# DEVELOPING A VYGOTSKIAN PRACTICE IN MATHEMATICS EDUCATION

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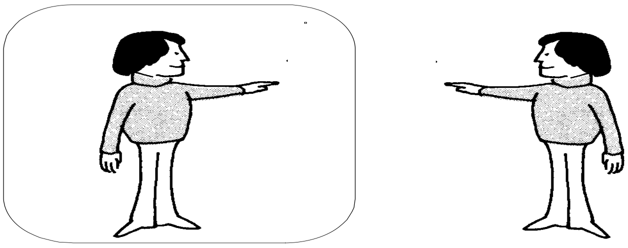
*This paper introduces some problems encountered during my research, which was based on a Vygotskian, Activity Theory approach to the teaching of mathematics. It attempts to focus attention on the dialectical complexities of practical problem solving activity.*

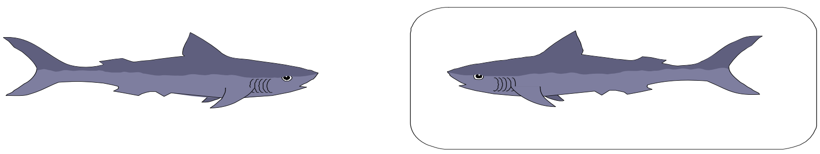
## INTRODUCTION



When the man in this picture looks into the mirror his image appears to be reversed from left to right. This is a pragmatic model of his activity. Pragmatic theory has been advocated by a wide range of prominent theoreticians (such as Dewey, Bruner, Piaget or Habermas) and will be familiar to most readers. The best pragmatists realise the limitations of their theories, however, and as Ricco has put it:

"There is no claim that conceptualisations are separable from matters of fact and thus might constitute true or false representations of 'real' facts in the world. The claim is only that it is useful to think of the matters of fact from within these conceptualisations. The utility of theories or models, however, is a thoroughly empirical question." Ricco (1993 p142).

The man above is not reversed from head to toe, for example, although light rays are unaffected by gravity and as we can see, the left/right reversal does not happen to this fish.



I will present some brief examples from my research to illustrate problems, which can arise when we try to develop such practical understanding in mathematics classrooms.

## DIALECTICAL LOGIC

Investigating the reflection problem further would be an exercise in dialectical logic. An apparent contradiction leads us to re-examine the theory that mirrors only transform images from left to right. From the perspective of dialectical logic, a thought directed to the immediate goal of explaining a reflection in a mirror is necessarily accompanied by a thought about that thought in terms of the nature of reflections in general. In this way we can move towards a deeper and therefore more truthful understanding of reflections in mirrors. The new understanding affects or mediates how we look at other similar problems. This mediation is carried out by means of speech and other semiotic objects and perhaps the key difference between Piagetian and Vygotskian theory concerns the importance attributed to the notion that the form of objects changes within language. While Piaget considered language unimportant, for Vygotsky, language was the key attribute separating humans from animals.

## LINGUISTIC TRANSFORMATIONS AND FLUENCY

One important aspect of semiotic mediation is the *fluency* of the linguistic transformations that we carry out. Russian research (Talyzina 1981) suggests that fluency depends upon the amount of abbreviation the verbal actions have undergone and determines the ease and the speed of their execution. Fluency is especially important because it also determines how much will be remembered in the longer term.

## MEDIATION THROUGH FORMAL AND DIALECTICAL LOGIC

The nature of the relationship between this mediational activity and material activity is itself an important and complex factor in human activity. Davydov (1999) has described two types of transformation that take place in the creation of the new ideal images that people form to guide practical activity. These are external and internal transformations and are differentiated in terms of a notion of essence. Those changes that relate to an object externally are concerned with the notion of essence as described in formal Kantian logic. In this system, the essence of an object is just something it has in common with other similar objects. Equations, for example, would be seen as essential elements of mathematics. Within this perspective, fluency in transforming equations could become a central goal for teachers and the practical relevance of the work could be ignored.

