils overall.

Pupils were allowed unlimited time for the 'calculation method' test. Pupils were given an alternative activity when they had completed the test so that all could have as much time as necessary to complete the calculations. The questions were written horizontally. Pupils were asked to write something to show how they had calculated if they chose to do it in their head.

RESULTS – ERRORS AND MISCONCEPTIONS

Only the most striking results are given here with a brief comment. The total percentage for all schools is given with the range across the five schools in brackets

Write in figures: four hundred thousand and seventy-three.

400073 63% (59-70) 40073 14% (11-19) 4073 10% (8-11)

This compares with 42% correct in the CSMS tests. Notice that in one school nearly 20% gave 40073.

Work out 0.8 + 0.71

1.51 56% (46-61) 0.79 28% (24-33)

A wide variation between schools and up to a third of pupils ignored the decimal point.

Ring the number which has the smallest value: 0.625 0.25 0.375 0.125 0.5

0.125 46% (39-51) 0.625 28% (22-39) 0.5 16% (10-22)

In the APU tests for 15 year olds: 0.125 37%, 0.625 34% 0.5 22%

Half of the pupils in one school gave the correct answer. 'Longest is smallest' and choosing the largest where decimals of the same length are sorted, identified in the APU surveys (Foxman, 1981), is clearly still a common misconception.

Add one tenth to 2.9.

3, 3.0 or 3.00	51% (46-55)	3.9	18% (15-22)	29	8% (5-15)

This again compares favourably with the 38% achieved in the CSMS test. Notice that nearly a quarter of the pupils in one school added one.

Work out 0.3 x 0.2

0.06	11% (5-27)	0.6	60% (40-80)	0.5	8% (4-15)

This question was remarkable for the low facility level and the wide variation between schools in the proportion of pupils giving 0.6 as an answer. This ranged from 40% of pupils in one school to 80% in another.

Round 6.748 to 1 decimal place

6.7	28% (19-35)	67.48	18 % (10-21)	6.8	10% (7-15)

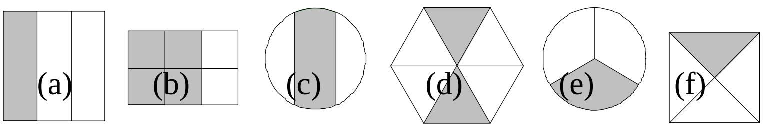
Over 20% of pupils in one school seem to have 'moved the decimal point one place'. Rounding successive decimal places was only used by 10% of pupils

Ring the one which gives the BIGGER answer in each pair:
(a) 8 × 4 or 8 ÷ 4 (b) 8 × 0.4 or 8 ÷ 0.4 (c) 0.8 × 0.4 or 0.8 ÷ 0.4

× , ÷ , ÷	12% (9-14)	× , × , ×	39% (31-46)	× , ÷ , ×	21% (18-26)

In the original CSMS test 13% gave the correct answer and 50% assumed 'multiplication makes bigger'. Pupils in this sample had a wider variety of errors.

Tick all the shapes which have ⅓ shaded.



ade	21% (17-25)	ace	29% (25-38)	acde	26% (22-29)
CSMS:	21%		30%		26%

Here proportions have remained similar over the last 20 years. The common error is to pick those shapes divided into three parts.

Zoë and Ahmed have pocket money. Zoë spends ¼ of hers and Ahmed spends ½ of his.

Is it possible for Zoë to have spent more than Ahmed? Why do you think this?

Yes (a)	55% (44-64)	Yes (other)	13% (6-19)	No (d)	21% (18-25)

55% indicated that the amount of money was important. 21% said No, because ½ is bigger than ¼. A total of 27% said No. In the CSMS test 42% gave No.

Write down a fraction between 1/2 and 2/3 .

Correct	20% (16-27)	1/3	24% (18-31)	1/4	7% (6-7)
CSMS (age 14)	26%		27%		10%

This question also illustrates that misunderstanding in ‘fractions’ has changed little but here the comparison is between 12-year-olds now and 14-year olds 20 years ago.

How many fractions are there between 1/4 and 1/2 ?

	infinite	6% (4-10)	1	18% (13-24)	2	16% (13-18)
CSMS (age 15)		16%		27%		10%

The school achieving highest here was the lowest scoring school overall. Perhaps this concept had featured in teaching more in this school than elsewhere.

Write 1/5 as a decimal

0.2	35% (25-41)	1.5	18% (13-26)	0.5	17% (14-25)

A quarter of the pupils in one school replaced the ‘/’ for a ‘.’, and a quarter in another school seem to have simply placed the denominator after the decimal point.

A piece of ribbon 17 cm long has to be cut into 4 equal pieces. Tick the answer you think is most accurate for the length of each piece: (a) 4 cm remainder 1 piece (b) 4 cm remainder 1cm (c) 4¼ cm (d) 4/17 cm

	(c)	31% (27-35)	(b)	39% (34-43)	(d) or (a)	14% (11-18)
CSMS		43%		37%		19%

Nearly 40% of pupils gave 4 rem 1 as the most accurate now, and in the CSMS test.

