HUNGARY AND ITS CHARACTERISTIC PEDAGOGICAL FLOW

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***Abstract**: this paper reports on almost one hundred observations of mathematics classes in Budapest, Hungary. Undertaken with regard to the survey of mathematics and science opportunities team's framework, the observations confirmed a characteristic pedagogical flow. Lessons tended to follow a well-defined pattern: a public review of homework was followed by a "warm-up" period which, in turn, was followed by several episodes involving the posing, individual working on and public sharing of solutions of problems before homework was set again. Significantly the mathematics studied and the problems posed presented a perspective on the subject which appeared to privilege mathematical problem-solving, complexity, generality, coherence, topic integration and mathematics as a cultural artefact. Some implications in respect of raising attainment in England are discussed.*

Introduction: much attention has focused recently on international comparisons of mathematical attainment (Beaton et al, 1996, Mullis et al, 2000) with less paid to supplementary studies concerned with the nature of curricula from the intended and implemented perspectives. Two of these, the TIMSS video study (Kawanaka et al, 1999) and the survey of mathematics and science opportunities (SMSO) (Schmidt et al. 1996), have shown how comparative research can offer meaningful insights into the ways in which mathematics is taught and conceptualised for teaching and learning.

Methodological framework: the SMSO's initial brief was to explore the pedagogic traditions of six countries through focusing on the nature of the mathematics being taught and the manner in which participants interacted with it and each other (Schmidt et al 1996). Observations were undertaken by home researchers and despite early problems concerning perspectives being clouded by implicit and contextually embedded assumptions about the nature of classroom activity, protocols were developed and a process of interrogation was adopted whereby representatives from one country were questioned by representatives from others to make explicit that which had been implicit. Eventually the expression "characteristic pedagogical flow" emerged to describe the study's findings because interest lay with "the pedagogical strategies and approaches typical of a set of lessons" which were "enacted repeatedly in a country's classrooms" and, which, appeared routine and almost "below the conscious level for most teachers" (Cogan and Schmidt 1999 p71/72).

Background: a fortunate consequence of a TEMPUS project between the Eötvös Loránd Tudomanyegyetem in Budapest and the Manchester Metropolitan University was that colleagues from both institutions were able to observe the teaching of mathematics overseas. A few accounts were published (Hatch, 1994, Andrews, 1995, 1997) which convinced us that much could be learnt from a more thorough study of the traditions of the two countries. In 1996 the British Council and the Hungarian Ministry of Culture invited collaborative proposals which led to a project for the academic years 1997/8 and 1998/9.

Method: the project was symmetrical in that the same number of visits was made in both directions by similar numbers of colleagues from both countries. The first visits were preparatory in that they were concerned with establishing a common understanding of the task, developing a shared vocabulary and gaining an initial perspective on each other's traditions. This process was facilitated by the relationships developed earlier during the TEMPUS project - colleagues had already developed a familiarity with each other's contexts, both professional and social.

Between February 1998 and May 1999 the equivalent of eight one week visits were made in each direction although we focus here solely on visits made to Budapest. Each lesson observation was made by at least one English and one English-speaking Hungarian colleague. Almost one hundred observations were made. The teachers observed had volunteered for interview subsequent to a questionnaire survey concerned with teachers' beliefs and practices. There are no guarantees that they were representative of Hungarian teachers although there was variation in the quality of the lessons seen and our Hungarian colleagues have argued that the schools visited formed a representative sample.

The first week's observations involved a schedule intended to facilitate the recording of details pertinent to the study. However, the enforcement of contemporaneous coding was found to compromise data collection. Consequently almost all lessons were recorded in extended note form. Thus, whatever was recorded was determined in the moment and constrained, despite our beginning to develop a rudimentary Hungarian mathematical vocabulary, by Hungarian colleagues' translations and their choices in respect of what they chose to describe. Three or four lessons were viewed each day and data gathered included details of the classroom discourse, the mathematical tasks and activities, the actions of participants, the resources used and the nature of the classroom. At the end of most days colleagues were able to discuss the lessons observed and agree an account of, but not an account for, what had been seen.

During pre-project discussions it became apparent that observing single lessons - as in the original TEMPUS project - yielded too partial a picture. We could not see, for example, how topics might be developed over time or how homework might link lessons. Therefore we viewed all the lessons taught by teachers to a class of students for the week of the respective visit.

