A CATEGORISATION OF UPPER SIXTH-FORM STUDENTS' BELIEFS ABOUT MATHEMATICS

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The purpose of this paper is to present a categorisation of high achieving upper sixth-form students' beliefs about mathematics as a discipline, about themselves as learners and when working in mathematics. The analysis of data suggests that there are three coherent categories of macro-beliefs among students and within each one there is a system of micro-beliefs. We will argue that these systems of macro-beliefs act as a potential medium of predicting students' working habits and approaches in a given mathematical problem and we will discuss possible ways in which coherent systems of beliefs may change over time.

INTRODUCTION

The study and analysis of students' mathematical behaviour is better interpreted though theories of cognition that integrate the affective domain. The focal point of this paper is the presentation of an interrelation between affect and cognition by describing students' beliefs about mathematics and by further examining a possible link between these beliefs and their mathematical performance.

We ground our research on the understanding that learners' affective responses in mathematics guide their learning and working habits and that this is at least as big an influence on their studies as the cognitive mechanisms that control the form of their learning. The existing literature disaggregates affect into attitudes, beliefs and emotions, which vary significantly in their level of intensity and in their stability. While aspects of all three factors will play a role in the research reported here, we will concentrate on belief.

Lester *et al* (1989) define beliefs as "the individual's *subjective knowledge* about self, mathematics, problem solving, and the topics dealt with in problem statements." (p. 77). Students form certain beliefs about mathematics during their route through the school environment and these beliefs vary in their domains of attention. Students develop beliefs about the nature of mathematics, about the setting of exercises, about themselves when working in mathematics and the role of the teacher according to their experiences in school. These beliefs are affected by external factors such as the teaching style, the syllabus structure, the exercises and exam requirements and by internal factors as students' own abilities in mathematics and achievement, their preferences of mathematical topics, their confidence and motivation.

The beliefs that students develop over time are structured in a hierarchical way according to their centrality within a given belief system. Adapting the theory of Rokeach (1975) concerning the organisation of beliefs within a "central-peripheral dimension" for the individual, we suggest that students' beliefs are constructed in central belief systems that we call *macro-beliefs* which in their turn can be seen as an inner structure of peripheral belief systems, varying in depth and stability that we call *micro-beliefs*. McLeod's suggestion that beliefs are "thought to be relatively stable and resistant to change" can be re-evaluated in the light of Rokeach's ideas and we suggest that the broader *macro-beliefs* change relatively slowly over time, while the more detailed *micro-beliefs* may be modified more easily in response to changed circumstances.

The analysis of our data suggests that there exist three dominant *macro-belief* systems among students, which act as a potential medium of predicting their mathematical behaviour in a given mathematical problem. The data presented in the following sections will exemplify the three prevalent *macro-belief* systems, the "systematic", the "exploratory" and the "utilitarian" and their links with students' respective manifested working habits in mathematics.

METHODOLOGY

The study presented here focuses on the systems of beliefs and mathematical behaviour of upper sixth-form students. This is the first phase of a longitudinal PhD project that aims to monitor students' development of attitudes towards mathematics in relation to cognitive aspects through the transition from school to university.

27 students participated in this phase of the study, chosen because they had each been offered a place at Warwick's Mathematics Department to study for a pure mathematics degree. Semi-structured interviews were conducted with them three months before their A-level examinations and a mathematical problem was given to the participants at the end of the interview. The selection of the problem was made in a way so that it was comprehensible by the students but without the need for specialist mathematical knowledge, in order to be open to a range of different approaches that we could interpret as the manifestations of different beliefs. This problem also provides us with an opportunity to compare the students' espoused beliefs with their mathematical behaviour.