My first example of difficulties arising during the introduction of Vygotskian theory into mathematics teaching concerns problems arising from a traditional tendency to use overly formal and abstract teaching methods. A teacher from School 2 in an experimental teaching program I recently carried out was a specialist in Mathematics and was responsible for the teaching of mathematics in the school. This specialist mathematics teacher decided to move prematurely away from material examples given in the research program to more abbreviated abstract operations on symbols. This caused problems because important information was left out which made his examples meaningless.

The teacher began a lesson with a series of examples he had prepared. He wrote a number of sets of symbols for rate such as V1, V2, V3 on the board and asked how, for example, Vo (overall rate) could be found from them. He was attempting to develop fluency of mathematical actions separately from their material origins. This caused some concern to the children because no contextual information was given. He did not indicate, for example, whether addition or subtraction actions were necessary. I passed him a note and he quickly readjusted his questioning in line with the ethos of previous lessons. When this was done answers came steadily and mostly correctly from the class. Some of his examples were wrong, however, and were not corrected. For example:

"One man walks at a speed of 5 miles per hour and a second walks at a speed of 7 miles per hour. Their combined speed is 12 miles per hour."

The speeds are clearly not additive in this example.

**Dialectical essence**, on the other hand, is a universal relation that traces a mediating object back to its social origins. It is a law of development of any system rooted in human activity. Formal logic constructs classifications in order to orient a person in future practical activity. Rearranging the existing order of things in formal logic is then extended when rearranging them in terms of dialectical essence. In dialectical logic it is also necessary to relate these objects to an essence which gives birth to all the terms that are only described abstractly in formal logic. This activity releases the practical potential of an object of study and we are then in a position to change it.

My second example refers to some classroom practice carried out naturally according to the logic of dialectical essence. The teacher from school 3 of the same teaching program was keen to make connections between material reality and algebraic symbols.

Teacher: *(Writing on the board as he is talking)*

> "If Jim and I have 25 apples, together. Jim has 12. How many have I got?". "Nice and simple, isn't it. What value is this? .. So = 25 .. S1 = 12 .. I need to find S2, don't I? You can work it out in your head .. the value is 13 .. but how do you get it? Its So - S1 isn't it. .. yes. Some of you are messing this up. Its so easy isn't it .. on the first question .. I know some of you have got it. Then some of you are messing it up on question three. It really is that simple. Think about it. Its not hard."

In this example symbolic mental actions involving thoughts such as such as S1 and S2 introduced in the teaching program were fluently combined with more concrete notions of apples belonging respectively to Jim and the teacher. By moving fluently from the general to the particular in this way, the teacher connected the formal notions with their essence in practical relations between people.

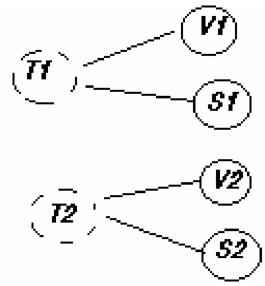
## GENERALISATION

My final example of problems arising during my attempts to develop a vygotskian practice in mathematics education concerns another important notion in the development of dialectical understanding. The degree of generalisation is determined by the extent to which an action is distinguished from other similar objects. In this example the notion of rate of pumping water is being generalised by applying it to a similar practical situation involving children running. The following extracts from a video transcript will illustrate some difficulties that can arise in a mathematics classroom when more complex practical problems are generalised in this way. My example involves a class teacher who was a specialist in Information Technology and a keen amateur actor. He clearly had problems understanding some of the ideas we were introducing. The problems which arose within the substantive mathematical content of more complex problems would suggest that while a non specialist mathematics teacher might be less resistant to the practical nature of the innovation presented in this program, training in the new dialectical techniques of mathematical education will be an important requirement.

Khalid, a student from schoo1 in the program, has written a problem for the class to discuss:

> "Two boys run towards each other. The first runs at speed V1 and covers distance S1. The second runs at speed V2 and covers distance S2 before they meet. How long did they run for?"

*(Khalid returns to his place and Ranjit sits down in the swivel chair)*



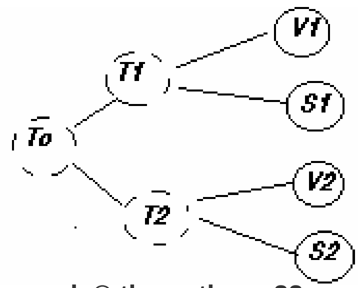

Teacher: "Um .. put down the bits. .. What do you know?"

