STRATEGIES FOR MENTAL CALCULATION

Tom Macintyre and Ruth Forrester

Edinburgh Centre for Mathematical Education (ECME)

University of Edinburgh

Researchers at the Edinburgh Centre for Mathematical Education are currently investigating use of the soroban (Japanese abacus) to develop strategies used for mental calculation. The classroom-based project involving year 8 pupils in two schools is funded by the Nuffield Foundation. This paper focuses on the initial assessments of students' mental computation abilities. Deficiencies, particularly in subtraction of two digit numbers, are striking in the light of recent emphasis on developing mental calculation strategies. The findings have surprised teachers and researchers involved in the project, raising questions about assumptions made of students' competence in mental calculation.

BACKGROUND

Recent policy initiatives (SOEID, 1997 & DfEE, 1998) require schools to pay greater attention to mental calculation in mathematics. Current research at ECME investigates the use of a Japanese abacus, the soroban, as a means of promoting number sense and developing mental strategies for computation. The classroom-based project involving year 8 pupils in two schools is funded by the Nuffield Foundation.

METHODS

Part of the research project required an initial assessment of students' abilities in mental computation and range of adopted strategies. Two short tests were constructed to provide opportunities for use of a range of mental calculation strategies. Each test attempted to include measures of speed and accuracy by asking students to answer as many questions as they could in three minutes. Pupils were asked to do the calculations mentally and write down the answers on the sheet; jottings were allowed. Both tests included a variety of questions involving all four operations.

Once each timed session was concluded, teachers led a very brief discussion asking pupils to describe the methods they used to calculate, say, 43+39 in order to model ways of explaining mental strategies adopted by individuals. Pupils were then asked to return to the initial six questions of the test and record how they worked out the answers. Test 1 began with six addition sums and Test 2 began with six subtractions, selected to promote a variety of strategies. The items that required an explanation were:

Test 1	36+23	56+39	37+64	47+26	183+55	685+46.
Test 2	85-41	96-59	64-25	83-27	219-55	635-67.

A sample of pupils from each class was interviewed to further clarify the strategies described and to explore pupils' thinking.

FINDINGS

The first notable feature of the data concerns the pupils' performance in terms of speed and accuracy. Many respondents only completed a small proportion of the set tasks, with very few pupils tackling over 60 questions in two three-minute periods (including errors and missed items). Some respondents didn't attempt many of the subtraction items beyond those six that are detailed above – a worrying feature for Year 8 pupils (Data having been collected in February of the Scottish Secondary 1 stage).

A closer analysis of the performance in addition and subtraction problems involved studying the opening six questions on each set. Out of a possible 6 correct responses the frequencies of success are noted in Table 1. Addition tasks are clearly completed in a much more confident manner than the subtraction items, with over 50% of the study group gaining maximum credit. Subtraction items appear to have presented a much bigger challenge to the pupils, with around 50% of them having 3 or more questions wrong. This result surprised the teachers and researchers taking part in the study.

Table 1 Frequency and Percentage of pupils recording correct responses across the 6 Addition and 6 Subtraction items (N=135)

Number of correct responses	**Addition**: Number of pupils (Percentage)	**Subtraction**: Number of pupils (Percentage)
0	0 (0.0)	6 (4.4)
1	0 (0.0)	28 (20.7)
2	2 (1.5)	16 (11.9)
3	7 (5.2)	18 (13.3)
4	16 (11.9)	29 (21.5)
5	39 (28.9)	26 (19.3)
6	71 (52.6)	12 (8.9)
Total	135 (100.0)	135 (100.0)

STRATEGIES

A classification of strategies used by pupils was developed, based on the work of Thomson & Smith (1999) and Klein & Beishuizen's (1998). A category of 'enhanced flexibility' in number work was introduced to capture sophisticated approaches including compensation strategies. Table 2 shows our coding for strategies used by pupils.

Table 2 Categories for Strategies used by pupils

Code	Description	Example: 37+64
1	C - Counting on (Ones and Tens)	
2	Digits - Manipulating digits (including standard algorithmic approach)	7+4=11, 3+6 +1=10 =101
3	10 10 - Mainly Left to Right computation but can be Right to Left if explicit reference to values is evident	30+60=90, 7+4=11 90+11=101
4	Mixed method involving 10 10 and then sequential adding rather than a combination of the Ones being added to the sub-total (10 10 N)	30+60=90, 90+7=97 97+4=101
5	N10 - Sequencing with Tens and Ones or multiples	37+60=97, 97+4=101 64+30=94, 94+7=101
6	Flexible thinking being demonstrated through: • inventive use of number • enhanced number sense • compensation method (N10C) • adjustment and variation on sequential procedure (A10)	See Figure 1 below

Figure 1 Demonstration of 'flexibility' in pupil's thinking (Pupil 517)

85-41 = 86 - 40 - 2 = 46-2 = 44

96-59 = 97-60 = 37

64-25 = 65-25 = 40-1 = 39

If possible, a strategy code was determined whether or not the question was answered correctly. Table 3 and Table 4 show the breakdown of adopted strategies for Addition and Subtraction items, representing the percentage of pupils favouring the strategy codes outlined above. Note that none of the respondents used Strategy 1 ('counting on').

