# BELIEFS OVERHANG: THE TRANSITION FROM SCHOOL TO UNIVERSITY

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***Abstract:*** *In this paper we introduce the idea of 'beliefs overhang' - the continuation of views about the nature of mathematics which students develop in school but which are carried forward (often inappropriately) to mathematics in university. As part of a larger project considering the role of the affective in the transition from school to university, we consider two of the many distinct belief types we have identified - the systematic and the utilitarian. We outline the characteristics of these beliefs as built in school and the consequences of them during the first few weeks of the students' university mathematics course. We speculate about the ways in which these beliefs can cause difficulties and possible actions that teachers and lecturers might take to ameliorate the problems.*

## Introduction

University mathematics isn't school mathematics - but what is it? As well as bringing various forms of mathematical knowledge, students moving from school to study for a mathematics degree bring with them beliefs about the nature of the subject, which have been built up from their experiences of school mathematics. In this paper we will explore this phenomenon, which we call *beliefs overhang*, and the consequences of it in the case of two particular types of belief.

The main focus of much existing literature in the field of advanced mathematical thinking has been on cognitive factors. The shift from A-level mathematics to University mathematics confronts first-year mathematics undergraduates with formal abstraction, mathematical rigour and formal mathematical reasoning. As Tall (1991) remarks, the transition to advanced mathematical thinking involves a move "from *describing* to *defining*, from *convincing* to *proving* in a logical manner based on those definitions"(p.20). As many studies (Alcock and Simpson, 1999; Bills and Tall, 1998; Moore, 1994; Nardi, 1996) have reported, first-year maths undergraduates encounter difficulties with formal proof, with the use of definitions and with mathematical abstraction in general.

However, affective factors such as beliefs, attitudes and emotions, have a vital role in the integration of cognitive factors to the explanation of students' behaviour in a mathematical setting. In a previous article (Daskalogianni and Simpson, 2000) we referred to the focal influence of *attitudes* in students' expression of mathematical behaviour but in the present extract from our study we will attempt to shed light on the difficulties first-year undergraduates face in mathematical studies due to the preservation of their school *beliefs* when they go to university.

Lester *et al* (1989) define beliefs as "the individual's *subjective knowledge* about self, mathematics, problem solving, and the topics dealt with in problem statements." (p. 77). There are a number of different beliefs about mathematics that students develop during their school years, which are shaped by endogenous factors (such as students' mathematical skills, students' preferences of mathematical topics, students' confidence and intrinsic motivation) and exogenous factors (such as teaching practices, curriculum style and assessment requirements). The most commonly held views about mathematics include mathematics being a coherent subject characterised by a set of rules and steps to follow in which absolute and unique truth dominates, mathematics being a subject that is intimately related to real life situations in which truth remain to be discovered and mathematics being an abstract, rigid and static body of knowledge.

One would expect that the changes in the subject matter of mathematics and in the social environment during the first term at the university would result in a reaction from the students concerning the way they approach learning and studying in order to adapt to the new settings. It is natural to infer from this assumption that these changes should initiate a subsequent change of students' affective reactions to mathematics and therefore influence their behaviour in the new mathematical environment of instruction and learning.

In our study the data, however, we see the converse of this idea: even though students know that university will be different, their initial responses to educational experiences at university are guided by beliefs developed in school, which are preserved at least for the first three weeks in the course. The majority of these beliefs are often inappropriate for students' adjustment to advanced mathematics, and result in a problematic move from school to university. We refer to this phenomenon as *beliefs overhang*.

As reported in literature, the central components of the affective domain, beliefs, attitudes and emotions, vary significantly in their degree of intensity and in their stability. McLeod (1989) describes beliefs to be the component "thought to be relatively stable and resistant to change". In addition, Rokeach (1975) who has developed a hierarchy of beliefs according to a central-peripheral dimension for the individual, states that "the more central a belief, the more it will resist change" (p. 3).

The evidence from data presented in this paper will describe two of the dominant belief profiles of the first-year undergraduates, in an attempt to enhance our understanding of students' difficulties in mathematics during their transition from school to their mathematical studies due to *beliefs overhang*.

## The Methodology and Methods of data collection

The passage of our study on which this paper is focused on is taken from the first part of an extended PhD project, which investigates the evolution of students' attitudes towards mathematics and their influence on cognitive aspects through the transition from school to university.

Students were selected according to their responses to attitudinal questionnaires out of ¾ of all A-level students who were planning to apply for the single honours mathematics degree at Warwick. Semi-structured interviews were conducted with them while they were in their last year at school with 12 students eventually gaining a place at Warwick. The central data of the project comes from a set of interviews across school and the first term of university with each of these students. In addition, a second attitudinal questionnaire was distributed to all first-year pure mathematics undergraduates during their second week of term with comparable questions to the first one. There was also an open-ended question at the end of each questionnaire. Finally additional data was gathered by observing students' supervision groups while they were at the university and keeping a diary of them as a means of getting access to a more natural setting of the students' mathematical experiences.

