BUILDING A HOLISTIC VIEW OF MATHEMATICAL THINKING - *DATA EVALUATION OF IMPROVING ATTAINMENT IN MATHEMATICS PROJECT*

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This paper describes how the thinking process behind the data evaluation of a research project led to considering a holistic view of mathematical thinking. Improving Attainment in Mathematics Project (IAMP) is funded by the Esmee Fairbairn Foundation (grant number 01-1415) and involves three academic researchers and nine teachers-co-researchers. The aim of the project is to explore and develop ways of teaching and learning of below average attainers, focussing on stimulating mathematical thinking and understanding of key ideas in mathematics.

BACKGROUND OF THE PROJECT

The aim of the project is to improve attainment at Key Stage 3 of “below-average” learner by identifying key ideas in the curriculum and teaching in ways which focus on development of mathematical thinking rather than just teaching separate bits of mathematical knowledge. “Below average” learners are defined in this project as students whose achievement falls below the government target as they enter secondary schools i.e. on pupils who have not achieved level 4 in the KS2 SAT score nor in the KS3 optional tests, or who achieved level 4 but have since “slid back” to level 3.

Research by Boaler,1997; Ahmed *et al,* 1987; Watson, 2000 and Harries, 2001 show that low attainers often exhibit abilities to think mathematically. For example, low attainers have shown the ability to use examples and counter-examples, to generalise, to develop efficient methods of working, to move to higher levels of abstraction. Some of these are seen as features of advanced mathematical thinking (Tall, 1991; Krutetskii, 1976) because they relate very closely to the way that mathematics is internally structured. Research associated with the development of thinking skills in mathematics suggests that achievement can be improved, such as by explicit use of such skills in particular "Thinking Maths" lessons, resourced by particular materials (Ahmed *et al*, 1987). However, the project suggests a different approach, which is to develop teachers' and pupils’ abilities to incorporate appropriate thinking skills into *every* lesson, rather than depending on special activities. We work with nine voluntary teachers-co-researchers who are explicitly trying to develop the mathematical thinking of their pupils. They do this based on their personal aims and beliefs, their ideas of what mathematical thinking means.

THE DATA

The data collected include teacher diaries, lesson plans and evaluations, pupils' work, interviews, recorded lessons and observations, reflecting how teachers put ideas into practice in the specific context of low attainment. We also have SAT's progress tests, Key Stage 2 results and CAT scores where available .

Because the focus of the project is improving *attainment* of learners in mathematics we started looking at ways to 'measure' this. To make this profound, it was the intention to evaluate the effect of the project teachers' developing practice on three levels: by evaluating the scores of national tests of the project classes to comparative national tests scores; through teachers' normal assessment practices and informal reports; by devising project-specific performance indicators to assess developments in students' mathematics.

Using national test scores seemed at the beginning to be the obvious and 'easy' way of measuring improvement in attainment, it turned out to be rather complex and illusive. There seems to be a lack of robustness in the tests itself and in the levels awarded. There are cases of students taking the progress tests and not achieving level 4 but then taking the optional tests, which are on harder material, and achieving a "comfortable" level 4 (Wiliam, 2001). Not all our students sat the Progress Test. One teacher refused to set these tests for her students because she believed that the experience of doing so would undo the good work she had done all year in building self esteem. The students would have been faced with trying to answer questions on topics at which they had failed in the past and not had any subsequent teaching. Some teachers did not teach Year 7. We intended to use national statistics fro comparative analysis. However, so far, we have not been able to find any national statistics with which to compare results.

Overall, there was certainly agreement among teachers that the tests were not a good thing to be doing, nevertheless most students did do them. We also feel these tests do not tell us anything about mathematical thinking so far, although we are currently classifying questions to see how students did on different types of questions.

Other data included teacher diaries, pupils' work, interviews, recorded lessons and observations. We realised that looking for evidence of improving attainment was not necessarily the same as looking for evidence of mathematical thinking. One of the earliest discussions of the project focussed around the meaning of mathematical thinking. We soon sensed there would be no agreement or disagreement on what mathematical thinking is, just many interpretations and we would have to incorporate this lack of definition in our data analysis methods. But data from classroom observation, teacher notes and group discussions led to the identification of the range of pupil activity that the teachers in the group were encouraging, and we all agreed that these provided evidence of mathematical thinking.

These are summarised in Table 1 into two types, prompted and unprompted which might match against notions of dependence and independence.

Prompted	**Unprompted**
Choosing from a variety of techniques one which is appropriate Posing own questions Dealing with unfamiliar problems Making something more difficult Describing a connection or relationship in maths using prior knowledge Predicting problems Identifying what can be changed Working on extended tasks over time, generating own enquiry Identifying similarities beyond superficial appearance Making comparisons	Changing their mind in the face of new experiences Choosing from a variety of techniques one which is appropriate Initiating a mathematical idea or question Looking for connections and relationships in maths Dealing with unfamiliar problems Making something more difficult Using prior knowledge Predicting problems Identifying what can be changed Creating shortcuts Creating own methods Generalising a structure from a diagram, or from examples Finding similarities or differences beyond superficial appearance Making other kinds of comparison Contributing examples, especially where these need to be constructed Generating own enquiry

Table 1: Evidence of mathematical thought

However we soon found out that if the learners were not engaged in the lesson in an active way, there was little *observable* evidence of mathematical thinking. What we all agreed on was that a pre-requisite for mathematical thought, is engagement. At the other hand, being engaged does not guarantee learning (Helme and Clarke, 2001), nor mathematical thinking.

