

OBSERVING SUBJECT KNOWLEDGE IN PRIMARY MATHEMATICS TEACHING¹

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The mathematics subject matter knowledge of primary school teachers has in recent years become a high profile issue in the UK and beyond. This paper describes a videotape study of mathematics lessons prepared and conducted by trainee teachers. The aim was to identify ways in which their subject knowledge, or the lack of it, was evident in their teaching. We set out a framework which has emerged from the data, in which classroom events and episodes can be viewed as representative of one (or more) of four broad categories.

CONTEXT

The seminal work of Lee Shulman conceptualises the diversity of the knowledge required for teaching. His seven categories of teacher knowledge include three with an explicit focus on ‘content’ knowledge: subject matter knowledge, pedagogical content knowledge and curricular knowledge. Shulman (1986) notes that the ways of discussing subject matter knowledge (SMK) will be different for different subject matter areas, but adds to his generic account Schwab’s (1978) notions of substantive knowledge (the key facts, concepts, principles and explanatory frameworks in a discipline) and syntactic knowledge (the nature of enquiry in the field, and how new knowledge is introduced and accepted in that community). For Shulman, pedagogical content knowledge (PCK) consists of “the ways of representing the subject which makes it comprehensible to others...[it] also includes an understanding of what makes the learning of specific topics easy or difficult ...” (Shulman, 1986, p. 9). PCK is particularly difficult to define and characterise, but seems essentially to conceptualise the hitherto missing link between knowing something for oneself and being able to enable others to know it.

In 1998, the UK government specified for the first time a curriculum for Initial Teacher Training (ITT) in England (DfEE, 1998), setting out what was deemed to be the “knowledge and understanding of mathematics that trainees need in order to underpin effective teaching of mathematics at primary level”. There is now a growing body of research on prospective primary teachers’ mathematics subject knowledge, which has undeniably been facilitated by the necessity of some process of audit and remediation within ITT (e.g. Rowland, Martyn, Barber and Heal, 2002; Goulding and Suggate, 2001; Morris, 2001).

This paper is representative of ongoing collaborative work in this field between researchers at the universities of Cambridge, London, Durham and York under the

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acronym *SKIMA* (subject knowledge in mathematics). The conceptualisation of subject knowledge and its relation to teaching which informed the project has been detailed extensively elsewhere (Goulding, Rowland and Barber, 2002).

The focus of the research reported in this paper is on ways that trainees' mathematics content knowledge can be observed to 'play out' in practical teaching during school-based placements. This study took place in the context of a one-year (three term), full-time Post-Graduate Certificate in Education course for prospective primary school teachers in a university faculty of education. The primary 'trainees' are prepared to be generalist teachers of the whole primary curriculum. In this particular course, each of the 149 trainees followed a route focusing either on the 'lower primary (LP)' years (3-8) or the 'upper primary (UP)' (7-11).

METHOD

About one month into the second term of the course, a 16-item audit instrument was administered to all the trainees, under semi-formal conditions. These audits were marked by tutors, who gave individual feedback to trainees indicating any further targets for self-study.

For the purpose of this research, the total scores for each paper (maximum 64) were used to identify groups with 'high', 'medium' and 'low' scores. Two trainees were then assigned to each cell of a 2×3 matrix, with rows corresponding to age-phase specialism (UP, LP) and columns to level of mathematics content knowledge as assessed by the audit (high, medium, low). Two mathematics lessons taught by each of the trainees were observed and videotaped. These took place approximately in the 5th and 7th weeks of the 8-week placement; school half term occurred between the two observed lessons. Trainees were asked to provide a copy of their planning for the observed lesson. As soon as possible after the lesson (usually the same day) the observer/researcher wrote a *Descriptive Synopsis* of the lesson. This was a brief (4-500 words) account of what happened in the lesson, so that a reader might immediately be able to contextualise subsequent discussion of any events within it. From that point, we took a grounded approach to the data for the purpose of generating theory (Glaser and Strauss, 1987). In particular, we identified aspects of trainees' actions in the classroom that seemed to be significant in the limited sense that it could be construed to be informed by the trainee's knowledge of *mathematics* subject matter knowledge or *mathematics* pedagogy as opposed to other more general kinds of pedagogical awareness or expertise. Next, each of us viewed about five videotapes, and elaborated the *Descriptive Synopsis* into an *Analytical Account* of each lesson. In this account, significant moments and episodes were identified and coded, with appropriate justification and analysis.

It should be noted that the dimensions of age-phase (LP, UP) and audit score category (high, medium, low) do *not* feature in the analysis of lessons which is the focus of this paper. For our present purposes, it is sufficient to note that both dimensions were properly represented in the sample of trainees.