## The case of systematic and utilitarian beliefs

It is a very difficult task from a methodological point of view to trace beliefs and place them in a coherent framework for further analysis. Students' conceptions about mathematics are not always consciously espoused; rather, they are views that are developed through a long period of time from the school syllabus, setting and instruction, from learners' idiosyncratic ways of studying and by social parameters. That is why they "must be inferred as best as one can, with whatever psychological devices available, from all the things the believer says or does" (Rokeach, 1975, p. 2). In this study we attempt not only to infer students' mathematical beliefs but also to identify the extent in which these beliefs remain unchanged and hinder students' successful accommodation in university mathematics, describing the *beliefs overhang* experience.

The analysis of our data suggests that there are at least three significant categories of students' pre-university belief profiles as it appears from the collection of data at school, but in this paper we chose to focus on the two most common ones. In the first category, which we call *systematic*, students' views about mathematics could be summarised as mathematics being a methodical subject with exact and definite answers, systematic working and solutions which require not more than two steps. In the second category, *utilitarian*, we have students whose views about mathematics involve mathematics being a subject with a direct application to real life and other topics.

In the following sections we will expand these descriptions of the characteristics of these two types of beliefs by illustrating extracts from interviews with two students, Katherine and Mark, whose belief profiles are somewhat stereotypical of the two main categories described above. We will then portray the consequences of the maintenance of their beliefs when they come to university.

**Systematic believer: the case of Katherine**

Systematic believers generally like the structured way of working they perceive in mathematics and a methodical way of working towards a correct answer. As Katherine wrote in the open-ended question of the first questionnaire "*I enjoy maths because it is very logical and there is an exact answer to be found at the end*"[1]. They like working towards definite answers and they consider mathematics to be a rigid science in which exercises are usually solved in a few steps.

K: I'd always liked maths. I think I just, uhm during the lower school and GCSEs I just got sick of writing essays! ☺ And I preferred the scientific approach, just an answer and sort of short explanation answer rather than 3 pages essay! ☺

[1] For recognition reasons we write students' quotations in the questionnaires in italics to distinguish them from the quotations in the interview transcripts.

For example Katherine prefers Algebra and Calculus to Statistics since "[Statistics] is not like solving a problem really; it's just generating a numeric answer." In this category of believers, the exploratory nature of mathematics does not appeal to them. They feel quite confident and "secure" with exercises they had worked before and as Katherine pointed out in the questionnaire "*once you have learnt the basics of a type of a problem you can use those ideas to tackle other problems*". Finally, one of the prevalent characteristics of a systematic believer is that her views about the nature of mathematics are focused mainly on the "logical" (by which they appear to mean methodical) aspect of it, which is shown below in the case of Katherine and it's evident throughout the whole interview with her:

K: It sort of encourages logical way of thinking, help to solve problems. Uhm, I'm not really sure! ☺

I: To rephrase the question a little bit, what do you think that you gain from learning maths?

K: It meant to be some methodical, work through and logical, uhm, it just teaches you how to see things through systematically and finding different ways of attacking the problems.

## Utilitarian believer: the case of Mark

The utilitarian believers are concerned mainly about the utility of mathematics to other subjects and to its real life applications. For example Mark prefers the mathematical topics that are more applied and as he says:

M: I just like to understand a bit more about how things work you know. I don't like to be sitting in situations where the teacher says "this actually comes from that but you don't actually know it" and then hammer down the idea. I like to have a bit of an idea, a bit of understanding.

These types of believers prefer more mathematical topics that are interesting and easy and in which they enjoy working with. In the case of Mark he focuses more on the workload, on study techniques, on the teaching style and on his success at the exams rather than on mathematics topics and that is mainly because he prefers to put the least possible effort in his studying and because he believes that maths is about practice when you're already good at it. Finally, a utilitarian believer's views about mathematics form the basis of a very practical and numerically oriented approach. When Mark is asked what he thinks mathematics is he replies:

> M: Maths is related to everything. […] I think that maths is incredibly useful because I mean for example I'm really into computing and if I didn't do some maths, I mean for example converting from base 10 to decimal, OK? You can do it on a calculator but it's useful to have a bit of maths understanding to know how it works.

### *Belief overhang* for a systematic believer: the case of Katherine

For a systematic believer, the persistence of systematic beliefs from school to university affects the way he/she experiences advanced mathematics. In the case of Katherine, the maths she encounters in university is difficult to reconcile with her beliefs about what maths should be especially because maths is much harder than she expected as she states in the open-ended question about university mathematics.

> K: Yeah. Analysis especially seems a bit strange. […] 'Cause you have to prove everything. You can't just write something and say "well, this is this 'cause well it just is!"; you have to try and prove it and that's quite difficult. […] Using all the definitions for proving; it's quite hard. […] It's just we haven't really had to prove things like that before.

One of the reasons why Katherine finds university maths in general and Analysis in particular difficult is the fact that she is not used to exploring mathematics; she prefers to solve mathematical questions in which the solution technique is known.

