Williams, J. (Ed.) Proceedings of the British Society for Research into Learning Mathematics 23(3) November 2003

AMBIGUITY IN MATHEMATICS CLASSROOM DISCOURSE

Richard Barwell

University of Bristol

Ambiguity is generally seen as problematic in mathematics and this view may also arise in mathematics classrooms. The national numeracy strategy, for example, advises teachers to 'sort out' any ambiguities in students' mathematical language. In this paper, I offer a discursive analysis of a transcript in which ambiguity can be seen as an important resource for making sense of the concept of dimension. This in turn raises questions about the role of ambiguity in doing mathematics.

INTRODUCTION: AMBIGUITY AND THE NNS

Mathematics, particularly school mathematics, is seen as a subject that avoids ambiguity. If ambiguity is the appearance of two or more possible meanings for a word or other linguistic expression, then mathematicians aim for precision, clarity and unique meanings for the words and expressions they use. Pimm (1987) characterises this perception of mathematics as follows:

> …there are right and wrong answers to everything, together with clear-cut methods to be taught and learnt for finding them. So how can mathematics be discussed when there is no place for opinion, informed or otherwise? While there might be open problems at the frontiers of mathematics, it is all sorted out and written down at the school level (p. 47).

The 'clear-cut methods' Pimm refers to include the careful definition of terms and notation. Definitions are designed to precisely delineate what a term does and does not refer to.

The *National Numeracy Strategy* (DfEE, 1999) appears to subscribe to the preceding perception of mathematics as an ambiguity-free zone. Guidance on teachers' use of mathematical language (DfEE, 2000) appears as part of a booklet entitled *Mathematical Vocabulary*, which consists largely of lists of words designed to be suitable for each year-group from Reception to Year 6. In a brief introductory section entitled 'How do children develop their understanding of mathematical vocabulary?', teachers are provided with three paragraphs of advice to support them in their use of language in the mathematics classroom. The main points of this advice are:

- children need supporting to move on from 'informal' to 'technical' language in mathematics, and from hearing and speaking new vocabulary to reading and writing,
- teachers should ascertain the extent of children's mathematical vocabulary and the depth of their understanding,
- the introduction of new vocabulary is important and teachers should structure carefully how they do this.

The underlying model on which the guidance appears to be based sees mathematical meaning as separate from language and views technical terms as being used to convey precise mathematical meanings. This model is illustrated by the following extract:

> You need to plan the introduction of new words in a suitable context...Explain their meanings carefully and rehearse them several times...sort out any ambiguities or misconceptions your pupils may have through a range of...questions (DfEE, 2000, p. 2).

Thus, whilst ambiguity may arise in the mathematics classroom, it is with the students, rather than as an inherent part of mathematical talk. Ambiguity, furthermore, is similar to misconception, with connotations of error, of misunderstanding, of not having fully mastered the discourse.

A DISCURSIVE PERSPECTIVE

The perspective outlined above is based on a rather formal model of the relationship between language and learning. An alternative view looks at language as it is used (see, for example, Edwards, 1997; Gee, 1999). From this perspective, for example, rather than examining whether students or teachers use a mathematical term 'correctly' or not, in accordance with its definition, interest is in *how* participants use such terms, and what they use them to do. Thus, mathematics classroom interaction is examined in terms of the discursive practices of the participants. In considering the role of ambiguity in mathematics classroom interaction, therefore, the aim is to understand how ambiguity arises for participants, how they deal with it and what it does in relation to the mathematical (or other) work of the discussion.

In the conference session, I offered an opportunity to explore the role of ambiguity in the teaching and learning of mathematics, through examination of an example of mathematics classroom interaction in which ambiguity becomes a relevant concern for the participants. The interaction was presented in the form of a transcript of the first few minutes of a Year 5 mathematics lesson, in which the concept of 'dimension' is discussed. The full transcript is reproduced in an earlier BSRLM proceedings (Barwell *et al.*, 2002). In this paper, I present and briefly comment on one short exchange from the transcript, before outlining some aspects of the discussion that emerged at the conference.