CATEGORISATION OF BELIEFS

Analysing data that contain valuable information on the affective domain required the use of an analysis tool that could form a solid theoretical framework for explaining the "delicate" and dynamic characteristics of students' beliefs. Thus we adopted a "Grounded Theory" approach (Strauss and Corbin, 1998) from which three predominant categories of *macro-beliefs* emerged. In addition the analysis of students' behaviour on the mathematical problem indicated a connection between students' espoused beliefs and expressed practices when working in mathematics. In the following figure we present a synopsis of the emerged m*acro-beliefs* along with their peripheral *micro-belief* characteristics.

		MACRO - BELIEFS		
		Systematic	**Exploratory**	**Utilitarian**
MICRO- BELIEFS	**Nature of mathematics**	methodical, logical	problem-solving, linking things	tool for other subjects, applied in life
	Focus of exercises	follow a series of steps	understand different ways of thinking	obtain correct exam answer
	Working in mathematics	exact answer, similar exercises	explore things, enjoy challenge	known algorithms, study techniques
	Didactical contract	dependence on notes and teacher	dependence on own abilities	dependence on teacher

In the following paragraphs we will illustrate the belief profile of three students, Katherine, Lara and Andy, who we see as representative of learners belonging to the three key *macro-belief* systems, "systematic", "exploratory" and utilitarian respectively. It is important to note that the categorisation we have given is one of tendency. For example, some of those categorised as 'systematic' may well accept some of the micro-beliefs from other categories, but it is through the 'systematic' lens that these students come most sharply into focus.

***Macro-belief 1*: "Systematic". The case of Katherine.**

Students whose beliefs fall into the "systematic" system of macro-beliefs view mathematics as a static and rigid body of knowledge and they like what they perceive as the systematic and methodical way of working in it. Their micro-beliefs also include mathematical exercises having a definite answer that can be approached through following a series of steps. They feel confident with exercises where they have to apply strategies that they have used before in previous problems. Throughout the interview with Katherine we note expressions of views representative of a "systematic" believer and we cite below some indicative extracts from her interview.

I: To rephrase the question a little bit, what do you think that you gain from learning maths?

K: It meant to be some methodical, work through and logical, uhm, it just teaches you how to see things through systematically.

I: Why do you think you prefer maths in relation to other subjects?

K: I'd always liked maths. I think I just, uhm during the lower school and GCSEs I just got sick of writing essays! ☺ And I preferred the scientific approach, just an answer and sort of short explanation answer rather than 3 pages essay! ☺

When it comes to the approach of the given mathematical problem, Katherine firstly attempts a specialisation by trying out specific numeric examples but fails to generalise a pattern either algebraic or numeric without being prompted to do so. When she was asked about how she found the exercise she replied:

K: Uhm, it's quite interesting […] I didn't know where to start really.

The 'systematic' macro-belief seems at odds with risk taking and exploration. While attempting a few numerical examples fits, the beliefs about the nature of mathematics leave her with no strategy for tackling an open problem.

Macro-belief 2: "Exploratory". The case of Lara.

Students whose *macro-belief* systems are better explained by the "exploratory" category believe that mathematics is a dynamic subject, characterised by the discovery of truths and the exploration of new concepts and approaches to exercises. Their initial views about working in mathematics are focused on the problem-solving nature of it and on the existence of more than one correct answer. They also like the challenge of a new exercise and they are always looking to make interesting links between the concepts involved. Lara, as a typical student of the "exploratory" category, expresses these views in the following passage from the interview with her.

I: Is there any topic that you prefer the most among the others?

L: The question is that when I'm doing it, it tends to be the one that there always isn't a straight way obviously, but the more you think about it, the more it makes sense, which is the normal when you link things together rather one particular topic area…I like interesting questions the ones that stick with algebra rather than being difficult because you have to play with the numbers.

When Lara was presented with the mathematical problem she attacked it by considering two different approaches to it. She could immediately make the connections between all the elements of the exercise and she finally proceeded with an algebraic one, which led her to the correct answer. In contrast to students whose *macro-beliefs* are "systematic" or "utilitarian", Lara felt challenged by the exercise and she tried to understand the meaning of it without immediately complaining that she could not solve it because she had not seen a similar one before.

