TEACHING YOUNG CHILDREN TO MAKE EXPLICIT THEIR UNDERSTANDING OF NEGATIVE MEASURES AND NEGATIVE RELATIONS

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A previous study (Borba and Nunes, 1999; 2000) showed that young children solve negative number problems significantly better orally than when requested to provide an explicit representation for negative numbers before solving the problems. This paper presents an intervention study aimed at discussing with young children how they can explicitly represent the understanding they already have about negative numbers. It was also investigated if instruction on one number meaning- measure or relation - is transferred to another. All instructed groups improved in performance in comparison to a control group. Understanding of relation problems was transferred to measure problems but understanding of measures was not transferred to understanding of relations.

The aim of this study was to test an instructional programme designed to help children overcome difficulties in representing negative numbers and in reasoning about different number meanings for negative numbers.

The instructional programme was based on findings of a previous study that will be reported briefly followed by a more detailed report of how the intervention study was performed and the results obtained.

THE EXPLORATORY STUDY

Borba and Nunes (1999, 2000) investigated the effect of number meanings, systems of signs and conceptual invariants on reasoning about negative numbers.

Method

Participants

Students from a North London school took part in the exploratory study. All 60 children (mean age: 8y 4m old) attended Year 3.

Design

The children were randomly assigned to four experimental groups as shown in Table 1 and individually interviewed.

Half of the students solved problems that involved the number meaning of *measure*, half solved problems about *relations*. Half of the children solved the problems *orally*, half were requested to provide an *explicit representation* of the numbers involved in the problem situation and to operate on these representations. The explicit representation could be either in writing or by use of manipulative material chosen amongst those made available (coloured cards, marbles, rulers or sticks). Half of the problems the children solved were *direct* (final measure or final relation unknown) and half were *inverse* (initial transformation unknown).

	FORM OF REPRESENTATION	
NUMBER MEANING	Implicit (Oral)	Explicit (writing or manipulatives)
Measure	G1 (n = 15)	G3 (n = 15)
Relation	G2 (n = 15)	G4 (n = 15)

Table 1: The experimental groups of the exploratory study

Each child solved 12 problems, presented in the context of a pinball game and that involved three transformations (points won or lost in the game). An example of a measure problem was:

> "I won one point, then I won three points, then I lost six points. How many points am I winning or how many points am I losing at the end?"

An example of a relation problem was:

> "I had some points yesterday. Today I played again and I won one point, then I won three points, then I lost six points. How many points more or how many points fewer do I have now?"

Results

The exploratory study showed that problems about measures were significantly easier than problems about relations (F (1,52) = 10.07, p < .005). Significant differences were also observed when the pupils solved problems with versus without explicit representations (F (1,52) = 16.88, p < .001). Solving direct problems was significantly easier than solving inverse problems (F (1,52) = 27.65, p < .001). No significant differences were observed between boys 'and girls' performances, nor significant interactions between main effects.

Qualitative analysis showed that children that performed well when asked to reason about the more difficult number meaning – relations – were those children that understood that if the transformations occurred during the game were known, it was not necessary to know how many points a player had at the beginning of a game to find out if he/she had more or fewer points at the end of a game.

Qualitative analysis also showed that the difficulties with having to make explicit the negative numbers involved in the problems could be overcome when children marked positive and negative numbers differently; when negative numbers were differentiated from the operation of subtraction; and when children correctly interpreted the results obtained from operating on the explicit representations they had generated.

These results were used to design the intervention programme.

THE INSTRUCTIONAL STUDY

The intervention study aimed to work on children's representational difficulties and also difficulties related to number meanings.

Method

Participants

Participants were 80 children in Year 3 (mean age: 7y 8m old) of a North London school.

Design

At pre-test the children were interviewed individually and requested to solve measure and relation problems after providing an explicit representation of the data. They could either write down the numbers or use manipulative material for their representations.

Pre-test results were used to match the children, who were then randomly assigned to one of the five groups shown in Table 2. All five groups had the same mean of correct answers in the pre-test.

	FORM OF EXPLICIT REPRESENTATION	
NUMBER MEANING	Use of coloured cards	Writing
Measure	G1 (n = 15)	G3 (n = 15)
Relation	G2 (n = 15)	G4 (n = 15)
Control: Multiplication: one-to-many correspondence	GC (n = 16)	

Table 2: The experimental groups of the intervention study

Successful procedures had been observed in the exploratory study amongst children that used manipulative material to explicitly represent and operate on the numbers involved in the problems and children that wrote down the numbers. For the intervention two systems of signs were selected. Children were requested to either explicitly represent the numbers by *use of coloured cards* or to *write down the numbers*. This would enable the further investigation of the role of systems of signs on reasoning about negative numbers.

The same number meanings for negative numbers investigated in the exploratory study were used as base of the instruction: two groups were instructed about *measures* and two groups about *relations*. This would enable the investigation of the possibility of transferring the understanding of one number meaning to another.

