AN INVESTIGATION INTO THE MATHEMATICAL KNOWLEDGE OF PRIMARY TEACHER TRAINEES

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The U.K. government's National Curriculum for initial teacher training and the associated set of assessment standards both include a focus on mathematical subject knowledge. This paper reports collaborative research into the mathematical subject knowledge of primary student teachers by a group of researchers from four English Universities.

INTRODUCTION

Prospective primary school teachers in England and Wales need to have certain minimum qualifications in mathematics before entry to teacher training courses. However, students may have attained such qualifications several years before training when syllabuses were different, or may have been entered for tiers with less curriculum content, or attained the grade without consistency across all aspects of the curriculum. Additionally, the knowledge required to meet the public qualification standard, even if it is judged to be very good by that standard, may need to be transformed and enriched in order to support the act of teaching.

The SKIMA (Subject Knowledge in Mathematics) group is a collaboration between researchers in the Universities of Cambridge, Durham, York and the Institute of Education at the University of London. It grew out of a common interest in primary teacher trainees' subject knowledge in mathematics predating the introduction of the government's (DfEE, 1998) National Curriculum for Initial Teacher Training. This paper will discuss some of the specified knowledge and understanding which the government deemed to underpin the effective teaching of primary mathematics, the way in which the institutions investigated and addressed weaknesses in this knowledge, the self assessments made by the trainees and the link between this knowledge and teaching competence

Building upon previous work (Rowland, Martin, Barber and Heal, 2000; Goulding and Suggate, 2001) the researchers devised a common procedure for use with over 400 primary trainees in the different universities. It involved an early self-audit, a period when specific teaching was given and/or students could follow up areas of weakness, an audit taken in formal conditions and a follow up period when peer teaching was used. The process was designed in part to yield research data, which would give further insights into students' strengths and weaknesses and their feelings about the process.

SUBJECT KNOWLEDGE IN MATHEMATICS

The conceptualisation of subject knowledge and its relation to teaching which informed the project has been detailed extensively elsewhere (Goulding, Rowland and Barber, 2002). For the purposes of the self-audit and the audit, Shulman's construct of *subject matter knowledge (SMK)* ‘the amount and organisation of the knowledge per se in the mind of the teacher’ (Shulman, 1986, p9) later analysed further (Shulman and Grossman, 1988) into *substantive knowledge* (the key facts, concepts, principles and explanatory frameworks in a discipline) and *syntactic knowledge* (the rules of evidence and proof within a discipline) were influential.

Clearly we were constrained into using the government’s list of content. For instance, one of the substantive topic areas, equations, functions and graphs, included ‘understanding the significance of gradients and intercepts’ (DfEE, 1998, p61). Although primary children would not be expected to cover the equation of the straight line, probing trainees’ understanding of this equation *could* reveal their appreciation of the relationships between symbols and graphs, and the power of visual representations. It could be argued that such appreciation is relevant to the task of teaching primary mathematics. Similarly, teachers are now required to sow the seeds of proof and algebraic thinking in the primary years. Trainees, therefore, were expected to ‘[follow] rigorous mathematical argument’ and ‘[be] familiar with methods of proof’. (DfEE, 1998, p62) In devising the audits and planning teaching we were able to choose our own questions and activities and in so doing tried to keep the relationship between these and the primary curriculum in mind.

Elsewhere (Goulding, Rowland and Barber, 2002) we have highlighted weaknesses identified in substantive knowledge and also the particular difficulties which trainees in previous cohorts had with generalisation and proof. We interpreted these difficulties as a weakness in syntactic knowledge, an inability or unwillingness to make and test conjectures by personal investigation. Elements of weaknesses in other substantive areas were found by other researchers working with the same requirement to audit and remediate primary UK teacher trainees’ mathematical knowledge, but with different audit instruments. For example, Sanders and Morris (2000) found problems in all areas of the curriculum and Jones and Mooney (2002) found particular weaknesses in geometry.

The relationship between SMK and the pedagogical content knowledge (PCK) (Shulman, 1986) required for teaching is still not fully understood. For instance, in the United States, Ball et al (2001) acknowledge that we have ‘an insufficient understanding of the mathematical knowledge it takes to teach well.’ In the UK, Carol Aubrey’s (1997) small scale but in-depth study led her to argue for the ‘central importance of disciplinary knowledge to good elementary (primary) teaching’. In the larger Effective Teachers of Numeracy Project at Kings’ College, London (Askew et al. 1997) the teachers whose pupils made the greatest gains in test scores were described as having ‘knowledge and awareness of conceptual connections between the areas which they taught’, without necessarily having advanced mathematical qualifications. In one of the SKIMA institutions, the Institute of Education, the relationship between SMK and teaching performance in number as judged by observing tutors was investigated for two cohorts of primary trainees in 1999 and 2000. In both years, an association between mathematics subject knowledge as assessed by the audit and competence in teaching number was found, with a particular risk associated with trainees with low audit scores (Rowland, Martyn, Barber and Heal, 2000, 2001).

Promising insights into the way in which a combination of SMK and PCK can inform teaching are now emerging (Huckstep, Rowland and Thwaites, 2002). Trainees who have several representations for mathematical ideas and whose knowledge is already richly linked will be able to draw upon these both in planning and in spontaneous teaching interactions. In such cases we have argued (Goulding et al. 2002) that the trainees' SMK is ripe for exploitation and that insights gained by such trainees in their teaching will feed back into and enrich SMK. The boundaries between SMK and PCK may well be blurred.

