LEARNING EFFECT AS A STRUCTURING RESOURCE IN ALGEBRAIC PROBLEM SOLVING ACTIVITY

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The present study looked at the problem solving actions of a group of high achieving secondary school students on algebra problems. In this paper, I report on the solution activity of Year 12 students (aged 16 years) on non-standard algebra problems, looking in particular at how that activity was shaped by recent experience on similar problems.

INTRODUCTION

Within the Mathematics National Curriculum for England and Wales, the learning of algebra is compulsory for all students, and a particular reference is given to the way algebra can be used to model and solve problems. Despite this emphasis, many students rarely turn to algebra to solve problems. They exhibit a preference for arithmetic calculation or numeric trial and error.

The almost complete absence of school algebraic methods when solving algebra problems has been reported in a number of research studies (e.g., Bednarz *et al*., 1992; Stacey and MacGregor, 1995; Filloy and Rubio, 1993; Hall *et al*., 1989; Lee, 1987). Few studies, however, have gone on to investigate the structuring resources that can trigger the students' internal algebraic resources. An example of such research is the work by Brown and Coles (1999) in which they linked students' 'needing to use algebra' to them being able to ask and answer their own questions related to contexts.

In an exploratory study two Year 9 high achieving students were interviewed while solving four algebra problems in think-aloud fashion. Both were found 'automatically' working with arithmetic but they could illustrate algebraic arguments when probed (Doraisamy, 2001). They had the required algebraic mental resources but did not use them spontaneously. This sparked my interest in looking at when they do in fact get used.

The present study explored the structure and resources in the setting (physical as well as mental) that trigger students use of algebra in problem solving situations. High achieving Years 9 and 12 students from one English school participated in the study. In this paper, the focus is on the Year 12 students, their solution activity on the nonstandard algebra problems.

METHODOLOGY

The study of the Year 12 students was in two phases. Phase 1 comprised five students (average age 16.5 years), and phase 2 a further five students (average age 16.4 years).

Six of them had secured an A and four a B grade in the GCSE O-level mathematics examination last year. All were taking AS-level mathematics at the time of the study.

The students were tested on their understanding of algebraic letters, and interviewed individually while solving algebra problems in think-aloud fashion. The phase 1 students solved four standard and four nonstandard problems, differing in algebraic explicitness (symbol letters present or absent in the wording) and structure (solution guessable or not easily guessable). The phase 2 students solved two nonstandard problems. All were given the opportunity to structure the task in the way they saw fit. The time for problem solving was open-ended. The researcher acted as a participant observer. In phase 1 interviews, interventions were made on occasions to clarify the problem, tease out the reasons for a particular action, perturb their thinking, and/or explore their maximal level of performance. In phase 2 interviews, there was limited intervention. Once students had completed the two given problems, they were taken back to the first problem, and if not done by algebra, prompted towards algebra. On completion, they were invited to try again the second problem, if not done previously by algebra.

All interviews were audiotaped and transcribed. Pauses in speech less than 2 seconds were represented by a slash (/), and a double slash (//) indicated a pause more than 2 seconds. The complete transcripts of the tapes, the written work of the students and field notes were integrated to produce extended narrative accounts (Ainley, Nardi and Pratt, 2000), describing the work of each student on each problem. The narrative account for each student as well as their paper-and-pencil test were used in the analysis.

RESULTS AND DISCUSSION

Phase 1

The case of Geof is presented first followed by overall findings. Geof's case illustrates the solution processes, and explanations and justifications typical of many students in this phase. Geof recognised the standard problems as fitting in with stereotype of algebraic questions. In these problems, he spontaneously used his internal algebraic resource and produced algebraic representations and solutions. In the nonstandard problems (Pyramids, Magic Squares, Arithmogons, and Pentagrams), however, that resource was not triggered initially. Therefore, the resource was not adequate. It was only after exploring the problems numerically that Geof himself, with no prompting, used that resource. He said, after three unsuccessful attempts from the top in Pyramids: *"…Right/ I'm about to use algebra now … Because of x and y. That makes me think of algebra/ I'll start from the bottom now…"*, after six unsuccessful trials in Magic Squares: *"…Right// I am going to find x. If I can find x, then I can do it … Because of x and y … It's just that letters mean algebra. I use to see them in equations. It's the way I've always seen them"*, and after one unsuccessful trial in Arithmogons: *"Right. In my experience now, I'm going to put that as x, that as y, and that as z … From the experience here, the previous two* *questions [Pyramids and Magic Squares]"*. In Pentagrams, a problem Geof had not seen before (*"it looks completely alien to me"*), he was much more free to use that resource himself: *"…I guarantee algebra for this one"*. His spontaneous use of the resource was attributed, once again, to the learning effect from earlier solved problems: *"It's the way I did in Arithmogons. I didn't use letters to start with, and I didn't get anywhere without using them. I only found the answers when I did use them"*.

