# PROSPECTIVE TEACHERS' SUBJECT MATTER AND PEDAGOGICAL CONTENT KNOWLEDGE OF VARIABLES

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*The knowledge base of teaching is an amalgamation of different forms of knowledge. In this paper I focus on prospective teachers' subject matter knowledge as a source of pedagogical content knowledge. I illustrate how subject matter knowledge affects prospective teachers' pedagogical decisions in the context of variables.*

## INTRODUCTION

The concept of variable is one of the most fundamental concepts in mathematics from elementary school through to university. However, research conducted in many countries indicates that students experience difficulties on their journey to learning the concept of variable. Although it is so fundamental and so difficult to learn for some, we do not know enough about teachers' or prospective teachers' knowledge base for teaching this concept; in particular subject matter knowledge and pedagogical content knowledge. This paper concentrates on prospective secondary mathematics teachers' knowledge and understanding of this fundamental concept- the concept of variable. It is based on a study of the relationships between subject matter knowledge and pedagogical content knowledge of variables.

## THEORETICAL BACKGROUND

It is commonly agreed that teachers' professional knowledge which is the knowledge base of teaching is an amalgamation of different forms of knowledge. There are different ways of classifying the knowledge base of teaching. One of the most influential classifications is suggested by Shulman (1986), who distinguishes several components of the knowledge base of teaching: subject matter knowledge; pedagogical content knowledge; general pedagogical knowledge; knowledge of educational aims. He describes pedagogical content knowledge, as "in a word, the ways of representing and formulating the subject that makes it comprehensible to others" (p.9), and says that it includes:

> an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons (p. 9).

Shulman (1986) also argues that teachers need to have two kinds of understanding of subject matter- knowing "that" and knowing "why":

> We expect that the subject-matter content understanding of the teacher be at least equal to that of his or her lay colleague, the mere subject-matter major. The teacher need not only understand *that* something is so; the teacher must further understand *why* it is so (p. 9).

Even and Tirosh (1995) illustrate how “knowing that” and “knowing why” affect teachers’ decisions about the presentation of the subject matter in the context of *functions* and *undefined mathematical operations*.

I will discuss how these types of understandings affect prospective teachers’ responses to pupils’ questions related to *variables*.

## VARIABLES

In this paper, the definition of the term variable is taken as the literal symbols that are used to represent numbers. As many others point out; there are different uses and conceptions of variables: variables as *generalized numbers*, variables as *unknowns* or *constants*, variables as *parameters* or *arguments*, variables as *abstract symbols* (Usiskin, 1988; Kuchemann, 1978; Ursini and Trigueros, 1997). However, the principal uses of variables that are part of the school curriculum are variables as unknowns, as generalized numbers and as varying values. (Kieran, 1990; Ursini and Trigueros, 1997).

Hence “knowing that” in the context of variables includes symbolization, manipulation and interpretation of each one of these uses in different mathematical situations. This makes the “basic repertoire” of subject matter knowledge of variables. Even (1993) argues that teachers should acquire the “basic repertoire” which gives insights into, and a deeper understanding of general and more complicated knowledge. General and more complicated knowledge of variables is integrating all these uses into one concept, and shifting from one to another in a flexible way.

In the context of variables, “knowing why” includes comprehension of why rules in manipulation of literal symbols work and anticipation of the consequences of using these rules.

## METHODOLOGY

Participants in the study were 184 prospective secondary mathematics teachers in the second, third and fourth years of their preservice education at three universities in Turkey. Data were gathered in two phases. During the first phase, 184 prospective teachers completed an open-ended questionnaire. The first part of this questionnaire which consists of 10 questions dealt with several aspects of subject matter knowledge of variables that are considered to be essential for teaching variables. The second part which consists of 6 questions addresses components of pedagogical content knowledge of variables. In the second phase of data collection, ten prospective teachers were interviewed after analysis of the questionnaires.

## RESULTS

The report here is based on one of the questions which were asked in the first phase of the study. In this question prospective teachers are presented with a scenario in which they have to respond to a student's question which may be asked when they teach about variables:

> How would you react to your students' questions as below in the classroom? Explain!
>
> a) "Teacher, why does 2a+5b not equal 7ab?"
>
> b) "While solving equations, why does *x* change its sign when it is brought to the other side?"

**Table 1- categorisation and distribution of "knowing why" of part a**

| | | Frequency | Percent |
|---|---|---|---|
| Valid | Number | 33 | 17.9 |
| | Object | 68 | 37.0 |
| | Invalid explanation | 58 | 31.5 |
| | Total | 159 | 86.4 |
| Missing | | 25 | 13.6 |
| Total | | 184 | 100.0 |

For both parts of this question more than 80% of the respondents write that they would try to explain it to the student. Responses to part (a) were categorised as seen from the table on the left.

A few students 33 (out of 159 who answered part (a)) gave a valid explanation to part (a) by making an analogy with numbers and/or explaining it by a counter example. For example:

— "Assume that the two expressions are equal and then substitute values for *a* and *b*; and see they are not equal, i.e. for $a$=2, $b$=3, $2a + 5b = 19 \neq 42 = 7ab$"

68 of the respondents preferred to explain this part by giving meanings to *a* and *b* as objects. *e. g.*

— "2*a*+5*b* is 2 of *a* and 5 of *b*, like we say 2 apples and 5 pears."

— "We can add quantities of the same kind. We can't add *a* & *b* together since they are of different kinds."