DISCUSSING DIMENSIONS

The teacher (T) began the lesson by introducing the topic of 2 and 3 dimensions. After some rehearsal of properties of 2D and 3D objects (e.g. length, breadth etc.) there is a discussion of the nature of 1D and 0D objects. In the following short excerpt, the teacher has just turned to a set of brightly coloured plastic shapes (see [1] for transcript conventions).

41	T	I don't like these (…) coz they look like three dimensional don't they. They're thick but they're not meant to be, they're meant to be two dimensional. Okay, they're flat shapes (*picks up a square*)
42	?	A cylinder
43	T	Yeah that's a cylinder (*laughs, waves circle*) (and that's a)

44 ? a cuboid

45 T cuboid (waves square). But it’s not meant to be it’s meant to be flat. Yes K.

46 K There’s no such thing as a one dimensional shape coz a line is kind of like a rectangle filled in

47 T Yeah. What just a line? (*points to board*)

48 K Yeah

49 T Like a- what like [(…) (*gestures thinness*)

50 K [a rectangle filled in

51 T (*Giggles*) Very clever. Like a dot (*draws dot*) oops (*erases, does again*) like that. It’s interesting isn’t it. Yes H?

In using the flat plastic shapes, the teacher introduces an ambiguity. She points out that the shapes “look like three dimensional”, even though they are “meant to be two dimensional” (turn 41). For this discussion of ambiguity, the issue here is not whether or not the shapes should really be seen as 2D or 3D. Rather, what is observable, is that the teacher has explicitly constructed an ambiguity around the shapes. They are ‘meant’ to be 2D, but can be viewed as 3D. This ambiguity is in terms of what is ‘meant’, so that the teacher’s alternative reading can be seen as mildly subversive, in the sense that it goes against what is ‘meant’. The students in the class are receptive to the teacher’s ambiguity, describing two of the shapes as solids rather than planes. A ‘circle’ is described as a cylinder. A ‘square’ is described as a cuboid. The teacher reasserts, however, that the latter is “meant to be flat” (turn 45).

The teacher’s ambiguity is taken up by a student, K. Continuing an earlier discussion in which a 1D shape was characterised as “a line”, K constructs an ambiguity similar to that made by the teacher when describing the plastic shapes. He portrays a line as “like a rectangle filled in” (turn 46). In the same way that a ‘flat’ plastic shape is actually three-dimensional, a representation of a ‘line’ has some thickness. A line can be seen as both one-dimensional *and* two-dimensional, and is therefore ambiguous.

K’s statement about one dimensional shapes (line 46), and the ensuing discussion, can be seen as an exploration of what it is possible to say using the word dimension. In short, K, and elsewhere in the discussion, the class, are probing and developing aspects of mathematical discourse. In so doing, the use of the term dimension becomes more complex, encompassing more mathematically mature discursive practices. Among these are that any object can be described as having additional dimensions; and consequently, that the language of dimension offers different ways of describing mathematical objects, each of which is equally applicable.

THE DOG THAT DOES NOT BARK AND OTHER MATTERS

Taking the transcript as a starting point, participants in the conference session explored a number of aspects of the role of ambiguity in mathematics classroom discourse. I will briefly discuss three:

- the dog that does not bark in the night: the representation is not the object
- ambiguity as a moment of learning
- ambiguity and mathematicians

The dog that does not bark in the night

Mike Askew [2] used the expression 'the dog that does not bark in the night' to describe a key mathematical discursive practice, by which representations of objects can only *stand for* that which they represent. Mathematical discourse is frequently concerned with objects which cannot be produced in a tangible sense. Hence the 2D shapes used by the teacher in the transcript cannot ever be 2D. Even as 3D objects, they can never be perfectly cuboid or cylindrical. The dog is not confined to geometry. Relations between numbers can only be represented. The notion of 'double', for example, cannot be shown in any absolute sense. It may be represented by pairs of numbers, or collections of objects, but the relation itself cannot be conjured up. A key aspect of mathematics discourse, then, is that it concerns matters which can be alluded to, exemplified, explained, defined or described, but not created in any tangible way. The 'gap' between what is meant and what is said or represented provides for the possibility of ambiguity. The students in the transcript are able to explore this possibility through its acknowledgement by the teacher.