(Ranjit begins her solution):

Teacher: "Right, so we've got a V1. You've got an S1, from which you can find T1 .. yes .. and why not. So that's how long the first boy was running. We know his speed. We know how far he went .. so we can get how long it took him."

Teacher: "Now .. second person .. we know his speed and we know how far he ran .. so we'll know how long it took him. You are now about to trip up over your shoe laces."

The teacher has seen Khalid's alternative solution and assumes Ranjit is going wrong. (Ranjit completes her solution):



*There are a number of unclear parts to this question. If the boys did not start and finish together, so that T1 and T2 were different, the time they were running could be seen as the time of whoever started first. If they did start and finish together the time running would be the same for each.* $\left(\frac{S1}{V1}=\frac{S2}{V2}=T1=T2=To\right)$

*In either case Ranjit's answer would be correct since she does not suggest adding the two times together. (She may have been tempted to carry this out if she had been asked how to find To).*

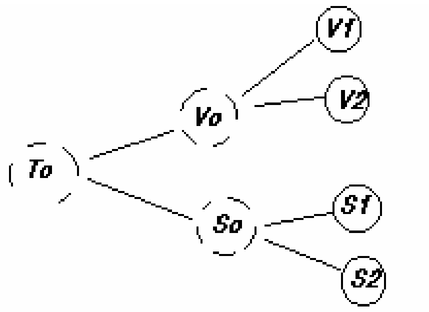
Teacher: "Now .. the point is .. let's just think about this ..stand up Ranjit .. by that door .. just to show you where this problem becomes difficult .. in terms of real time .. Ranjit starts running towards me (general laughter). She covers the distance .. to there in .. what .. three seconds. So run to there please. Right .. so .. that's how long it's taken her. At the same time I'm running towards her (general laughter). We collide, but .. go back to where you were .. that's all right. She took this long and I took this long. Wait a minute .. Go! (They run towards each other again amid general laughter) ..bumf .. right .. so what .. it didn't take that amount plus that amount did it? It took ..however long the whole thing was .. which is not quite the same."

*The teacher has constructed a situation in which the actions do actually start and finish together and thus demonstrates that he has missed the point of Khalid's example. He is thus unable to be part of the zone of proximal development that Khalid was attempting to establish. He is aware of the need to clarify things, however and calls upon Khalid to give his solution to the problem, perhaps partly to give himself time to think.*

Teacher: "The amount of distance we covered is that distance plus that distance. Our combined .. our impact speed is her speed plus my speed, but the time we took is not the time Ranjit took and the time that I took. Its a different concept. So its not really To. Its how long did they run for .. its not exactly what we found out. That's why I say you are going to trip over your shoe laces."

Teacher: "In the green corner Khalid has seen the solution to his own problem. Off you go ..(laughter) .. you and your big mouth Khalid."

*Khalid puts in his solution. He has arranged the diagram to find So and Vo directly from their components:*




*Khalid's solution does not actually fit his question. He appeared to be looking for a problem like the ones on rate of pumping in the worksheet he had been working on and which could have ended:*

"How long would they have taken to cover this distance, running at these speeds, if they started and finished together?"

*This question would have led to the answer:* $\left(\frac{S1+S2}{V1+V2}\right)=To$

Since they started and finished together in is question, To and T1 are equal and so both answers are correct.

Teacher: Right ..excellent! .. so .. the approach then .. If you want to find To .. you've got to find Vo and So .. then you can find To. (Points to Ranjit's solution). That looks totally logical ..but it doesn't actually find us To .. because of this thing about time.

*The complexity Khalid wanted to introduce into his problem thus uncovered a serious lack of clarity in the teacher's own understanding.*

## CONCLUSION

In this presentation I have viewed dialectical logic, not simply as a system of subjective laws, but as a developing process of teaching and learning practice. In my view this involves more than thinking logically. It involves both the logical development of the science of teaching and learning and the reflection of the development of teaching and learning practice in thought. The discussion that follows my presentation will be part of this process.

## BIBLIOGRAPHY

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