MECHANICS AND DIRECT MANIPULATION ENVIRONMENTS: A ROLE FOR PROOF?

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Abstract: *This paper examines the relationship between explanations, proof and the use of computational resources in the context of developing mathematical models for mechanics. It reports on three workshops with pre-university students using the Direct manipulation Environment, Interactive Physics, to explore connected particle systems, and analyses the style of reasoning used by the participants. The paper concludes with a discussion of issues raised by the workshops.*

Introduction

Modelling with mechanics involves analysing a given system in terms of the Newtonian system, structured by the key relation - Force = mass x acceleration. Usually a specific form of the equation, related to the particular type of system being studied, must be formed, often incorporating models of common objects and phenomena (e.g Hooke's Law, Law of restitution, Friction). The resulting equations are then solved using a range of mathematical techniques, and the solutions are compared with the original system to examine their correctness. If necessary, the equations or initial conditions are modified to obtain a solution that agrees more closely with the observed behaviour of the system being studied.

From a logical point of view, the modelling process can be split into two phases corresponding to Hanna and Jahnke's notion of static and dynamic proof. (1996). The process of deriving a solution from a given set of equations, largely through algebraic manipulation and transformation of the equations, establishes deductively, the logical and structural connections between the model's statements, and propositions about observation and measurement. In essence, this is a process of making a conditional proof (equation of motion => solution), and then using initial conditions of the model to generate new results, deductively. Such syntactic manipulations of the model involve disengaging, to some extent, from the underlying semantic relations that generated original equations, focussing on the *mathematical* ontology of the model. (Stevenson, 2000). By contrast, model-makers, developing their own ideas and understanding of a specific mechanical system, engage in a dynamic, non-linear process of trial, evaluation and modification. Generating mathematical justifications for these created models and later, systematising the results that fall within their scope, form a more dynamic role for deductive thinking. (Hanna and Jahnke, *ibid*).

Learning to model with the Newtonian framework typically involves both exploring a model and expressing one's understanding using a model. (Bliss, 1994). Exploring the Newtonian framework entails relating it to one’s own experience and knowledge in an attempt to make it one’s own. In this sense, one is representing the model to oneself in an expressive mode. On the other hand, part of the process of making a model using the Newtonian framework implies that one has to “stand-back” from it to assess its efficacy, and this may lead to exploring aspects of the model which were not initially apparent or intended. (Ackermamm, 1991).

Typically, learners who focus on the mathematical aspects of Newtonian mechanics are principally concerned with manipulating mathematical objects, and are concerned with the syntactical dimension of modelling. There is little need to examine the semantics of specific situations, since they are often presented with partial analyses of those situations, which learners have to manipulate. Whether this lack of emphasis on the semantics dimension is significant in learning how to use Newtonian mechanics is an open question, although there is a large body of evidence which suggests that students find the semantics of modelling difficult. (Berry, 1990, Berry and Graham, 1991, 1996)). Examining the nature of transitions in an exploratory-expressive dialectic, however, can provide a means of understanding how learners appropriate and use the framework to develop specific formal mathematical models. Placing the Newtonian framework in a computational context also raises new possibilities for researching these transitions.

Direct manipulation environments

New types of computational environments are available which allow access to Newtonian Mechanics, but which do not need symbolic programming skills in the imperative sense used by Logo and other high-level languages. (Doerr, 1997). Using the Direct Manipulation metaphor, these computational environments (DME) provide an open learning context characterised by the continuous representation of the object of interest. Physical action and labelled buttons replace complex syntacs, and there is rapid incremental, reversible operations whose impact on the object of interest is immediately visible (Shneidermann, 1982 cited in Hutchins, Hollan, and Norman, 1986). DMEs are claimed to be easy to learn and use, even after a period away from the application, and, it is argued, they provide instant feedback on learners' actions, with relatively little need for error messages. DMEs are intended to produce a sense of direct engagement with a particular situation, so called 'naive realism', that gives learners the sense of working with the objects rather than the software.