Table 3 Distribution of strategies for Addition questions

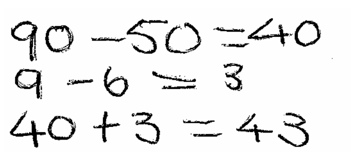
Strategy	36 + 23	56 + 39	37 + 64	47 + 26	183 + 55	685 + 46
	%age	%age	%age	%age	%age	%age
2	28.1	22.2	20.7	20.0	19.3	17.0
3	55.6	43.0	49.6	48.1	42.2	45.2
4	3.7	5.2	5.2	3.7	4.4	3.7
5	5.9	4.4	3.7	6.7	5.2	5.9
6	2.2	20.0	11.9	10.4	8.1	8.1
Missing	4.4	5.2	8.9	11.1	20.7	20.0
Total	100.0	100.0	100.0	100.0	100.0	100.0

Table 4 Distribution of strategies for Subtraction questions

Strategy	85 - 41	96 - 59	64 - 25	83 - 27	219 - 55	635 - 67
	%age	%age	%age	%age	%age	%age
2	24.4	22.2	23.0	20.0	20.0	16.3
3	43.7	22.2	20.0	21.5	14.1	8.9
4	2.2	1.5	0.7	1.5	1.5	1.5
5	12.6	10.4	10.4	11.1	7.4	8.1
6	5.2	14.1	13.3	6.7	12.6	5.2
Missing	11.9	29.6	32.6	39.3	44.4	60.0
Total	100.0	100.0	100.0	100.0	100.0	100.0

A predominant use of strategies 2 and 3 is evident in both the addition and subtraction problems. While use of a standard algorithm, Strategy 2 (Digits), is undesirable for mental calculation, the use of Strategy 3 (10 10) can be quite acceptable for addition. Sticking rigidly to this strategy for subtraction however, is likely to be the reason for many pupils' problems, as illustrated in Figure 2.

Figure 2 Incorrect use of Strategy 3 for '96-59' (Pupil 102)

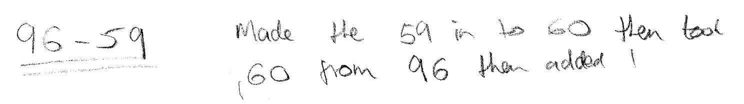


Only about 10% of pupils used Strategy 5 (N10), a more appropriate approach for subtraction problems requiring exchanges. A matter of concern was the increasing number of 'Missing' strategies across the six subtraction problems within Table 4. This reflects the pupils' inability to complete the calculation or after having completed it, successfully or otherwise, being unable to explain how they went about the mental process. These pupils may not be used to the process of discussing and articulating their methods in the classroom hence the difficulty being witnessed in this study.

A closer analysis of success on each question highlights the strategies attempted unsuccessfully. For example an analysis of the question 96-59, selected to promote the use of the compensation strategy in its most straightforward form of 'over-jump', shows that only 20% of the cohort chose to use a sophisticated approach along the lines expected (Strategy 6). Some 56% of the cohort gave an incorrect response to the calculation, and of those, the undesirable use of Strategy 2 (Digits) and Strategy 3 (1010), each accounted for 40% of the errors. A common response to this question was as illustrated in Figure 2 above, where clearly problems were caused by rigidly applying Strategy 3, which is perfectly valid elsewhere, but not desirable in subtractions involving an exchange.

In contrast, analyses of strategies categorised as '6' (exemplified in Figure 1) highlight the flexibility of response to such questions, albeit demonstrated by relatively few pupils. Compensation is often described simply as correction of an 'over-jump', as shown in Figure 3.

Figure 3 Compensation Strategy - 'over-jump' (Pupil 503)



A number of more sophisticated compensation-type strategies were evident:

E.g. 219-55= 220-55-1 64-25= 65-25-1

In some cases pupils did not show the compensation explicitly but found an equivalent calculation:

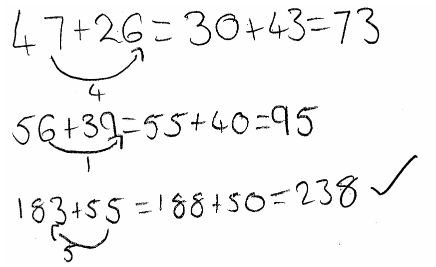
E.g. 685+46= 690+41= 700+31

96-59= 97-60

96-59= 90-53

The process of looking for an easier calculation and then compensating is one that requires considerable flexibility of thought, particularly when considering subtraction. There is a danger in teaching such a process in a very rigid way. For example in Figure 4, one pupil started to apply rather inappropriately the process she had been taught for finding an equivalent addition sum, to a subtraction situation. However, having realised her mistake she managed to compensate by adding 6 in one of the subtraction calculations tackled.

Figure 4 Additions (Pupil 501) Subtractions (Pupil 501)



CONCLUSION

Given a set of calculations, is there a particular method that should be promoted? It is probably desirable to move beyond the standard algorithm and digit manipulation in your head when attempting mental calculations. It is also desirable to develop sufficient flexibility so that pupils can select an appropriate method for the problems they are presented with, increasing the prevalence of Strategy 6 (Flexible). But are such strategies for mental calculation ones that can be taught in an explicit fashion? Simply teaching 'standard' compensation strategy clearly will not of itself solve pupils' problems. The situation is much more complex. There are many variations on the compensation strategy, all based on a similar concept but relying on pupil's inventive, creative and innovative interpretations as witnessed in the small sample of data gathered in this study.

Flexibility appears to be the key for success in mental calculation – not just being able to use a particular strategy, but being able to choose appropriately from a number of different strategies or to adapt thinking to suit the particular problem. One of the apparent difficulties in secondary schools is the underlying assumption that pupils will already have had sufficient exposure to a range of strategies. The findings suggest this assumption be not justified. Alternative approaches to mental calculation may be necessary - perhaps including a substantive input on the N10 strategy that appears to offer greater flexibility. Work on the *Soroban* may also enhance flexible skills in mental calculation – watch this space!

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