THE USE OF ORIGAMI IN THE TEACHING OF GEOMETRY

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This paper describes how Origami was used as a source of mathematical problem-solving in a series of lessons with Year 6 and Year 7 children. One of the strategies was to give groups of children an Origami object and allow them to discover for themselves how to make it. The children were asked to make posters to enable children in the other year group to make their object and were encouraged to reflect on the mathematics they used in completing the various challenges. Could origami be a starting point for geometrical activity which would be useful in primary-secondary liaison?

INTRODUCTION

There are members of the mathematics education community[1] who are convinced of the value of paper folding activities in the classroom to support the development of mathematical understanding. However, this is not widespread and there is certainly no mention of it as a strategy in official documentation. Internationally there have been a number of conferences on origami in education and therapy and on origami science, mathematics and education (see for example, Cornelius (1995) and Hull (2002)). In America many resources have been produced to encourage teachers to use origami throughout school and college teaching of mathematics (see for example Pearl (2002) and Jones, R. (2002)).

In a recent report by the Royal Society and Joint Mathematical Council (2001) on the teaching and learning of geometry the need for developing good models of pedagogy was highlighted (key principle 7). Recommendations of the report included opportunities for practical problem solving in geometry (recommendation 7) and the development of logical argument through geometry (recommendation 6). Theories of learning about geometry (for a concise summary see for example Jones, K (2002)) stress the importance of practical experience to support the development of visualisation and the ability to deal with relationships between geometrical properties. As an accessible practical activity, Origami also offered a potential bridge between primary and secondary mathematics.

A leading mathematics teacher working in a middle school with high attaining Y6 and Y7 students expressed concern that whilst her students were excellent at modelling number mentally and thinking logically to solve problems in number their

[1] Many members of the Association of Teachers of Mathematics and the Mathematical Association enjoy sessions at their annual conferences on mathematics and origami. Both associations have included articles on paper-folding in their journals: Mathematics Teaching and Mathematics in School respectively.

experience of practical mathematics and consequently their ability to work with geometrical concepts was very limited. Students would struggle with a practical task such as draw shapes with the same area as a given rectangle. Whereas if they were asked to find pairs of numbers whose product is 24 they would find it easy! I managed to persuade my colleague to work with me on some lessons using Origami as a starting point.

We planned lessons for Y6 and Y7 adopting strategies advocated by Wollring (2001).

- Only the very simplest folds are introduced to the whole class, students then investigate properties of the resultant shapes and justify their findings. Students are challenged to develop their shape into something more interesting.
- Students work in groups. Each group has two examples of a folded object. Advise students to dismantle just one object and figure out how it is made. As the objects are modular (i.e. made of more than one piece) once they have decided how one unit is made they can then work together to produce the units they need. By having one intact model they can figure out how to reconstruct the model.
- Students are asked to communicate their findings by preparing posters that could be used with other groups of students in order to make the same object. These posters should use as few words as possible.

We also asked students to reflect on the mathematics they had used and/or learnt during the lessons. This was something that the students were used to doing as their teacher regularly asked them to write about their learning of mathematics.

The purposes of the lessons were to investigate the extent to which Origami offers

- access to a range of geometrical concepts and opportunities to further develop understanding,
- opportunities to develop problem-solving and communication skills,
- appropriate challenge for students at the interface of primary and secondary mathematics.

THE PLANNED LESSONS

The first lesson for year 6 involved using A paper. Students were asked to fold the paper in half along the long mirror line, unfold, and to then fold one corner of the shortest edge onto the original crease to make a new crease through the adjacent corner. Students were asked to find out all they could about the resultant quadrilateral.

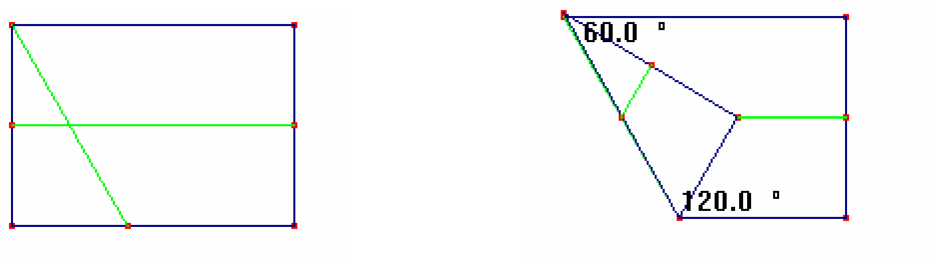

Figure 1: Crease lines on A paper, and trapezium with angles marked

Once students had agreed and come up with reasons for the properties of the trapezium, they were asked if it would be possible to make an equilateral triangle. Students found out how to fold their equilateral triangle into a triangle whose sides are half the length (what happens to the area?) and into a regular hexagon (what fraction of the area of the equilateral triangle is it's area?)