Interactive Physics (Knowledge Revolution 1987-2001) is a DME for mechanics, among other things. It allows learners to build a system directly on the screen using a variety of objects, set its initial conditions, and observe its development according to the laws of motion. Figure 1 shows a pendulum system with a mass connected to a string, and a meter to measure displacement in the vertical direction. It is also shows vectors for the gravitational force acting on the bob, and the tension in the string. Vectors can be attached to any object on the screen, provided they represent appropriate quantities.

Figure 1. A pendulum system in *Interactive Physics*.

From an exploratory point of view, *Interactive Physics* can reduce the ‘semantic distance’ between the system and its model by directly engaging learners with on-screen objects, through the sense of direct manipulation. As an expressive medium, *Interactive Physics* can enable learners to reduce the distance between their intentions and their ability to express those intentions with and in the environment. (Hutchins, Hollan and Norman, 1986). From a logical point of view, *Interactive Physics* may be regarded as a deductive system in action. Once the initial conditions have been set by the user, the software applies a numerical solution of the equations of motion to produce a dynamic representation of the system’s behaviour over time.

Instead of focussing on mathematical objects, learners can use the ontology of the DME to construct and express their understanding of mechanical systems. A central issue is whether, and to what extent, *Interactive Physics* can provide an exploratory and expressive medium for transitions between naive realism and formal mathematical models. To examine this issue, I will describe some preliminary outcomes of three workshops involving pre-university mathematics students working with *Interactive Physics*. The topic for their investigation was connected particles, since this system makes use of all the key concepts of mechanics in an apparently simple configuration, but one which causes conceptual difficulties for learners. (McDermott et al., 1994). Two aims guided the construction of the activity. The first was to explore with the students what they understood about connected particles in the context of building Newtonian models of the system. The second was to examine the nature of the arguments that the students constructed using the resources of *Interactive Physics*.

To make a preliminary examination of these issues, forty-one pre-university students from two schools took part in three workshops at the Institute of Education, University of London, each lasting approximately 5 hours. The first two workshops were composed of 25 students who had been studying mechanics for between 3 - 6 months, and the third contained 16 students who were just beginning to study mechanics. Two-thirds of the total group were male, studying mainly mathematics with physics, and the female students were taking a mix of other science and social science subjects.

Working in pairs, their task was first to explore, and then explain the behaviour of a simple connected particle system to the rest of the group. Prior to the activity, none of the students had met this kind of system. Each session had approximately the same structure. The first hour took the form of an introduction to the features of *Interactive Physics*, followed by about 3.5 hours on the connected particles task, and the workshop concluded with half-hour for student presentations and discussions. Data was collected using two sets of video cameras, positioned with different pairs of students throughout the modelling task. The students' presentations were recorded, and the acetates that they produced were collected, together with any written materials. The sessions took place in the same computer-lab, with two teachers supporting the students, and one participant observer. Before the sessions, the teachers agreed to adopt a supportive but non-directive role so that the students could develop their own responses to the situation. The teachers intervened to help students clarify their thinking about the modelling problem, but tried not to guide the students to solutions, a role that they found challenging but achievable!

Reflections

Two questions dominated the analysis of the workshops. First, how did the students appropriate and use the computational resources that *Interactive Physics* offered them, and second, what kind of reasoning about the system of particles did the students engage in?

Students' use of Interactive Physics

The students found *Interactive Physics* very easy to learn and use, typically creating and exploring situations within an hour. It has a rich ontology, with a variety of objects to manipulate on screen, and the students seemed to be able to use this ontology to construct mechanical systems easily. (Stevenson 2000). The extent to which this supported the student's expressivity with the system can be seen by their modification of the connected particles system. Figure 2a) shows the system that is obtained by using *Interactive Physics's* 'pulley' option. However, this proved to be too complicated for the students to analyse, since the circle masses kept bumping into one another as they moved through the pulley. Figure 2b) shows how students in each of the three workshops, quite independently, changed the system to provide a more manageable situation to analyse.