First, however, we offer a brief description of Hungarian education. Formal schooling starts at the age of six and passes through three four year phases - lower primary, upper primary and secondary. Our experience indicates that most children in primary grades are taught in single institutions called general schools (Általános Iskola) with lower primary taught by generalists and upper primary by specialists. At age 14 children generally transfer to one of three types of secondary school - an academic school (gimnázium), a vocational school (although we argue that the term technical might be a more recognisable description) or a trade school (Báthory, 1995). The school day begins at 08.00 and is followed by six periods of 45 minutes - we have seen no variation to this pattern which conflicts with earlier accounts of a five period day (Garden and Livingstone 1989). Between lessons is a gap of ten or fifteen minutes allowing for both physical and psychological breaks. Thus teachers frequently work through the bell confident that the next lesson will start on time as pupils have no excuses for lateness. Relationships and dress codes are informal. Children know and use teachers' forenames although they are preceded by the titles Bácsi and Néni - respectful titles given to older males and females respectively. Classrooms are clean but sparsely decorated. Almost all have wall-mounted blackboards with hinged side panels which allow teachers to write up work before the lesson and reveal it at the appointed time. The majority of observed lessons were in the upper primary phase - grades five to eight - although a small number in grades outside that range were viewed to help our understanding of the wider picture.

Results: we attempt here to describe the characteristics of mathematics teaching in Hungary. It is a task which we believe requires an examination of the tasks set to pupils, and the nature of participants' engagement with them. Therefore, most of this section comprises examples of activities and tasks offered to Hungarian pupils from which we have constructed our perception of the characteristic pedagogical flow of that country. Most of the lessons we have seen comprise four phases. The lesson starts with a review of the homework set the previous lesson. This is followed by a warm-up period after which comes the main part of the lesson where a problem is posed which, after a few minutes' individual work, is solved collaboratively and publicly (this phase may be repeated several times). Lastly, a new homework task is set.

The Homework review: in general teachers invite pupils to the board to share their solutions to problems set the previous lesson. The teacher's role is facilitatory in that he or she offers prompts at moments of hesitation, probes when clarification is thought necessary, summaries at the end and general encouragement and support for the active pupil.

Homework problem 1: this was one of three related problems given to a grade 7 class following some work on inequalities and their solution sets.

Solve $$2.x + (-4) \geq -3.x + \frac{6}{3}$$

An invited volunteer suggested that the fraction could be simplified because the addition of negative four could be viewed simply as a subtraction. He re-wrote the inequation as

$$2.x - 4 \geq -3.x + 2$$

before working through the problem, explaining as he went, what he was doing and why. The oral description was matched by what he wrote on the board.

$$\begin{array}{l|l} 2.x - 4 \geq -3.x + 2 & +3x \\ 5.x - 4 \geq 2 & +4 \\ 5.x \geq 6 & :5 \\ x \geq \frac{6}{5} & \end{array}$$

A second pupil was invited to check the work which was also done publicly.

Homework problem 2: was given to a grade 5 class after some introductory work on decimals and place value. Pupils had to place in ascending order the following:

$$0.4\dot{5} \quad 0,\dot{4}\dot{5} \quad 0,405 \quad 0,45 \quad 0,5\dot{4} \quad 0,54 \quad 0,45\dot{4} \quad 0,\dot{5}4\dot{5} \quad 0,4\dot{5}\dot{4} \quad 0,\dot{4}5\dot{4}$$

The response to the problem was collaborative with ten different pupils invited to share their perspectives. Each, in turn, wrote against each number its ordered position in the list. Thus the first pupil wrote a 1 against 0,405, the second a 2 against 0,45 and so on until the order was completed.

Homework problem 3: concerns some work on angle given to a grade 5 class which had been doing some introductory work on angles in plane shapes. The question was: I have two angles which sum to 92^{o}. If one is three times the other, what are they? A child was invited to explain what she had done. She wrote (on four lines rather than one):

$\alpha + \alpha.3 = 92^{o}$ $\alpha + \alpha + \alpha + \alpha = 92^{o} | :4$ $\alpha = 23^{o}$ $\alpha.3 = 69^{o}$

Homework problem 4: concerns a single question set to a grade 7 class following work on the area of simple plane figures. An isosceles triangle has an area of 9 units squared and a vertex at the point (2,3). If the remaining vertices are on integer points then how many such triangles are there? The discussion, which lasted around five minutes, involved more than half the class. Eventually an agreement emerged that the desired triangles were best viewed as rectangles of area 18 units squared, that the triangle's base should be even for the third vertex to lie on a grid point, that all possible integer-sided rectangles could accommodate an appropriate triangle, and that rotations and reflections yielded 36 possible solutions.

