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REFLECTIONS ON THE ROLE OF TASK STRUCTURE IN LEARNING THE PROPERTIES OF QUADRATIC FUNCTIONS WITH ICT – A DESIGN INITIATIVE

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This paper will draw on research being developed within the Teaching and Learning strand of the ESRC InterActive Education: Learning in the Information Age project which is examining the ways in which new technologies can be used in educational settings to enhance learning. It will focus on the learning and understanding of quadratic functions using a graphical software package and includes a discussion of how the structuring of the activities influences the nature of the learning environment and how it might influence student exploration of mathematical concepts.

INTRODUCTION

The study reported in this paper is a small part of a large ESRC funded project, Interactive Education: teaching and learning in the information age (www.interactiveeducation.ac.uk). The purpose of the project is to look at ways to use ICT effectively in teaching and learning from within the micro-context of the classroom, to macro-contexts such as policy management and teacher training. Within the Teaching and Learning theme of the project there are 52 teachers representing various subject areas. They work closely together with researchers in trialing and sometimes developing pre-existing uses of ICT in order to investigate what conditions are conducive for effective learning to take place. Any particular and peculiar affordances created by the mix of the software environments, the students, and the teacher facilitated learning structures are investigated. These interventions are called design initiatives.

This paper outlines the preliminary work and findings from one such design initiative. It is a case study focusing on the work of one student within a series of year 9 Mathematics lessons investigating the properties and behaviour of quadratic equations with the aid of the software package Omnigraph. Omnigraph is a relatively easy to use dynamic graphing package that allows the user to quickly produce graphs of functions from their equations. In the past, the teacher Rob Beswetherick, used Omnigraph for the teaching of quadratic equations from the front of the class with the aid of a data projector. Students were encouraged to come out and interact with the software but were not given access to the computers themselves. In the design initiative the students would spend some time on the computers during each of the lessons and data collected in order to focus on the processes of learning that might be occurring. The role that task structure may play in directing prescribed and experimental work is considered.

THEORETICAL DISCUSSION

There are now a number of software packages that can help with the learning of mathematics. These range greatly in their nature. Some may be considered tools, others revision aids, and others mathematical "microworlds". Very often the software package may combine a number of these or quite possibly have its function determined as a result of the nature of the tasks stuctured by the teacher. The term "learning environments" is often used to describe some of these packages and Mariotti (2002) talks about these as being environments that allow pupils to experiment with mathematical ideas.

It is important to distinguish clearly between a mathematical microworld and a mathematical tutoring package. A microworld is a representation in some way of a set of mathematical concepts. The user is able to enter this world and play around with mathematical relationships and representations. The world is truly interactive. The teacher plays the role of structuring and possibly designing the activities so that the students' activity becomes directed. The computer screen can also present a window on the student's thinking and provide a shared space of communication with others. Hoyles and Noss state:

> "We have glimpsed how the computer can, in ways we have yet to elaborate, combine elements of the "concrete" with the "abstract", the intuitive with the rigorous, the particular with the general. At the same time it affords the learner a support for thinking, a screen on which to construct and reconstruct emergent ideas, and on which we, as observers, can catch sight of the construction process as it takes shape." (Noss & Hoyles, 1996).

Mariotti (ibid) also states that "…the relationship between the pupil and the teacher may be transformed by the introduction of the computer making communication between them more efficient and reciprocal".

Omnigraph, the package used in this research, is an example of a dynamic graphing package. These have the advantages of allowing students to rapidly construct graphs of algebraic equations (Hennesy, Fung & Scanlon, 2001). This rapid construction makes it relatively easy to see the properties of families of functions such as quadratics. By fixing some parameters within an equation and varying others the student can be directed to experiment and discover for themselves their behaviours. The scaling facilities of the package can potentially allow students to gain an insight into the fact that the appearance of these visual representations can be deceptive and they can be directed to reflect on this with questions such as "What stays the same and what differs when scaling?".

There often seems to be concern about using such dynamic packages in that they are sometimes "accused" of doing the work for the students, for example, because the students no longer plot the graph they may lose sight of how the y values are generated from the x values. With any new system there needs to be a consideration of what is gained and what is lost, whether the old and new systems complement each other, what to change, get rid of or renew. One of the problems of traditionally plotting graphs is how time consuming it all is. A lot of time is wasted on undue accuracy and preparation; getting the ruler, choosing the colours, rubbings out, off-task talking (since these procedural activities do not require a large amount of concentration). It is also interesting to ask oneself what mathematics is actually going on within this process. There could be some understanding of the relationship between the y and the x, but rarely is there any talk about the representative nature of what the students are doing, or indeed why they are doing it. The plotting of graphs on paper does not always appear to be very time efficient in terms of learning. It is also very difficult, and slow, to ask "what if" types of questions e.g. What would be the effect of reducing the coefficient of x on the graphs appearance? A task such as investigating the properties of quadratic functions is arguably more quickly and possibly more effectively achieved with the use of dynamic graphing packages.