The other four students, like Geof, were inclined to use algebra on the standard problems. On the nonstandard problems, they similarly were less inclined to use algebra, but this was only true when it was the very first problem received. For example, Rosy in her first problem Pentagrams, needed prompting to go into algebra. As soon as it was not the first problem, she was using algebra because of an apparent learning effect. Britany too gave feedback that if Arithmogons were the very first problem she had received, she would have used non-algebraic rather than algebraic methods to solve it.

Learning effect seemed to be affecting students as far as nonstandard problems were concerned. This indicated a need to look at nonstandard problems, when it was the first problem received, and the students were responding spontaneously. This was explored in phase 2.

Phase 2

The cases of Samuel and Jamie is presented first followed by overall findings. Samuel solved his two nonstandard problems, Arithmogon and Pentagrams, originally by trial and error. He was then prompted towards algebraic symbolism in Arithmogons. On completion of the algebraic solution to this problem, he was invited to have a look again at Pentagrams. In response to: *"Do you think there is another method that you can use?"* he remarked straight off, *"Emm, put letters in this"*. He used five letters, and came up quickly with five equations. In commenting on his use of general letters, Samuel acknowledged the learning gained from solving Arithmogons, an analogous problem: *"From the last question … That's the same sort of questions, emm, using one number to get to another, by looking at the unknown … it was sort of similar question. So, I used the same method"*. The experience in Arithmogons allowed him to reattempt Pentagrams employing the algebraic method.

Jamie too solved his first problem Pentagrams using trial and error, but in his second problem Magic Squares he spontaneously used algebra. His algebraic resources was cued by the explicit x and y in the problem: *"It says what numbers should replace x and y … It signals to me that that's where I should put my concentration, on this bit [x and y] … It made me think of an equation, instead of numbers"*. When required to redo Pentagrams, Jamie remarked straight off, *"I don't know, I don't know if we can use the equation thing"*. When asked: *"Why do you want to use equations here?"* Jamie replied: *"Because it's another method that I've used in the other one … It was fresh in my mind as well because I've just done equations"*. He proceeded to write one equation, and added on: *"…If we did that for all of them, we will have similar things like what we had last time actually…"*. He continued to write four further equations. The experience in Magic Squares allowed Jamie to redo Pentagrams using algebraic representations.

The others, Sofie and Jake, like Samuel, originally solved both nonstandard problems presented by trial and error. In a revisit to the first problem, their algebraic resources was triggered, and the resulting learning gained allowed them to solve the second problem using algebraic methods. Elly, unlike the others, used algebra in her first problem. This experience allowed her to solve the second problem using algebraic methods. All three students articulations clearly distinguished the learning gained in the previous problem as a structuring resource for solving the later problem.

A distinction here is in the source of the students' learning effect. In Samuel, Sofie and Jake's case, it emanated from the researcher prompting them towards algebra. In Jamie and Elly's case, it was from the student him/herself, spontaneously using algebra. Whatsoever the source, learning effect is significant as a structuring resource for solving problems.

CONCLUSION

The Year 12 solvers demonstrated a spontaneity in using algebra on the standard problems. On the nonstandard ones, in quite a few cases, they were using algebra when there was an apparent learning effect, and not when there was no previous learning effect. Their internal algebraic resources were triggered by problems of the standard type, and by the learning effect from a previous problem tackled through algebra. This finding suggests a didactic setting in which student's use of algebra is explicitly triggered by these factors. However, the means by which the student's reliance on them is faded may demand careful attention to the scaffolding (Vygotsky, 1978) within the setting.

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