METHOD

Early in the course, all trainees undertook a self-audit (21 items) in their own time. They then consulted a commentary and support materials, and completed a self-report form with judgements of their responses to each item using a five-point scale from 0 ('I couldn't attempt this question without help') to 4 ('My response was completely secure'). At the end of the form they were asked to 'add any general comments about your mathematical subject knowledge that may be of help to your tutor'. Of the 432 trainees completing the self-report, 274 (64%) added such comments.

The audit consisted of 16 items on number and algebra, mathematical proof, measures, shape and space and probability and statistics, each marked on a five point scale from 0 (not attempted, no progress towards a final solution) to 4 (completely secure with convincing and rigorous explanations not necessarily using algebra). This ordinal scale coded responses for the purpose of formative feedback, with a crucial boundary between 2 and 3, since <3 advised further study. Criteria for 0 to 4 specific to each item were mutually agreed, piloted and then refined.

The data obtained from the self-audit and the audit were analysed statistically; the self assessment comments were read and re-read for common themes, from which coding categories were developed and checked by two researchers.

FINDINGS

In the self-audit, the items on reasoning and proof, identified as problematic in the previous research, did not have particularly low mean ratings and were rarely mentioned in the trainees' comments. The students were more concerned about terminology, shape and space, and the equations and graphs of straight lines. Many did not know the terms associativity and commutativity, which almost certainly accounted for the difficulty with the number operations question. Similarly the terminology of transformations may have accounted for difficulties with one of the Shape and Space questions. In the graph question the word gradient may have been the problem but it also seems likely that seeing the connection between the graph and the equation was a source of difficulty.

In the formal audit, two low scoring items on reasoning and proof did accord with the previous research. Perhaps students had not addressed reasoning and proof adequately because they had not identified this area as problematic in the self-audit. Perhaps the questions used in the audit were more probing than those on the self-audit. The item on transformations similar to that on the self-audit also had a low score, even though students had earlier identified this and the associated terminology as difficult.

The difficulties with terminology in the number operations, identified by students on the self-audit, had been resolved by the time of the formal audit, and the graph problem was tackled more successfully when set in a ‘real life’ context. In both cases this later success may have been a feature of students’ improved understanding or a feature of the item itself. In some cases, self-assessed difficulties seem to have been resolved and in others they persisted.

Most of the students who commented about their confidence were either confident in all or most areas or were confident that they could update their knowledge. In terms of knowledge, most felt rusty or out of date, or felt that their knowledge was patchy. Of those who commented, the majority acknowledged the need for revision but fewer said exactly how they intended to go about it. It seems likely that many were relying on course provision. In specifying difficulties, there were common patterns in the specific items identified and in the generic difficulties across items. This helped tutors in the course provision between the self-audit and the audit, but it is not clear if the students acted upon their own self-assessments.

There was an identifiable but small group (10%) of students who reported particular concerns in their comments on the self-audit. Their responses were characterised by emotional language and sometimes reflected negative learning experiences in the past. Half of these students also had low self-ratings, but there were also students with high self-ratings who expressed concern and students with low ratings who expressed no concern. Being able to express their concern at this stage in the process may have been helpful to some, since tutors were alerted and could respond accordingly. The choice of peer support groups and peer tutors was made with these considerations in mind and seemed to be successful in boosting the confidence of weak trainees and also that of the stronger trainees who acted as peer tutors (Barber, Heal and Martyn, 2002)

CONCLUSIONS

The audit items on reasoning and proof demanded very little technical expertise but did require the ability and willingness to investigate a situation, look for general patterns, make conjectures and try to justify them i.e. expertise in syntactic knowledge. This change in orientation may be too much to achieve in the one year PGCE course. The weaknesses in the shape and space items running through both the self-audit and the audit included transformations, an element of substantive knowledge. Difficulties with transformations should be of direct concern to primary teachers and clearly still needs addressing. Although we were technically able to pass all the students after follow up work we still have concerns about aspects of both substantive and syntactic knowledge.

The fact that so many students seem to take a fairly sanguine view of the whole auditing process is encouraging although some of them may be too complacent. The identifiable group with particular worries is still of concern. Voicing their concern may have been helpful to these students and it continues to remind tutors of the need for sensitivity when handling mathematical subject knowledge. In the same way that aspects of substantive and syntactic knowledge may take longer than one year to develop, developing confidence and positive attitudes will almost certainly be a long-term project for these students. It would be interesting to see if the students with low self-ratings who also express concern have more difficulty in developing confidence than those with high ratings.

Postscript

During the period of this study, government inspectors claimed to find that trainees' mathematical subject knowledge had improved substantially, attributing this partly to more systematic and less superficial auditing of subject knowledge. The new regulations (TTA, 2002a), however, do not specify a body of knowledge or require an audit, but one of the assessment standards is 'a secure knowledge and understanding of the subject(s) [the trainees] are trained to teach.' The non-statutory handbook (TTA, 2002b) suggests that the source of evidence for this standard 'is most likely to be found in trainees' teaching, particularly in how they present complex ideas, communicate subject knowledge, correct pupils' errors and in how confidently they answer subject-based questions' (part 1, para. 2.1, p11).

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