By allowing students to gain access to the computers there exists a greater possibility for experimentation. This, however, can be unproductive if the experimentation is undirected and random. Goldenberg (1995), writing within the context of dynamic geometry, suggests that children find it difficult to experiment in meaningful ways. He states that children often change too many variables at once. This means that any patterns, or predictive behaviours are difficult to spot. It is here that the teacher can structure the tasks so that the students experiment within a certain framework. Part of this structure is suggested by the focus of the questions that the teachers ask but also by creating scaffolded learning environments.

In any learning environment students will construct their own meanings. These meanings may be different from those of the teacher and other class members. It is arguable that by creating the right balance of structure and experimentation the meanings that are created within individuals are not too disparate but yet allow the development and input of the individual's own way of working, or interpreting. For example some may work in an experimental, trial and error way whilst others may work in a more intuitive way. Marriotti (ibid) refers to the interaction of the software environment and the student as the "instrument". This instrument, or student-learning environment complex, is different for everyone in the class. The teacher-learning environment complex might be very different from that of the student instruments and even of other mathematics teachers' instruments. This means that what actually happens when using a new mathematics learning environment is not predictable.

METHODOLOGY

The focus of this research is largely upon the learning process, partially because of the unpredictability suggested above. The greater question of how the teacher-class-software complex impacts on this learning is being investigated. The focus of this particular paper is on one of the students, Kay, who represents the start of a broader comparative study. How Kay responded to the structures of the tasks set, her degree of experimentation, and what she may have learned, are considered. Kay was one of six students that were interviewed in pairs before and after the design initiative took place. All the students within Rob Beswetherick's class were given pre- and post-initiative assessments which required the students to plot three graphs. Kay and Guy were given a second post-initiative interview with a shared laptop so that they could demonstrate what they had learned in the environment that they had become familiar with. Within the lessons there was intensive data collection. Two digital video cameras, a minidisk recorder and a digital camera were used throughout. When individual activity was recorded, one camera was focused on Kay for all the sessions. Worksheets produced by the students in the lessons were also photocopied for later analysis.

NATURE AND CONTENT OF THE LESSONS

The main purpose of the sessions was to develop an understanding of the properties of quadratic functions. Each lesson began with Rob Beswetherick giving an introduction. He used a combination of a data projector which was used to dynamically display Omnigraph and a traditional whiteboard to write down the odd equation. Rob would also ask questions of the class and sometimes they would come up to the front to point on the projections. Intermittently there would be a series of exercises that he would get them to do. These were referred to as “quiz graphs”. For instance he would ask them to sketch the graph of a particular function on their worksheets. After the introduction the students would go to the computers and follow worksheets that Rob had designed. These tended to involve a mix of clearly prescribed tasks and those that allowed a greater degree of experimentation. At the end of each lesson Rob would draw the students back for a plenary.

Omnigraph itself is fairly easy to master but a little time was allowed in the first session (about 10 minutes) for the students to play with the package. For example, Rob allowed them to play with the zooming function, which was important in later explorations by the students

ANALYSIS - FOCUS ON KAY

Kay was one of the six students chosen for more detailed observation and interviews. She represented someone slightly below the middle of class in terms of her present attainment levels (this was based on the teacher's personal assessment). She also attended both interviews and all of the sessions (this was not true of some of the others within the group of six). Other students within the six are to be compared and contrasted with Kay at a later date. Kay came from a fairly ICT rich home environment. Her father worked with IT and encouraged Kay to install programs herself because he felt this would help her learn. Part of the success of the use of Omnigraph with Kay might partially be due to her general confidence and familiarity with ICT.