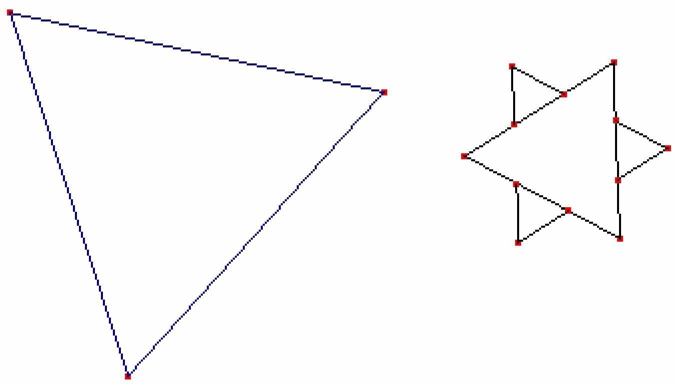

Figure 2: Fold a regular hexagram from an equilateral triangle

They were then shown a folded regular hexagram and asked how they might develop their equilateral triangle into such a shape. The second lesson involved students working in groups to develop a poster for other students (the year 7 class) to have a go at making the hexagram.

The first lesson for year 7 involved using pairs of pre-folded stars with small groups. The challenge was to figure out how to make the star. Students were advised to pull one star apart and to keep one intact. The star is a classic origami model which, unlike many modular origami objects, uses two symmetric units.

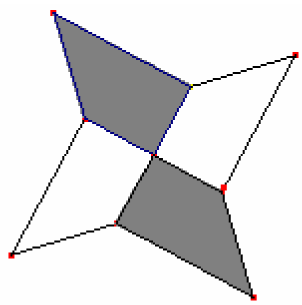

Figure 3. The 4-pointed star made from two symmetric pieces

Students were challenged to determine the properties of this star and then, in their groups to prepare a poster which showed how to make the star that could then be used with other students (the year 6 class).

Subsequent lessons were spent completing the posters, reviewing the mathematics done, using the posters prepared by the other class, critiquing these and preparing revised posters.

THE OUTCOMES

Students thoroughly enjoyed the practical work and the experience of working collaboratively in small groups. They showed considerable determination and perseverance; for example, by the end of the first lesson only a few in year 7 had managed to complete the 4-pointed star. By the end of lunchtime everyone could do it; there had been some serious paper-folding going on in the playground! Students were extremely supportive of one another in their groups and in the class. They were keen to show what they had figured out but were also patient in allowing others to figure things out for themselves.

In year 6 several students thought that the acute angle in the trapezium (see figure 1) was $45°$ because they had folded a right angle. Using a number of trapezia together, or folding the acute angle in half convinced students that this wasn't the case. Students used a range of approaches to show that the angle was indeed $60°$:

- When you halve the acute angle and unfold the right angle is split into three equal angles so the acute angle must be double 90 divided by 3, i.e. $60°$.
- The acute and the obtuse angle are a straight line and the obtuse angle is twice the acute angle so the angles must be $60°$ and $120°$.
- Three acute angles together make a straight line so the angle must be $60°$.
- Six acute angles fit together around a point so the angle must be $60°$.

When completing the posters students were encouraged to use partly folded pieces of paper to illustrate the process rather than lots of words. All groups in both classes successfully completed a poster to demonstrate how to make their object. During lessons students were encouraged to articulate and share their strategies. They also completed written tasks reflecting on the mathematics they had used.

As well as recognising the geometrical content: angles, symmetry, properties of shapes, names of shapes, properties of angles within parallel lines, angle sum of a triangle, angle sum of a polygon, many students identified aspects of problem-solving such as 'there is not just one way of doing something' and the importance of communication and co-operation. Several were concerned with producing a quality product and so commented on the importance of accuracy and taking care. Others were impressed that careful folding could produce particular angles and lengths — 'you don't need a ruler to measure accurately'. The year 7 students noted that ratio was important when producing their star as the starting piece of paper needs to be a rectangle with sides in the ratio 1:4. Students in year 6 also noted fractions having been challenged with the questions about what fraction of the equilateral triangle are the hexagon and hexagram.