Macro-belief 3: "Utilitarian". The case of Andy.

Students whose central belief systems follow the categorisation of the "utilitarian" *macro-beliefs* consider mathematics to be a tool for other subjects and are mainly concerned with its real life applications. They focus more on study techniques and they are interested in obtaining a correct answer in their homework and exams. Andy's appears characteristic of a utilitarian believer:

I: Right! So no problem at all so far?

A: No. Sometimes it's a little…deep, but I came out of it eventually. But some of the P1 Step, Special Paper questions I don't like. The P3 and P4 I can do fine. I've got the P1 that's strange. I don't get it. It just tends to be harder I think. Just 'cause P3 and P4 they just tend to be what it is in an A-level really but just with a little bit extra.

I: Imagine a student who's a GCSE student and he or she is asking you "Oh, could you tell me just a few words about A-levels maths?". What would you say to him or her?

A: …It's just the same but more homework!

I: What about the level of difficulty of maths?

A: It is more difficult. You do a lot more work.

Because utilitarian believers hold practical views of mathematics they tend to be based on known algorithms and numerical approaches on their working in mathematics. When Andy tries the problem he doesn't write anything on the given piece of paper; he is working the whole time in the calculator instead. And when he finds a pattern for the solution he can't move on and says "I can't see how to prove that they all are [divisible by 11]". He finally admits that

A: I just didn't know what you wanted me to do. […] Uhm, I prefer it gives me something I can do this to it, something to get the head on.

We suggest that it is his "utilitarian" beliefs about mathematics that drive him to work solely with his calculator and he neither works systematically in search of a pattern, nor relishes the exploratory nature of the problem.

DISCUSSION

In the above illustrations of the three distinct categories of *macro-belief* systems we can observe a relation between students' expressions of conceptions of mathematics and their engagement in a mathematical problem. This same general match between the espoused belief and the student's behaviour can be seen in all the students who participated in the study, suggesting we have a sound basis for the postulating that knowledge of students' systems of *macro-beliefs* can be a medium for predicting their mathematical behaviour.

In the case of a "systematic" believer, such as Katherine, a synopsis of *micro-beliefs* includes mathematics being systematic and methodical and mathematical exercises following a series of logical steps, which were used in previous exercises. This, along with her successful previous mathematical experiences while approaching a task by using subsequent valid steps, results in her idiosyncratic way of approaching the palindrome question by trying to apply numerical and then algebraic steps but without making all the necessary connections among them in order to have a successful strategy of solving the problem. For the "exploratory" believer Lara, her *micro-beliefs* of mathematics being a problem-solving activity of challenge, exploration and the search for meaning influence the way she approaches the problem first by assigning meaning to the given question and second by considering more than one approach to it. Finally Andy, an example of a "utilitarian" believer, whose views about mathematics are summarised as mathematics being a subject related and used by other subjects with exercises that are straightforward and easy to do if you have done some practice, finds himself in a difficult situation when faced with a problem he has never seen before and does not know how to approach, apart from trying some examples in his calculator.

Although the observations of this research are focused only in a short instance of students' mathematical behaviour, interesting findings emerged concerning the interrelation between students' affective expressions and their tendencies to respond in a mathematical problem. In some cases, "systematic" and "utilitarian" learners the *macro-beliefs* about mathematics they hold hinders the development of polymorphous and flexible styles of working in a mathematical environment, a finding that accords with Schoenfeld's (1989) exploration of students' beliefs and behaviour in mathematics. As Daskalogianni and Simpson (2000) have noted in a previous paper, students' "predisposition becomes activated in a mathematical task and influences their cognitive processes and consequently their mathematical behaviour while working in it." (p.223).

The question raised, of course, is how can we challenge and enrich students' existing *macro-belief* systems order to ameliorate their performance in mathematics?

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