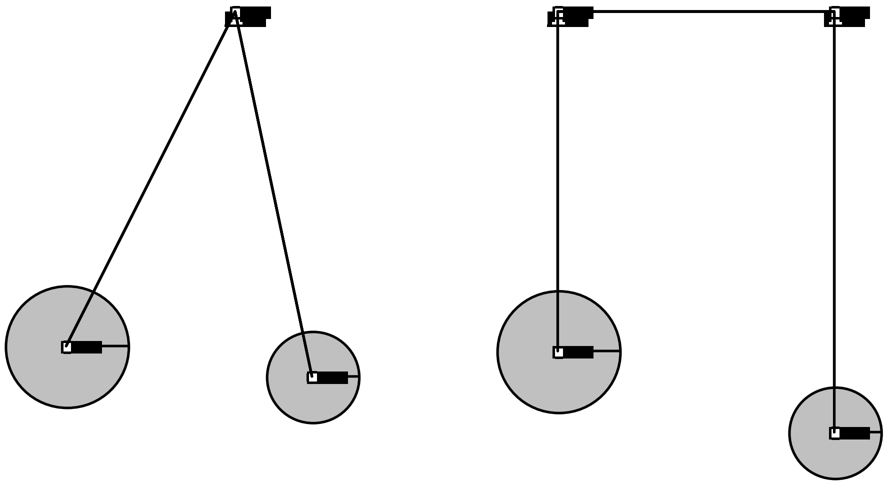

Fig 2 a) Original Problem Fig 2 b)Students' modification

The second observation is that the students made considerable use of the meters to measure velocity, acceleration, and force, often checking their guesses about how the on-screen system was behaving using calculators. However, they did not routinely use the option of overlaying forces to explore the underlying dynamics of the particles

An important aspect of the way in which the students used the software related to the setting. (Stevenson 2000). Working in pairs, the students discussed their difficulties and discoveries with one another, and also with students sitting at adjacent computers. Teachers provided varying degrees of intervention either by "dropping by" to see how pairs were doing, or by helping them to reflect on their models. Video cameras were moved from pair to pair, to capture the process. Clearly, this led to a complex set of interactions in this pedagogical setting which needs to be examined in more detail. Initial analysis do suggest some differences between the teacher-led and student-led discussions around the software. However, the students' presentations indicate that the pairs produced their own work in their own way, as the next section shows.

Student Models and Meanings

An interesting approach to modelling the connected particles emerged in two of the sessions. Figure 3 shows an extract from the presentation acetate of two students, Richard and Len, showing their approach to the problem.

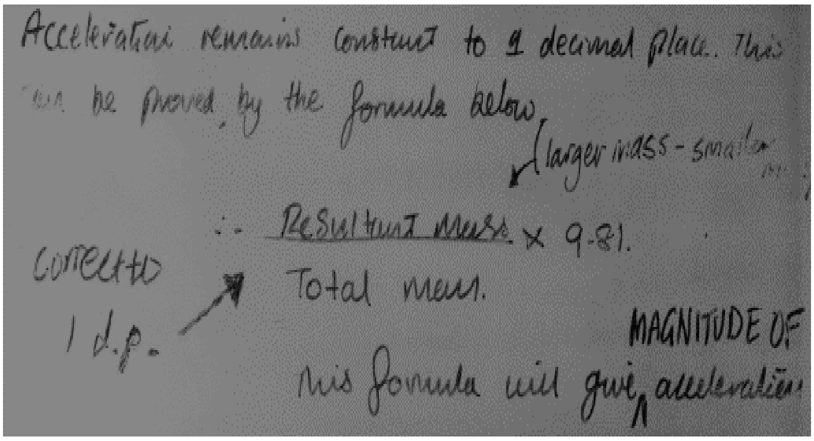

Figure 3. Richard and Len's Presentation acetate.