The warm-up period: many of the teachers we saw went through what our translators described as a warm-up period. Generally this involved children responding to four or five orally-posed questions. The purpose of this section of the lesson, we infer, is to shift pupils' attention from the just-discussed homework, to offer an opportunity for the practise of routine calculations and logical thinking, and to prepare pupils mentally for the lesson proper. On some occasions the questions allowed pupils to revisit topics taught earlier whilst on others they seemed closely focused on the topic of the lesson itself.

Examples of the sorts of questions we saw offered to pupils in grades 5, 6 and 7 included

- How many minutes is four fifths of an hour?
- 3.5 m of fabric cost ft700 what would 2m cost?
- How many kilometres would 9cm represent on a map with scale 1:300,000?
- How many diagonals has a pentagon?
- A square has a perimeter of 44cm. What is its area?

Answers to these questions were read out either by teachers themselves or by invited pupils.

The body of the lesson: generally this involves a problem being posed for private working before solutions are shared in a public fashion with teachers managing the contributions of individual children. Frequently several pupils are involved and there is little evidence of teachers telling their pupils the answers. Where difficulties arise teachers probe contributors' understanding or invite others to work alongside the pupil having difficulty. There may be several such episodes during the course of one lesson. Throughout teachers adopt a facilitatory role similar to that described above in respect of homework. We offer three episodes, from different teachers, which we regard as being representative of all episodes.

Lesson episode 1: Judit, working with a grade 5 class on some pre-functions work, wrote the following on the board

$$\Delta$$

and then asked someone to give her a number. A volunteer suggested 20 which was written alongside the rectangle. Judit then wrote 60 next to the triangle. She asked for a second number and got 3. She wrote 94 beneath it. She asked if anyone could see a relationship between the two sets of numbers. No one could and so the process was repeated. 97 was offered with -84 being written beneath.

	20	3	97	2	1
Δ	60	94	-84	96	98

A sense of system began to emerge with both 2 and 1 following. At this stage someone suggested that the rule was $2\square + \Delta = 100$. Judit asked if anyone could offer a different way of writing it. Offers of $2\square = 100 - \Delta$ and $\Delta = 100 - 2\square$ followed. Judit asked how they would find the value of $\square$. Another pupil came out and wrote $\square = 100 - \Delta :2$. A different pupil objected suggesting that it didn't make sense - she came to the board and inserted brackets: $\square = (100 - \Delta) :2$

Lesson episode 2: a grade 8 class had derived the theorem of Pythagoras from a geometric investigation but had not used it to solve a triangle in class although their homework had been to calculate the hypotenuse of two right-angled triangles. Having solved publicly the two triangles their teacher, Vera, offered the following. Construct the circumcircle of a right-angled triangle whose legs are 3cm and calculate the length of its radius and the perimeter of the triangle.

Pupils were left for a few minutes to attempt a solution. Vera asked what should be done first. A volunteer suggested constructing the triangle. A different child was invited to do this. It was clear that the conscripted girl could not solve the problem and, having used a board rule to draw a base line, turned to Vera for help. Vera, in a manner indicating that no judgements were being made, turned the question back on the class. What should she do? Gradually instructions emerged and the girl used a compass and straightedge to construct the required triangle. Further discussion showed that the circumcircle needed to be constructed. A second conscript was called forward and quickly constructed the perpendicular bisector to the hypotenuse. A different pupil announced that the construction was now effectively finished. In response to Vera's invitation the pupil showed how symmetry confirmed that the perpendicular bisectors of either leg would pass through the intersection of the hypotenuse and its bisector. Thus, the circumcircle was constructed. Next Vera asked how the length of the radius might be calculated. A different volunteer wrote up the solution using Pythagoras to calculate the hypotenuse and then halving the answer to find the radius of the circumcircle. Lastly, Vera asked for the perimeter of the triangle - someone pointed out that it was just the sum of 3cm, 3cm and $\sqrt{18}$cm. This problem was followed by Vera offering a range of related problems which were done in the same private then collaborative manner.