Initially the data collected from the survey was eyeballed. The graphical output and Kay's answers on the worksheet gave an indication of her activities during the lessons. It was possible to record almost all the graphs that she produced with

Omnigraph throughout these lessons. The graphs that she produced were related back to the worksheets and coded in the following ways:

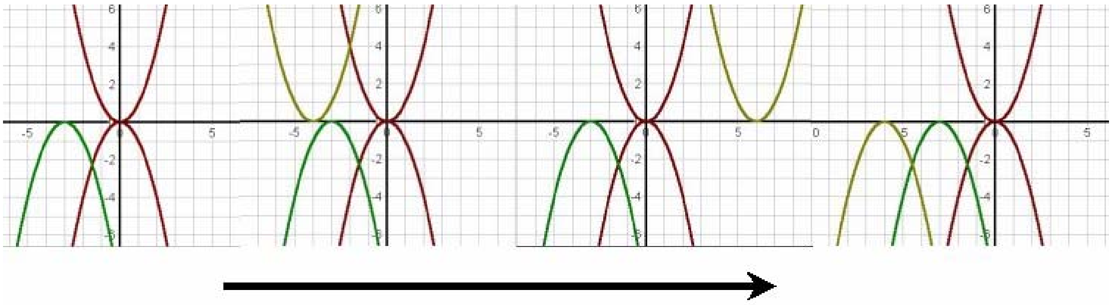
A - anchor graphs - these are graphs that act as a kind of base point e.g. $y = x$, $y = x^2$

P - prescribed graphs - graphs given by the teacher e.g. plot the graph of $y = 3 + x^2$

E - Experimental graphs - these may be in the form of more open ended questions

e.g. "What changes and what stays the same when you change the "a" in $y = -(x + a)^2$?"

Sometimes Kay produced a series of experimental graphs that were related to each other in some way. These were referred to as ‘runs’ (see figure 1 for an example). Throughout the sessions Kay produced a total of at least 6 anchor graphs, 27+ prescribed graphs and 38+ experimental graphs. A total of 9 runs were also identified. It is clear that a large amount of the graphs produced by Kay were experimental in that she played with different variables within the limits of the tasks set by the teacher.



$y=x^2$	$y=x^2$	$y=x^2$	$y=x^2$
$y=-x^2$	$y=-x^2$	$y=-x^2$	$y=-x^2$
$y=-(x+3)x^2$	$y=-(x+3)x^2$	$y=-(x+3)x^2$	$y=-(x+3)x^2$
	$y=(x+4)^2$	$y=(x-6)^2$	$y=-(x+6)^2$

Fig. 1: An example of an experimental run of graphs produced by Kay

It was clear after the lessons that Kay had progressed in her understanding of quadratic functions. In the pre-initiative assessment Kay was unable to plot any of the three graphs given. In the post-initiative assessment she sketched the three graphs accurately. It was interesting that she had sketched these graphs and not plotted them in the traditional way. This demonstrated an understanding of the effects of the various parameters on the behaviour of the graphical representation rather than merely using a series of calculations to generate the shape of the graph.

In the interview with Kay and Guy, Kay demonstrated a good understanding of the behaviour of the quadratic plots under varying conditions. The researcher showed the students sketches and they had to reproduce them. They were completely successful with two of these and successful with some gentle hinting by the researcher in the other two.

CONCLUSIONS

The focus of this paper has largely been to take a look at how the structuring of the tasks might effect successful learning with a mathematical microworld such as Omnigraph. Omnigraph is not in itself difficult to master but creating the right learning environment to foster and use it effectively requires planning and thought. When a new initiative is introduced into the classroom the results cannot always be predicted because researchers, teachers, pupils, and wider cultures all bring their own meanings and understanding to the event (Noss & Hoyles, 1996). It is well known that with many of these new microworlds teachers might simply try to do what they have always done within the confines of a curriculum that precedes these innovations. They will also bring to the software their own knowledge, experiences, interpretations and preconceptions. As well as this the students will all react to the software in different ways forming unique "instruments" or student-software complexes. Within this design initiative the teacher was asked to choose an area that they wished to work in. The researchers then worked with him to develop the ideas to try out in the classroom. Rob was taking a risk by moving from his normal way of working with Omnigraph to allowing the students to work at the computers on their own, with the potential anarchy that may sometimes occur. However by planning the tasks, allowing a mix of experimentation and prescription, the students could be kept to task and gain a useful learning experience. It was also clear that Kay found the tasks stimulating as demonstrated by her frequent interactions in whole class activities and in a comment to the teacher after one of the classes where she indicated to Rob her enjoyment of the tasks. Rob was shown the videos of his lessons and in his own post-initiative reflections he thought that some of the tasks could have been developed to be more open ended since some students finished the tasks and could have been give further, more challenges to work on.

It is also important that the tasks are designed to prompt the students to reflect on their actions, to think and predict, and help them to generate an insight into the mathematical concepts that they are learning.

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