They produced a result which indicates that the acceleration of the system is given by:

$\frac{\text{Resultant Mass}}{\text{Total Mass}} \text{x } 9.81$, where "Resultant Mass" is the difference between the two masses in the system, and "Total Mass" is the sum of the masses. A simple rearrangement of their equation shows that "Resultant Mass x 9.81 = Total Mass x Acceleration of the system". In effect, they have reduced the system to a single object with mass equal to the total mass of the two connected particles, and which is acted on by a force equivalent to the difference in the weight of the two objects. Tension becomes an internal force of the system, and so does not figure in their analysis of the connected particles. This approach also appeared in other students' presentations in one of the other groups, from another school. What this suggests is that students' experiences of working with a dynamic system *in Interactive Physics*, may lead them to think of the connected particles *as* a system, and analyse it in those terms. Dynamic systems are characterised by causal interdependencies which develop over time, and require that those who use them be clear about the boundaries and parameters which determine the behaviour of the dynamic systems. (Ossimitz, 1996).

This has significant consequences for the nature of the analysis that students undertake, and what becomes the focus of their investigations. It is also in marked contrast to the type of static analysis that is found in most mechanics text-books.

Discussion

To conclude, I will discuss some of the issues about the nature of the student's reasoning in the context of *Interactive Physics*. In particular, I will focus on the nature of proof in this context, and the relationship between the computational resources and mathematical modelling.

Richard and Len's acetate indicates that their approach was experimental ("correct to one decimal place"), although they use the language of proof—largely, one suspects, as a result of the task being stated in terms of "proving". Their result was also generated with minimal tutor intervention. (Stevenson in review). These two factors may be linked, although more work and analysis needs to be undertaken to assess what the nature of the link may be, if any. However, the two students seemed to regard their empirical result as an explanation of the system's behaviour, perhaps because it gave numerical values that agreed with the meters in the software. One may speculate whether, given more time, they might have created something which could be called an explanation, but it seems more likely that the students were not clear themselves about the difference between a description and an explanation. There is also the question of how they interpreted what the software was presenting to them, perhaps regarding the rule as true because the computer said so, and agreed with their calculations.

Logically, Richard and Len's result may be thought of as a conjecture, which relates to the idea of a ‘sufficient reason’ that is implied by Hanna and Jahnke's notion of dynamic proof. The students have produced a mathematical relationship that seemed to “fit” the behaviour of the screen system, in the sense that it gave the same numerical results as the meters in *Interactive Physics*. In the process, they have established semantic links between a situation and its mathematical expression in a modelling framework. What seems to be missing is the awareness that their relationship expresses a general situation. A hall-mark of a proof is that it provides reasons for the situation being examined, which establish links *between* elements of that situation and *to* theories or principles that are accepted as determining that situation. In mechanics this may involve being able to express a particular system in terms of mass, acceleration and force, using F=ma as a structuring principle. One may speculate that for students to appreciate the generality of their conjecture, and help them to relate it to a more formal and symbolic discourse, there needs to be some kind of awareness that the screen behaviour represents a general situation, and is a necessary consequence of the rules governing the Newtonian framework, relative to a numerical approximation.

On the face of it, *Interactive Physics* does not seem to offer much in supporting students as they try to form and solve formal mathematical models. The software is specifically designed to hide the construction of the equations of motion that are needed to control the behaviour of the screen objects over time, and gives only a partial clue that numerical (not symbolic methods) are used in animating the system. However, the DM metaphor does seem to support the construction of the links between physical systems and students' perceptions, as expressed through their models, which captures a sense of dynamism and direct access that they might experience with a "real system". The fact that the students produced a result that may be thought of as a systems description of the connected particles, but lacks the symbolic expression of a formal model, suggests two things. On the one hand it implies the need to find ways, at a formal level, of connecting the students' linguistic expressions of rules with the symbolic medium of mathematics. Through the mechanism of "proving" in the static sense, the various resources of the DME can used to recover the missing mathematics that govern the system. Given a certain set of assumptions and data, one can derive a conclusion that follows *necessarily* from the starting points, and is applicable in a variety of situations that satisfy those initial conditions. On the other hand, there is the need to explore how the ontology and syntax of the computational resources can be related to the "real system" that is being modelled. What is clear is that the computational metaphor of direct manipulation offers a number of possibilities for generating situations which motivate the need for proof, and the corresponding notion of generality.

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