However, giving meaning to letters as objects, sometimes labelled 'fruit-salad algebra' is criticised in the literature. One of the reasons for this criticism is that this approach encourages students to perceive letters as objects. Another reason is that this approach begins to be unsuccessful in situations where brackets or minus signs are used (How can a banana be minus?). Therefore, this type of responses can also be considered in "Invalid explanation" category.

Another large group of respondents (58) do not give a valid explanation for why 2a+5b is not equal to 7ab:

— "I have never taught. But, when they ask, I try to explain it patiently. I am here since I trust my patience and I like maths."

— "I begin with explaining the meanings of multiplication and summation"

For part b, 67 students (out of 149 who answered part b) do not give a valid mathematical explanation.

- "It is a rule"
- "everything that goes to the other side changes its sign, therefore *x* changes its sign"
- "Let's write an equation. 2*x*-5=0 ➔ here let's take *x* to the other side, we get -5=-2*x*. Because the sign of *x* is +. This passes to other side as -. The inverse of addition is subtraction"
- "First of all I would want them to think of the equation as a number line. If we think of '=' as zero, the numbers on opposite sides of zero have opposite signs. Therefore when we pass *x* to the other side of the symbol '=' it changes its sign"

**Table 2- categorisation and distribution of "knowing why" of part b**

| | | Frequency | Percent |
|---|---|---|---|
| Valid | Number Analogy | 14 | 7.6 |
| | Preserve the equality | 68 | 37.0 |
| | Invalid explanation | 67 | 36.4 |
| | Total | 149 | 81.0 |
| Missing | | 35 | 19.0 |
| Total | | 184 | 100.0 |

These prospective teachers "know that" a literal symbol changes its sign when it is brought to the other side. However, they seem to forget or don't know why this rule works. This seems to affect their explanations.

On the other hand, more than half of prospective teachers (82) "know why" this rule holds and therefore provides a valid mathematical explanation to the student. However there are differences in their given explanations. A small group of respondents (14) draw an analogy to numbers to explain the rule. e. g.

- "Like the numbers change their signs when they change side, since *x* as well represents a number it changes its sign when it changes its side."

Another group (68) mentions that we have to keep both sides of the equation equal and explains where this rule comes from:

- "Changing the sign means subtracting or adding *x* to both sides. On one side *x* cancels out –*x*, and on the other side we get –*x*."

This group of students "know why" the literal symbols change their signs and they make use of this knowledge in their reactions to students' questions. This may suggest that subject matter knowledge of prospective teachers affects their pedagogical decisions on reacting to students' comments and questions.

## DISCUSSION

In this paper I dealt with prospective teachers' presentation of subject matter knowledge of variables when faced with students' questions. As Even and Tirosh (1995) point out teachers may respond to such questions considering different aims, such as "encouraging cooperative work among students, making students feel good, etc" (p.17). I analysed prospective teachers' responses in the light of existing literature on subject matter knowledge as sources of pedagogical content knowledge.

**Table 3- SMK question * part a) Crosstabulation**

Count

| | | part a) | | |
|---|---|---|---|---|
| | | number | object | Total |
| SMK question | number | 52 | 42 | 94 |
| | object | 5 | 13 | 18 |
| Total | | 57 | 55 | 112 |

This crosstabulation is obtained after recategorisation of part a) just considering whether their explanation contains objects or numbers

I differentiate between "knowing that" and "knowing why" types of subject matter knowledge of variables. It is generally agreed within the mathematics education community that teachers should have both kinds of knowledge. (e.g. Even & Tirosh, 1995; Skemp, 1976). As we saw in this paper, prospective teachers "know that" there are certain rules about manipulation of literal symbols. When it comes to "know why" these rules hold, they do not show the same success. For part a) 80 % of respondents do not provide a valid explanation. About half of these responses treat letters as objects; the other half do not even provide any sort of valid explanation.

Inferences about their subject matter knowledge of variables are further facilitated by interviews, and also by cross tabulating of this question with other questions which assess their subject matter knowledge. The figures in contingency tables show connections between SMK questions and PCK questions. For example, the content of the contingency table (Table 3) of part a) with one of SMK questions which reads "*What different things might an algebraic expression such as, say 2x+1, mean? What can x stand for?"* shows: most of the students writing *x* can be a number in 2*x*+1 employed explanations, for part a, involving numbers while most of those who did not write *x* can be a number used objects in their explanations. The evaluation of this connection by chi-square gives $p$=0.03, which is statistically significant at 0.05 level. This may suggest that these prospective teachers themselves treat letters as objects. In fact, in the interview when I asked "*How could an apple be minus*?" to one of the students after she said *x* can be an apple, she answered:

> Let's say there are some apples or pears in a box. Saying minus b means, for example, if we mean pear by b, taking a certain amount of pears from that box, that is, reducing it…

This student teacher regards minus sign as an action (taking out) on objects and sees no danger in considering "–b" as minus pear. However, we see that most of the students do not give such an explanation for part b).

For part b) more respondents give a valid explanation. 14 students draw analogies to numbers and none of them use objects to explain why literal symbols change their signs when they are brought to the other side.

The concept of variable has already been studied by these prospective teachers during their own secondary and high school years. They have managed to overcome the difficulties in manipulating literal symbols. They are well versed in algebraic manipulation. However, as we saw in this paper, they have forgotten or never learnt the reasons "why" rules in these manipulations work. Therefore, this affects their pedagogical explanations to students' questions.

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