Lesson episode 3: concerns a grade 7 class working on elementary functions. Pupils were asked to think about the appearance of the graph of function $x \to |x| - 5$. After a minute or so, during which pupils discussed the problem in pairs, a volunteer was called forward. The teacher, Eva, switched on an overhead projector, placed on it an acetate showing some axes and directed the volunteer to a pile of drinking straws. The girl placed one straw in a place commensurate with the graph of $x \to x - 5$.

Eva queried the attempt and asked for additional contributions. A boy suggested that negative x had not been considered because the effect of the modulus was to make it positive. He came to the front, took a second straw, repositioned the original so that it extended upwards in the positive direction from the point (0,-5) and then placed the second straw symmetrically about the y-axis. Eva asked another pupil to explain what had been done. Eva left the projector, with its correct image of $x \to |x| - 5$, switched on and wrote the following on the board.

$x \to x$	$x \to 3x - 10$	$x \to x - 10$
$x \to -x$	$x \to 3$	$x \to x/3$

The class was invited to find the number of intersections of $x \to |x| - 5$ with each of the functions in turn. Pupils worked briefly in pairs before volunteers were called to the front and asked to share their solutions. Where children were hesitant the whole class became involved in offering suggestions in a supportive and unthreatening way. During these discussions pupils checked and corrected their solutions - there was no sense of embarrassment at failure. Once this was done Eva asked if anyone could offer a function which would yield more than two intersections with $x \to |x| - 5$. Within seconds a boy volunteered that $x \to x - 5$ would yield an infinity and was invited to the front to demonstrate. The lesson continued with several similar problems concerning elementary functions (not necessarily linear) and their intersections.

Discussion: the episodes described above are indicative of a particular characteristic pedagogical flow. Homework, usually comprising no more than three or four questions, seems to serve several purposes. At its most basic it consolidates skills or techniques taught the previous lesson. Hungarian teachers appear to value lesson time and regard routine exercises done in class as a squandering of learning opportunities. Thus homework becomes the only place where such activity is acceptable. However, the problems we describe encourage, in varying degrees, the development of problem-solving skills, the revisitation of topics taught previously, and logical thinking and mathematical justification. Even when homework focuses on routine practice the numbers used reflect the generality of numbers rather than just, say, positive integers. Frequently homework links successive lessons or topics. In short, it seems that homework problems are not repetitive exercises but worthwhile mathematical problems that provide pupils with opportunities for mathematical thinking within a framework of topic re-visitation.

When speaking about the warm-up, teachers invoke the sorts of metaphors common to teachers around the world. Questions are generally closed and offer opportunities for pupils to rehearse facts learned earlier. Indeed, on one occasion we saw a teacher spend four or five minutes working through a comprehensive barrage of questions on the properties and vocabulary of triangles and quadrilaterals. However, the majority, though simple in statement, are anything but simple in their processes and force pupils to engage mentally with all manner of mathematical ideas. In short, the warm up serves not only to get pupils thinking but also to ready them for an engagement with problem-solving in the main part of the lesson.

The problems used for the body of the lesson seem to embody the generality of the mathematical ideas under consideration and provide pupils with opportunities to develop understanding not only of the topic itself but also its interrelationships with other topics. So strong is this sense of generality that Hungarian teachers alert their pupils to the problems of the particular (Andrews, 1999). This contrasts with the approach of English teachers of beginning with the simplest case and the hope that the more general will emerge as if from the ether. It appears that teachers see the problems they pose as devices to develop the skills of logical thinking, mathematical problem-solving and proof (Hatch 1999).