OVERVIEW OF FINDINGS: THE CATEGORIES

As a result of the process described above, 18 aspects of the trainees' teaching were identified, and a code assigned to each of them. The 18 codes identified *include*:

AC	anticipation of complexity	MC	making connections
AP	awareness of purpose	RCA	recognition of conceptual appropriateness
ATB	adherence to textbook	RCI	responding to children's ideas
CE	choice of examples	TU	theoretical underpinning
COP	concentration on procedures	UT	use of terminology
IE	identifying errors		

The code name should, in most cases, indicate the aspect of teaching intended to be associated with it. In principle, an event or episode in a lesson might be identified and coded as a *positive* or a *negative* instance of the any of the above aspects. Thus CE+, CE- would indicate respectively a good or poor choice of example(s). For other codes the existence of both possibilities is less apparent. Illustrated accounts of three of these aspects of teaching are given in Huckstep, Rowland and Thwaites (2002).

The focus of this paper is the emergence of four broad categories into which we have grouped these 18 codes. These four categories represent more comprehensive, higher-order concepts (Strauss and Corbin, 1998, p. 113). At present our groupings are tentative and our conceptualisation of the four categories is subject to ongoing deliberation and modification. Therefore we would especially welcome any comments on the accounts of the categories which now follow.

Theoretical background and beliefs

First there is the trainees' *theoretical background and beliefs*. This category would include, for example, overt subject knowledge and the use of terminology such as the readiness to distinguish between 'digits' and 'numbers' and 'numerals'. Trainees' avoidance of common errors such as their treating squares and rectangles as though they were necessarily disjoint sets would provide an example of evidence of essential background of this kind. Evidence of the 'theoretical knowledge' that trainees possess would also be found both in their formal understanding of, say, mathematical concepts and in their ability to articulate pedagogical issues concerning the teaching of such concepts. So that a trainee, who slavishly adhered to textbooks or NNS unit plans without consideration for their quality or suitability might be deemed to have a weak theoretical background. The same would be true of those who exclusively (or largely) pursue instrumental understanding with excessive concentration on procedures. However, understanding ideas *within* mathematics and the ability to review and critique suitable pedagogy for teaching does not exhaust the scope of this category. A trainees' beliefs about the *purpose* of mathematics education - the reasons why their pupils are compelled to learn it - would also lie within the scope of this category.

Transformation presentation and explanation

The second of these four categories is typically displayed by someone who is either planning to teach or in the act of teaching since they bear down on the *presentation* of the lesson. Shulman has referred to this switch from learner knowledge to considerations involved in actual teaching as *transformation*. We have augmented the definition of this category slightly as *transformation presentation and explanation*. This allows us not only to draw out the appropriateness of a trainee's choices of examples, or of a particular representation, but also the manner in which a demonstration² is carried out. In this way, our second category, unlike the first, picks out behaviour that is directed towards a pupil (or a group of pupils) which follows from *deliberation* and *judgement*.

As far as the choice of examples given is concerned, it is instructive to consider three kinds of ways in which we observed that examples fell short of perfection. One is the where the choice of examples *obscures the role of the variables* within it. One such case concerns Michael, in a lesson with a Year 4 class. The main activity was about telling the time with analogue and digital clocks. One group was having difficulty with analogue quarter past, half past and quarter to. Michael intervened with this group, showing them first an analogue clock set at six o'clock. He then showed them a quarter past six and half past six. When asked to show half past seven on their clocks, one child put both hands on the 7. The child's inference from Michael's demonstration example (half past six) is clear. Of the twelve possible examples available to exemplify half-past, half past six is arguably the most unhelpful.

A second category of poor choices of examples arises from the selection of calculations made to illustrate a particular procedure, when *another procedure would have been more sensible* to perform those particular calculations. Finally, we note that the videotapes offer copious instances of *examples being randomly generated*, typically by dice.

Coherence

The next category is evident not so much in certain choices and decisions that are made for a more or less discrete part of mathematical content – the learning, perhaps, of a concept or procedure etc. It concerns the *coherence* of the planning or teaching displayed across an episode, lesson or series of lessons. Such coherence, like the previous category of *transformation, presentation and explanation* is typically displayed by someone who is either planning to teach, or is in the act of teaching, but, additionally, it is marked out by its provision for what Shuard and Rothery (1984)

² Demonstration can be of a procedure, so that a teacher might be showing *how* to do something. But it also covers cases where what is demonstrated is that something is the case and why it is so. In this sense a demonstration can be a kind of explanation

once called the ‘flow of meaning’. So this category will be especially associated with decisions made about the *sequencing* and *connectivity* of content. Such decisions will typically follow from a trainee’s ability to anticipate what is complex and hence, or otherwise, what is conceptually appropriate for a given pupil or group of pupils.