Ambiguity as a moment of learning

Harry Grainger argued that ambiguity was closely linked to learning. In the part of the transcript, for example, where there was no ambiguity, or rather, in which no ambiguity was explicitly assumed by the participants, the exchanges have a more rehearsed feel. Students recite 'length, breadth, height' or name parts of a circle. The teacher affirms the students' utterances, and so maintains the appearance that meanings are clear, unambiguous and shared by all. Once some degree of ambiguity is constructed, as in the above extract, a space opens up for the students to explore. Where a plastic shape was 'a square', it becomes 'a square' or 'a cuboid'. By describing the shapes in two ways, the teacher highlights different ways of seeing the objects, ways of seeing which can be applied to other objects or representations, as some of the students then do. In exploring the relationship between these different ways of seeing, the students learn different ways of talking about geometric forms, as well as how they relate to each other. This observation turns the NNS guidance on its head. Rather than 'sorting out' ambiguities, teacher should see them as opportunities for mathematical exploration.

Ambiguity and mathematicians

During the discussion, I was prompted to wonder whether ambiguity played a part in ‘professional’ mathematics. My wondering came from the idea that ambiguity plays a part in learning. The students in the transcript are learning to do mathematics and to talk about mathematics and ambiguity has a role in that learning: could this not also apply to ‘professional’ mathematicians? Peter Johnston-Wilder suggested one class of problems which could be seen as depending on ambiguity, exemplified by Russell’s Paradox. The paradox is summarised by Clark (1975), who gives the example of Epimenides the Cretan:

> who had awkwardly said that all Cretans were liars. Was this statement in fact a lie? The question appeared to have only one answer: if it was it wasn’t, and if it wasn’t it was (p. 79).

Statements such as ‘all Cretans are liars’, if uttered by Cretans, could be seen as ambiguous, and, since such statements became crucial to Russell’s early work, mathematically significant. Russell’s Paradox perhaps provides an example of a role for ambiguity in mathematics and in the doing of mathematics, since the ambiguity had to be understood for the work to continue.

CONCLUDING REMARKS

Whilst mathematics may popularly be seen as essentially unambiguous, the exploration of mathematical discourse, stimulated by the ‘dimensions’ transcript, suggests that this is not the case. Ambiguity forms an important discursive resource in school mathematics discourse, and perhaps in all mathematics discourse. It is the potential for ambiguity inherent in all language, that allows students to investigate what it is possible to do with mathematical language, and so to explore mathematics itself. If, as suggested by the *National Numeracy Strategy*, all ambiguity is ‘sorted out’ as soon as it arises, valuable opportunities for students to learn the subtleties of mathematics could be lost.

NOTES

1 Transcript conventions: Bold indicates emphasis. / is a pause < 2 secs. // is a pause > 2 secs. (...) indicates untranscribable. ? is for question intonation. ( ) for where transcription is uncertain. [indicates overlaps.

2 This section includes my interpretations of the discussion, including the contributions of particular individuals. I offer no guarantee that my interpretation corresponds with anyone else’s recollections of the session.

ACKNOWLEDGEMENTS

I am grateful to the teacher and her students for inviting me into their classroom. The data collection was supported by ESRC R00429934027. I also acknowledge the participation of Constant Leung, Candia Morgan and Brian Street in the ongoing conversation concerning the data discussed in this paper.

REFERENCES

Barwell, R., Leung, C., Morgan, C. and Street, B. (2002) Mathematics education and applied linguistics: working group report. In Goodchild, S. (Ed.) *Proceedings of the British Society for Research into Learning Mathematics* 22 (1&2) 89-94.

Clark, R. W. (1975) *The Life of Bertrand Russell*. London: Jonathan Cape/ Weidenfeld & Nicolson.

DfEE (1999) *The National Numeracy Strategy: Framework for Teaching Mathematics from Reception to Year 6*. Sudbury: DfEE Publications.

DfEE (2000) *National Numeracy Strategy Mathematical Vocabulary Book*. Sudbury: DfEE Publications.

Edwards, D. (1997) *Discourse and Cognition*. London: Sage.

Gee, J. P. (1999) *An Introduction to Discourse Analysis: Theory and Method.* London: Routledge.

Pimm, D. (1987) *Speaking Mathematically: Communication in the Mathematics Classroom*. London: Routledge.