Katherine believes that mathematics is about following logical steps and during her first encounters with proof she did not find the expected structure of the logical steps mainly because the steps are not fixed anymore and mainly because she doesn't feel that confident in that context of proving by using definitions.

### *Belief overhang* for a utilitarian believer: the case of Mark

For a utilitarian believer, the abstraction of advanced mathematics causes conflicts with his views about mathematics being practical and applied to real life situations. Mark's first impression of university is that "they seem to be able to make clearly simple problems looking very, very complicated". In saying this he is referring to the mathematical language in theorems and proofs, to the rigour of advanced mathematics and to the "huge list of notations!". He believes that university mathematics is "*about complex problems and complex ways of solving simple problems*" and he doesn't see the reason for that.

I: So how are you doing with proofs in general?

M: Proof, uhm…I hate it. Because in my A-level they said we'll do more rigorous proofs at the university and I'm sorry, I don't believe them. […] Yeah, because at the A-level I used to do to prove bang, bang, bang (*he taps on the table as well*), proof! […] I think it's less organised the maths in ways. And I was expecting it to be you know more uhm, rigorous, just literally bang, bang, bang, bang. Whereas it's not. It's got loads and loads of English in it and I wasn't expecting that at all!

We suggest that Mark expresses a discomfort about university mathematics because he holds the view that working in mathematics is a process that involves little calculations and little explanations, a view that it's mainly influenced by the character of A-level mathematics and his working habits in mathematics in the past. This is also conveyed by his behaviour during the supervision group sessions where it is evident that although "he has a fair knowledge of the lecture notes, he doesn't want to put too much effort in problems".

## Discussion

In the above two illustrations of belief profiles through the interviews from school to university we can observe a perseverance of students' beliefs about mathematics. The same characteristics of behaviour can also be observed in all 12 students who participated in the main study, even when they belong to different belief types.

In the case of Katherine, a representative student of a systematic believer category, we could argue that the beliefs she holds about mathematics are carried over when she enters university and during the first two weeks of term. Her initial views about mathematics include that mathematics is a subject in which the exercises' solutions are structured in logical steps and there is a definite answer at the end. She hasn't expressed any view that shows interest in discovering mathematical truths or things by herself and she feels rather confident with, and used to, exercises where mathematical fragments from other exercises could be used for their solution. So Katherine entered university with certain beliefs about the nature of mathematics, about mathematics teaching, about the setting of mathematical problems and about herself in relation to mathematics. Even when she experienced the difference in all the aspects of teaching and learning in the academic environment, her beliefs about mathematics remained and they unconsciously influenced her expectations about mathematics at university and her working in it. She couldn't get used to working with concepts she hadn't met before and to proving things based on theorems and definitions because this was not the way she thought mathematics was.

In the case of Mark, an example of a utilitarian believer, his initial views about mathematics could be outlined as its relevance to other subjects and its applicatory nature. He also believes that mathematics is an easy subject with exercises he can do without putting in too much effort and a subject that he can easily understand as long as it is explained to him. Therefore, he finds strange all the changes in the character of mathematics that he recognises at university because his beliefs about what mathematics is and how mathematics should be taught are in contrast with what he comes into contact with. He believes that mathematics should not involve "loads and loads of English" and that is why he has difficulties in coping with proofs and he was expecting exercises to be more difficult but still straightforward like in school.

There are many factors that contribute to the formation of students' beliefs towards mathematics prior to coming to university. Students' everyday experiences in the school environment as these are enhanced by teaching methods, syllabus structure, examinations requirements and studying practices shape the formation of students' main beliefs about mathematics. These beliefs along with the nature of mathematical concepts to a certain extent determine students' mathematical behaviour. Because beliefs are changing slowly over time, some of the beliefs are preserved for the first weeks at university causing difficulties to students' mathematical development - that is, beliefs overhang.

The central beliefs that students hold from school (e.g. mathematics is about a set of rules or a set of steps to follow) are very difficult to modify in order to include new aspects of university mathematics, such as the role of axiomatic definitions of concepts. Hence, these beliefs predispose students to behave in analogous ways in their learning process and their studying behaviour even when the learning and working styles that they have adopted from school are no longer appropriate for the advanced mathematics environment. Some less central beliefs, especially the ones that concern the didactical contract and the change of context, still remain unchanged during the move from school to university but our research shows that students are likely to accept changes in these beliefs faster.

This begs the question of how these students might come to challenge their existing beliefs about mathematics and, perhaps, make the school/university transition smoother.

A report from London Mathematical Society (1995) recognises that one of the serious problems in first-year mathematics undergraduates is "a changed perception of what mathematics is - in particular of the essential place within it of precision and proof" (p.2). Therefore, although many universities have broadened their mathematical courses in order to include more applications of mathematics and have added more introductory courses in the first year planning as a means of a smoother transition of students from school to university (Kahn and Hoyles, 1997), these courses focus mainly on the conceptual level but not on the affective. This research suggests that explicit work on beliefs about mathematics in Y13 at school and an emphasis on negotiating new didactic contracts at university through group discussions about mathematics and its nature may help with this problem.

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