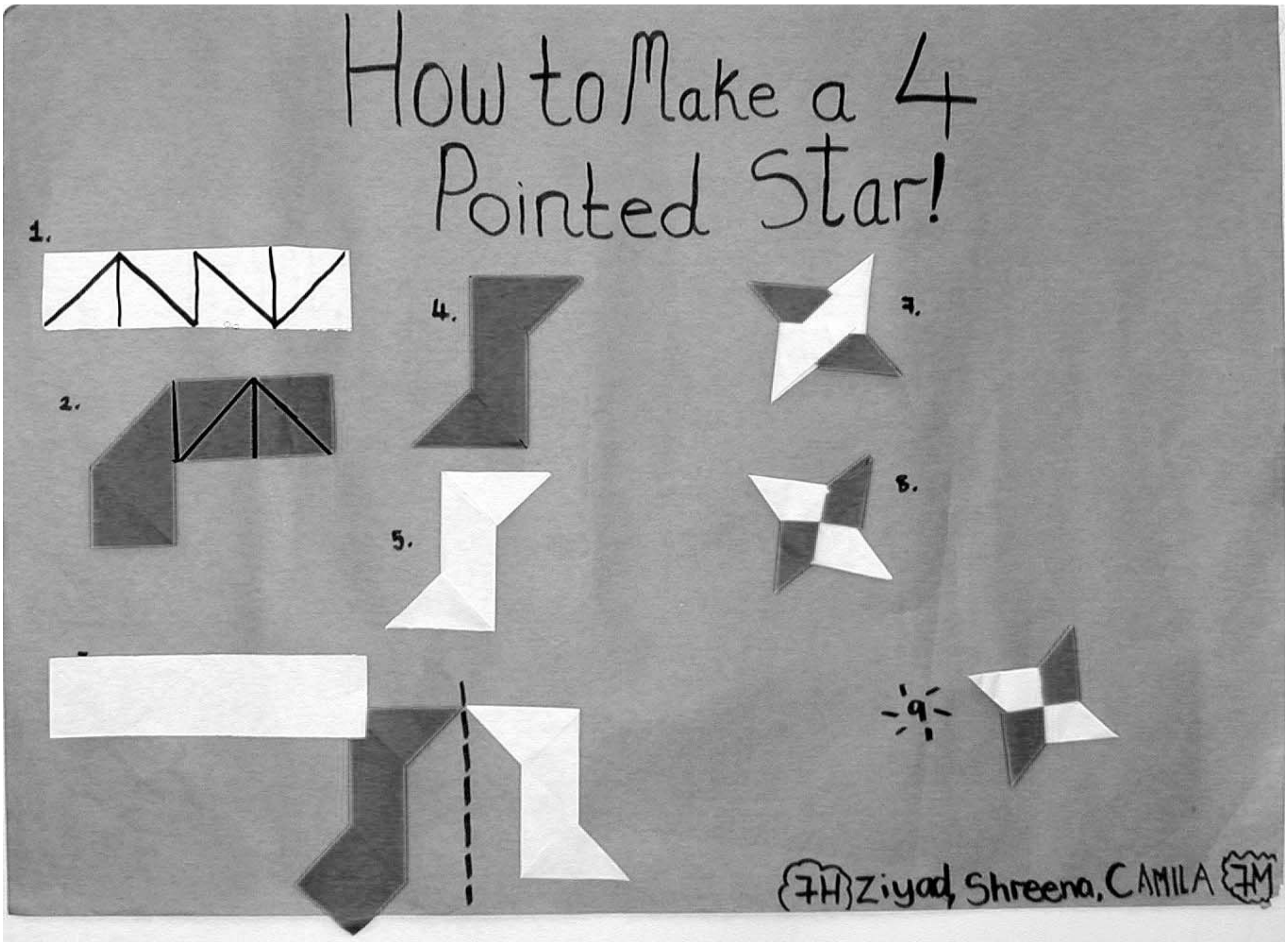

Figure 4. Poster designed by year 7 students

Figure 4 shows a typical poster by a group of year 7 students which was used by a group of year 6 students with no further instruction or guidance to recreate the star. Using the posters in this way with the different classes offered a suitable challenge across the two year groups. Each class had become expert in their original model but using a poster for a completely new model revealed weaknesses in the way that they had communicated their method for making the model. For example few posters made by year 7 showed that any rectangle with sides in the ratio 1:4 would do, consequently some children in year 6 measured their rectangles rather than finding a folding strategy to make a rectangle with the required property. None of the year 6 posters showed finding the centre of the original triangle as a step in making the hexagram, consequently it took year 7 some time to discover why their hexagrams 'looked odd'. Students found that being able to interact with the poster display was very helpful, consequently when revised posters were made students stuck down their interim models far more thoughtfully as they tried to envisage someone wanting to undo part of the folding to gain more insight into the process.

Despite this the students successfully recreated the other year group's model, critiqued the poster they had used and produced a revised version. Although the revised versions were easier to interact with they didn't necessarily address the other issues mentioned above. Although the students could make the models, communicating the process any more clearly than on the original poster was not easy.

The teacher was pleased with the way her students worked on these tasks and the mathematics they were learning. Some time after the lessons she gave the Year 6 students some typical KS2 level 6 Shape and Space extension questions on angles. The class had had no formal teaching on angles during the year apart from the work on Origami. She was delighted that all the children completed the questions successfully, and attributed the success to the learning that had occurred earlier.

IMPLICATIONS FOR THE CLASSROOM

The use of Origami in the lessons described allowed a wide range of mathematics to be covered in a stimulating and enjoyable way. Alongside the development of mathematical understanding the students were developing their problem-solving skills and their ability to work co-operatively with others. Producing posters challenged students to communicate their method succinctly to an audience. The students rose to this challenge even though not wholly successfully.

This approach straddled Primary and Secondary mathematics effectively. In this particular series of lessons the same teacher taught both groups but it would be interesting to find out if the same approach could be used in different schools with different teachers. Were it effective, then it could make an excellent liaison activity rich in mathematical opportunities.

ACKNOWLEDGEMENT

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