The language and syntax of mathematics is evident in all that pupils do. The episode concerning the relationships between functions not only expected pupils to transform and compare functions - highly generalised expectations - but also served to reinforce the importance of mathematical language. Pupils are expected to show their working - something evident in, for example, the inequalities homework which showed pupils writing in a logical manner their solution processes. Unlike those found in many British and American classrooms, the sorts of problems where the solutions are so obvious as to make the systematic recording of solution processes an absurd and unnecessary demand are rare. Hungarian teachers justify such expectations on the basis of necessity - because the problems are not trivial - and the expectation that pupils need to learn how to communicate mathematics meaningfully. In short, we did not see Hungarian teachers asking their pupils to document their solutions to equations like $x + 2 = 6$. The Pythagoras episode exemplifies the expectation that pupils acknowledge in their solutions the units necessary to communicate meaning - the problem concerned lengths and so units were stated in the solution. Significantly, in addition to expectations of developing high level thinking, Hungarian teachers explore and invoke equivalence. The episode concerning function machines explicitly forced pupils to engage with equivalent forms of the same function and emphasised the transformative nature of algebraic manipulation. Other problems, like the Pythagoras questions which linked the explicit material concerning right angled triangles to work done earlier on circles, showed revisitation in deliberate and systematic way.

There are other not insignificant issues which emerged from our observations. When Hungarian teachers test their pupils, which they do infrequently, tests are short and accord with teachers' wider objectives concerning worthwhile learning experiences. Hungarian teachers make rarely allude to the applications of mathematics to the real world although occasionally they may introduce illustrative real-world contexts. The manner in which Hungarian teachers work indicated to us that Hungarian pupils, unlike many in England, cannot hide from challenge in lengthy and largely meaningless exercises. We observed many pupils working publicly with confidence - not all were confident in their mathematics but all seemed confident that any lack of understanding would not be ridiculed.

Conclusion: the TIMSS and TIMSS-R indicated that the attainment of Hungarian children exceeds that of their English peers. That is, the attained curriculum in Hungary differs substantially from that in England. The fact that the recently introduced Hungarian National Curriculum is not dissimilar in content to the English, but lacks the latter's emphasis on regular formal assessment, supports a conjecture that differences in the attained curricula may be more the result of differences in implementation than intention.

It seems to us that Hungarian mathematics teaching is framed by three inter-related, sets of beliefs which create a coherent pedagogy substantially different from that found in England. The first concerns the philosophical and pedagogic basis for the curricular inclusion of mathematics, the second the nature of the problems posed in Hungarian classrooms and the third the interactive manner in which those problems are solved.

In respect of the first, our perception is that in Hungary there seems to be no ambiguity in respect of the rationale for the curricular inclusion of mathematics. It is popularly perceived as a high-status form of thinking. It is as a much part of Hungarian cultural heritage as literature, art and music. That is, mathematics may have applications to the wider world of commerce, industry, science and technology but for Hungarian teachers such perspectives have little influence on what is taught and how it is taught.

With regard to the second, our perception is that in Hungary the ability to solve mathematical rather than real world problems is highly regarded and viewed as one of the major criteria for claims made by an individual to being educated. This perspective has informed, and continues to inform, the classroom manifestation of the curriculum. It seems, irrespective of the content of that which teachers are required to teach, Hungarian teachers see mathematics as a coherent set of mathematically meaningful problems which emphasise process, logical thinking and links between topics.

In terms of the third, due to its being more a cultural artefact than tool, the teaching of mathematics in Hungary seems to occur in much the same way as would that of music or literature - something to be experienced together. Respect for mathematics seems to be passed down as any other aspect of folklore. Consequently Hungarian teachers create relaxed and friendly environments for the sharing of worthwhile mathematical experiences.

The above leads us to conclude that while notions of intended, implemented and attained curricula are helpful constructs we cannot ignore the cultural bases underpinning the manner in which implementation occurs. Our conjecture is that while the intended curriculum informs the implemented and ultimately the attained, it is the implemented curriculum, underpinned by implicit philosophical and cultural traditions, that wields greatest influence. If attainment is to be raised in those countries where achievement is perceived to be below acceptable levels then an understanding of the traditions of a country like Hungary may be helpful. However, worthwhile change will require more than an acknowledgement of the overtly manifested public sharing of solutions - something on which most recent British curriculum projects have focused - it will need an endorsement of the problem-solving emphases, explicit recognition of complexity and generality, revisitation through integrated problems and clear sense of mathematics as worthy of study independent of its applications found in that country. In England, with its tradition of exposition and practice, its mathematics curriculum of ambivalent provenance spawned of diverse interest groups and the rhetorical common sense of politicians, this will not be an easily attained goal with the consequence that few boundaries will be removed from children's opportunities to learn.

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