PEDAGOGY INVOLVED IN DEVELOPING MATHEMATICAL THINKING

We moved as a project to consider ways in which mathematical thinking might be supported in the classroom, i.e. the pedagogy. The teachers have quite a few common features in their pedagogy and in their aims and beliefs. What seems to be important here is not the actual pedagogic decision that is made, but the *purpose* of the decision. It is possible for two teachers to make apparently contradictory decisions which achieve similar aims. An example is given by the management of pencils: for one teacher a student bringing suitable equipment to class is seen as connected to a developing sense of identity as a learner; for another, the lack of a pencil is an unnecessary obstacle to learning so she gives out new ones without comment. Common features are:

Taking children to mathematics. Those of us listening to the teachers gained a strong picture that teachers were seeing their task in terms of structuring teaching in ways which enable students to make contact with and explore mathematics using their powers of thought and previous knowledge. We saw this as a contrast to restructuring or 'dressing up' mathematics in order to take it to students who are seen to be deficient by not knowing, or by having to be told how to think about it.

Connectivism. All the teachers wanted students to view mathematics as a connected, holistic way of working rather than as separate topics. Students have been asked to make connections in mathematics and to experience these connections, checking validity. An example: Becky gave students long exploration environments in which they could connect mathematical ideas for themselves as one question arose from another. Research by Askew et al (1997) shows that the most successful learners of mathematics at primary level are those whose teachers make connections within the subject; they called these "connectionist" teachers. At secondary level, it is possible to identify certain central themes in mathematics, which need to be understood as they appear in various mathematical contexts. This also relates to one of the aims of the project of improving attainment by identifying key ideas in mathematics.

Preparing to go with the flow. All the teachers 'go with the flow' of student response or mood and prepare for this deliberately. There is no point in battling against the moods and responses which tell teachers what the students are bringing to the task.

Creating own examples. All our teachers used the 'create your own example' type of task. This is very much part of their everyday lesson structure. Sometimes, it is a more separate activity For example, tasks of the type 'This is the answer, what is the question?' One student said:

> Making my own examples makes me think. I think about half the time in class now.

Allowing thinking time. All also found they were giving students a long time to think, including long wait-times with whole-class questions, but also in general throughout their work and interactions.

Duration of tasks. There has been in general a shift towards longer tasks in the project, if for no other reason than the fact that teachers are building more thinking time into their expectations. However, for a few teachers this is a deliberate major move in order to create an atmosphere in which students are embedded, surrounded, by a mathematical situation for several lessons. This goes completely against the belief that such students 'cannot concentrate' and that short concentration spans are a given characteristic of such students, rather than an effect of the task, and need to be 'treated' with task variety. This is particularly well-developed in Becky's case; she works on extending concentration spans of low attaining students by extending the time given to tasks.

Discussing mathematics. All teachers thought it was important for students to discuss mathematics with their peers, be it in pairs, in groups, or whole class discussions. All the teachers have a strong belief that everything said in class is valuable and everyone should hear it. For one, this leads to the practice of repeating everything which is said by students (ensuring everyone hears); for another this leads to the practice of repeating nothing and orchestrating discussion around what each student says (ensuring everyone listens).

The role of writing. All the teachers had thought about the place and role of writing. One teacher sees writing as sometimes a distraction from thought. For another teacher, the act of writing is seen as forcing thinking because it has to be expressed in a linear form. Others believe that writing gives you something of your own to look back at; a way to remind yourself what it is that you know.

Emotional responses and emotional security. There is a common recognition that students need emotional security, including respect for learners and trust in their ability to think. With some of the students there was the noticeable emotional development from extreme refusal to think at the start of the year to enthusiastic participation by Christmas.

Meta-cognitive learning. It is also seen as important that students know what they know. They encourage them to learn to learn, to handle problems, to reflect on learning, and to challenge ideas.

The visible effects on the classroom. The combination of respect, 'going with the flow' and holistic subject approaches has led to classrooms characterised by more discussion and more giving and taking of responsibility, choice and independence than before. All found themselves, either deliberately or incidentally, using fewer worksheets and textbooks and more activities, developments from starter tasks and students' own questions. The giving of more time, creating space rather than imposing pace, and offering choice by letting children devise their own questions, has been seen to have positive emotional, behavioural and cognitive effects. A student in Sara's class said:

> It's boring to be told what to do. It's nice to have time to choose … kind of relaxes me. When you are told what to do you don't want to do the work, you are bossed about … when you're relaxed you want to sit down and do the work.

CONCLUSION

We started by looking for evidence of improving attainment by using a variety of data. We accepted there is no definition of mathematical thinking, rather a multiplicity of agreed outcomes. Mathematical thinking is a process and that the teacher can play a role in facilitating its occurrence. Although the teachers are all different, they seem to share common aims, beliefs and pedagogy. We evolved project-specific performance indicators to assess developments in students' mathematics, herewith painting a holistic picture of mathematical thinking, finding indication of mathematical thinking and of the scaffolding offered by teachers, forming the cocoon that helps nurture this thinking and keeps distracting influences to a minimum. The teachers belief that students, whatever their attainment in mathematics, can think in a mathematical way. They let their students construct their own meaning and make sense of mathematics. They provide environments that encourage students to do so, such as the 'common features': giving time to think, extending duration of tasks, creating own examples, providing an emotional secure environment, etc.

The teachers *listen* to the students. They do not guess where the students are in their mathematical development, they ask and listen. The teachers still make inevitable assumptions when scaffolding, but the magnitude of the assumptions is reduced.

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