Work out 1/5 + 3/10

5/10 or 1/2	27% (14-40)	4/15	44% (28-59)	omitted	15% (9-21)

A high proportion of pupils added numerators and added denominators, but with wide variation between schools. In the CSMS test 19% of pupils did this in a similar question.

Work out 5 ÷ 13. Give your answer as a fraction

5/13	8% (5-18)	2/3	10% (8-19)	omitted	37% (27-58)

Pupils had a wide variety of wrong answers and many chose not to answer.

$^{-}8 + {}^{-}3$ = ……….

$^{-}11$	40% (36-48)	$^{-}5$	30% (26-39)	11	8% (2-10)

Over a third of pupils in some schools ignored the sign of the numbers in some way.

$^{-}8 - {}^{-}3$ = ……….

-5	45% (42-50)	-11	30% (26-42)	5	5% (2-7)

In a similar CSMS item 44% gave -5 and 37% one of the other answers.

RESULTS – CALCULATION METHODS

The test was administered in only one school. The percentages of pupils giving correct answers, and the methods used, were as follows:

Question	Correct	Algorithm	Other method	No attempt
376 + 248	90%	79%	20%	1%
376 – 248	72%	76%	23%	1%
46 × 57	18%	30%	64%	6%
1000 ÷ 7	27%	39%	34%	27%

It is worth noting that although 96 % of pupils attempted the multiplication 11% gave no answer. For division 40% of pupils gave no answer even though they had attempted it.

The percentages of correct answers using the different methods were as follows

Question	Correct - Algorithm	Correct -Other
376 + 248	89%	96%
376 – 248	81%	44%
46 × 57	53%	12%
1000 ÷ 7	69%	24%

This suggests that pupils are more accurate when using the formal methods.

Common errors (over 7% of sample) occurred in the multiplication question. The most common non-algorithm strategy (26% of the pupils) was to multiply tens by tens and units by units. This gave the answer 2042 for 11% of the pupils. The answer 242 was given by 40 pupils (15% of the sample). Of these 11 used the compact vertical algorithm whilst 19 attempting to multiply tens by tens and units by units and made an error in place value.

There was a tendency for low sets to be less accurate but the lowest groups, who had concentrated on ‘number’ in lessons, performed as well as the highest groups in multiplication. Nearly 60% of pupils in the lowest group in each half year adopted the ‘grid method’ for multiplication in comparison to 5% of the rest of the pupils.

For division nearly three quarters of pupils in the two highest sets chose to use the compact algorithm whilst a quarter of pupils in the other sets did this. There were no common errors – the most frequently occurring wrong answer was 142.6, given by 4% of pupils

CONCLUSION

This set of results gives an indication that pupils are prone to common errors even when teachers have adopted KS3 Strategy approaches to calculation. Performance in calculations with fractions, decimals and directed numbers was generally better than that achieved by pupils 20 years ago but the sample in this study was above national average achievement in national tests. Where details of errors made in the CSMS and APU tests are available they show similar proportions of pupils making the same mistakes then as now.

Written calculation has not been a high priority in the school taking part in this study and teachers in this school were of the opinion that the low levels of accuracy in multiplication and division reflected the low time allocation to this part of the curriculum. Since pupils had not practised their written calculation skills (with the exception of the lowest two sets) it is interesting to note that addition and subtraction were most frequently attempted using a standard written algorithm even though these might have been done using informal jottings. In contrast multiplication and division were more frequently attempted using informal methods.

There are implications for the schools who took part in the tests but other teachers could try the same items in order to ascertain levels of misunderstanding prevalent in their own schools. The questions and their answers provide teachers and pupils with opportunities for learning.

The test and analysis sheet for the ‘errors and misconceptions’ test are available on request (chris.bills@uce.ac.uk).

REFERENCES

Bell, A., Costello, J. (1983). *A review of research in mathematical education / prepared for the Committee of Inquiry into the Teaching of Mathematics in Schools. - Part A : Research on learning and teaching*. Windsor: NFER-Nelson

Dickson, L., Brown, M., Gibson, O. (1984). *Children Learning Mathematics*. Eastbourne: Holt, Rinehart & Winston.

Foxman, D.D (1981) *Mathematical development : report on the 1979 secondary survey from the National Foundation for Educational Research in England and Wales to the Department of Education and Science, the Department of Education for Northern Ireland and the Welsh Office*. London: HMSO.

Hart K. M. (1980) *Secondary school children’s understanding of mathematics: a report of the mathematics component of the concepts in secondary Mathematics and Science Programme*. London : Chelsea College of Science and Technology

Hart K. M. (Ed) 1981, *Children's Understanding of Mathematics 11-16*. London: John Murray