A control group was instructed on the multiplication of natural numbers, by use of the schema *one-to-many correspondence*. This control group was included to observe if just by being asked some time later to reason about negative numbers was sufficient for children to improve in their understanding of negative numbers.

After one individual intervention session, each child was individually post-tested.

The pre- and the post- test were identical: six problems about measures and six problems about relations. All the problems involved six transformations (points won or lost) and this number of transformations was a good reason for the children to feel the need to explicitly represent the numbers involved in the problem situation.

Results

The performance of each group in the pre- and post-test is shown in Table 3.

	PRE-TEST	POST-TEST
G1 (Cards/Measure)	m = 1.50 SD = 2.63	m = 6.75 SD = 4.70
G2 (Cards/Relation)	m = 1.50 SD = 2.68	m = 9.25 SD = 4.85
G3 (Writing/Measure)	m = 1.50 SD = 2.39	m = 6.56 SD = 3.81
G4 (Writing/Relation)	m = 1.50 SD = 2.45	m = 9.81 SD = 4.15
GC (Multiplication)	m = 1.50 SD = 2.53	m = 1.19 SD = 2.43

Table 3: Mean of correct answers (out of 12) in the pre- and in the post-test by experimental group in the intervention study

Significant differences were observed between performances on pre- and on post-tests amongst all groups, except the control group.

No significant differences were observed in the form of explicit representation. Holding number meaning constant, those that used coloured cards performed as well as those that wrote down the numbers.

Holding form of representation constant, significant differences were observed between number meanings used in the instructional programme. Those instructed on relation problems improved in the post-test not only on relation problems, but also on measure problems. Those instructed on measure problems improved mostly on measure problems.

The improvement on children's use of explicit representations

In order to examine how children's use of explicit representations improved an eight-year old girl's procedures will be examined.

In the pre-test she chose to use the marbles and answered only two of the problems correctly. She was told of a game in which initially there had been a loss of six points. She did not use any marbles to represent this initial loss saying, "because there are no points to lose". Then the child was told of a gain of two points. She represented this gain by putting two marbles on the table. Putting the two marbles on the table back with the original group of marbles saying, "because he lost more points than he won", represented a subsequent loss of three points. When told of a gain of five points she represented this positive transformation by putting five marbles on the table. Putting back the five marbles she had put on the table represented a subsequent loss of seven points. Putting four marbles on the table represented the final gain of four points. The child replied that the player's score at the end was "winning four".

In this girl's explicit representation all negative transformations were represented as subtractions and when a greater number was being subtracted from a smaller one the result would always be 'zero'.

During instruction, the girl learned to represent all number before solving the problem and to use cards of different colours to indicate whether the number was positive or negative.

At post-test she chose to use the marbles as she had done in the pre-test. She put the 'winning points' in one of the lids of the box used to keep the examiner's material and the 'losing points' in the other lid. The problem describing a loss of six points, followed by a gain of two points, a loss of three points, a gain of five points, a loss of seven points and a gain of four points was now represented as:

Because the marbles were approximately the same size and colour this was the way the child chose to mark distinctively positive and negative transformations. The child then counted all the gains (11 points won) and all the losses (16 points lost) and correctly answered that the player had at the end of the game a 'losing score of 5'.

Children from the 'cards groups' (G1 and G2) and children from the 'writing groups' (G3 and G4) devised efficient ways of representing positive and negative transformations and operated correctly on these representations. The basic procedure was that of 'cancelling out' positive values with correspondent negative values, a procedure observed in other instructional programmes, such as the one proposed by Linchevski and Williams (1999).

The improvement on children's understanding of number meanings

Learning about a more difficult number meaning – relation - helped the children in their understanding of a less difficult meaning - measure. The children who were taught about measures did not improve much on their understanding of relations. The incorrect procedures to deal with relations, used by the children taught about measures were:

a) guessing (because they said they could not possibly answer unless they guessed);

b) separately adding all the gains and adding all the losses, and presenting the greatest of these sums as final answer (realising that if one wins more than loses, the relation is positive; or if one loses more than gains, the relation is negative; but not being able to correctly quantify the relation);

c) assigning an initial value to start with, obtaining the correct relation by composing the transformations involved, then comparing this relation with the initial value assigned , instead of presenting the composition as the final answer;

d) comparing the result of one problem with the result obtained in a previous problem.

CONCLUSION

The instructional programme enabled children to overcome difficulties in making explicit their understanding of negative numbers, and reasoning about a more difficult meaning of negative numbers (negative relation) enabled children to reason about a less difficult meaning (negative measure).

One educational implication concerning the results obtained in this intervention study is that a careful assessment must be made concerning what dimensions of a concept – number meanings, systems of signs used to represent the concept, or conceptual properties - are not yet understood by students before initiating an instructional programme. This assessment can enable the teacher to focus on the dimensions the students do not yet understand.

Another implication would be that students that are cognitively challenged – as the children that were asked to think about a more difficult number meaning – can also improve in their understanding of less difficult aspects of mathematical concepts.

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