Caroline, another of the trainees observed, displayed such coherence in a lesson with a Year 2 class. She linked the work in the introductory section (reciting the two times table) with the main activity, and also attempted, with some success, to make a crucial link between fractions of shapes and fractions of numbers. Having carried out some divisions of regular shapes into congruent regions, she placed on the board a circular card with four smaller circles attached at equal distances round the circumference. It was supposed to represent a flower. She encouraged the children to see that by halving this configuration she simultaneously halved the number of petals.

Contingent action

There remains, however, an important category that is distinguished both from the mere possession of a theoretical background, on the one hand, and from the deliberation and judgement involved in making learning meaningful and coherent for pupils, on the other. Our final category, whilst favouring the informed trainee, concerns classroom events that are almost impossible to plan for. In commonplace language it is the ability to ‘think on one’s feet’: we call it *contingent action*. Such action involves both the readiness to *respond to children’s ideas* and a preparedness, for various reasons, to *deviate from an agenda* set out when the lesson was prepared.

The videotaped lessons gave rise to a large number of instances in this category of contingent action, and many of these involving contingent responses to children’s ideas. For example, Colin, working with a reception class, asked the children which coins could be used to pay for a toy dog priced at 5p. The activity was a vehicle for understanding the notion of value as opposed to simple cardinality, and was expected to entail addition of different values. After two children had given anticipated answers involving 1p and 2p coins, Niall suggested 10p minus 5p. Rather than reject or sideline Niall’s idea, Colin welcomed it, and shared and developed it with the class.

CONCLUSION

Our grounded approach to the analysis of the lessons has highlighted several normative notions of teaching practice, which we currently find it helpful to group into the four broad categories set out in this paper. We emphasise here our narrow attention to mathematics *content* knowledge, both SMK and PCK, in our scrutiny of the lessons. Whilst our research has not provided a direct mapping between mathematical knowledge and competence at teaching the subject, it does throw considerable light on this link by way of reference to particular moments and episodes. The grounded approach that we chose to take has the advantage that ‘theory’ can always, of necessity, be exemplified in this way: our research suggests to

us new ways of working with prospective teachers to develop their teaching by reference to this theory.

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REFERENCES

- DfEE (1998) *Teaching: High Status, High Standards: Circular 4/98*. London: HMSO.
- Glaser, B. G. and Strauss, A. L. (1967) *The Discovery of Grounded Theory: Strategies for Qualitative Research*. New York: Aldine de Gruyter.
- Goulding, M. and Suggate, J. (2001) 'Opening a can of worms: investigating primary teachers' subject knowledge in mathematics.' *Mathematics Education Review* 13, pp. 41-54.
- Goulding, M., Rowland, T., and Barber, P. (2002) 'Does it matter? Primary teacher trainees' subject knowledge in mathematics'. *British Educational Research Journal*, 28(5) pp. 689-704.
- Huckstep, P. Rowland, T. and Thwaites, A. (2002) 'Primary Teachers' Mathematics content knowledge: what does it look like in the classroom?' Symposium paper presented at the Annual Conference of the British Educational Research Association, University of Exeter, September 2002.
- Morris H. J. (2001) 'Issues raised by testing trainee primary teachers' mathematical knowledge'. *Mathematics Teacher Education and Development* 3, pp. 37-48.
- Rowland, T., Martyn, S., Barber, P. and Heal, C. (2002) 'The mathematics subject knowledge of pre-service elementary school teachers'. In D. Edge and B. H. Yeap (Eds.) *Proceedings of the Second East Asian Regional Conference on Mathematics Education and Ninth Southeast Asian Conference on Mathematics Education* Volume 2, pp. 274-280. Singapore: Nanyang Technological University.
- Schwab, J. J. (1978) 'Education and the structure of the disciplines.' In I. Westbury and N. J. Wilkof (Eds.), *Science, Curriculum and Liberal Education* (pp. 229-272). Chicago: University of Chicago Press.
- Shulman, L. (1986) 'Those who understand, knowledge growth in teaching'. *Educational Researcher* 15(2), pp 4-14.
- Shuard, H. and Rothery, A (1984) *Children Reading Mathematics*. London: John Murray
- Strauss, A. L. and Corbin, J. M. (1998) *Basics of qualitative research: Techniques and Procedures for Developing Grounded Theory*. Sage, London.