EVALUATING TEACHERS' KNOWLEDGE IN RELATION TO THEIR CHILDREN'S LEARNING

Constantia Hadjidemetriou and Julian Williams

University of Manchester

We report a study of 12 secondary school teachers' knowledge of their pupils' errors and misconception in graphical reasoning. A diagnostic test, previously given to their pupils, was used as a questionnaire to these teachers with instructions that they should record their perception of the difficulties of the items on a Likert scale, and suggest misconceptions students might have that would cause difficulty. We built a rating scale and the item-perception-difficulty measures that resulted were correlated with the children's actual difficulty as estimated by the test analysis. In addition we sought to confirm the teachers' responses through informal interviews. The teachers' mis-estimation of (relative) difficulties could be explained by one of two reasons: sometimes teachers apparently misunderstood the actual question themselves thus underestimated the difficulty of the item. At other times teachers overestimated the difficulty because they did not realise that children could answer the question without a sophisticated understanding of some concepts i.e the gradient.

INTRODUCTION AND BACKGROUND

Extended research in Mathematics Education (Clement, 1985; Bell et al., 1987; Even, 1998; Janvier, 1981; Kerslake, 1993; Sharma, 1993; Swan, 1985) has identified common errors and misconceptions in pupils' graphical thinking which are significant for their learning. These misconceptions are valuable indications and should not be avoided. Cornu (1991) argued that we should lead students to meet them and treat them as 'constituent parts of the revised mathematical concepts which are to be acquired'. However, as Leinhardt et al said about graphical research:

> 'of the many articles we reviewed almost 75% had an obligatory section at the end called something like 'Implications for teaching' but few dealt directly with research on the study of teaching these topics' (Leinhardt et al, 1990, pp. 45).

We would add that the 'teaching implications' drawn from research on the psychology of learning mathematics are in any case in general problematic: for many reasons these implications rarely impact on practice. Williams and Ryan (2000) argued that research knowledge about students' misconceptions and learning generally needs to be located within the curriculum and associated with relevant teaching strategies if it is to be made useful for teachers. This involves a significant transformation and development of such knowledge into pedagogical content knowledge (Even, 1998) which requires its own study. Shulman (1986) refers to the pedagogical content knowledge as knowledge 'which goes beyond knowledge of subject matter per se to the dimension of subject-matter knowledge for teaching' (p.9), which includes 'the ways of representing and formatting the subject that make it comprehensible to others' (p.10).

This study:

- developed an instrument from the research literature to assess children's learning and misconceptions on a scale related to their curriculum, which we suggest is a prerequisite for transforming this knowledge into professional practice, and
- explored the development of this into an instrument for assessing this aspect of teachers' pedagogical content knowledge.

The development of the assessment instrument involved the tuning of, or the development of, diagnostic items from the research literature on graphicacy to fit the school curriculum. This developed from an analysis of the key work in the field of children’s thinking, identifying items which related appropriately to:

1. Slope-height confusion: pupils failure to distinguish between two graphical features, the slope and the highest value (Clement, 1985);
2. The Linearity prototype: pupils tendency to sketch linear graphs in situations were they are not supposed to (Leinhardt et al, 1990);
3. The ‘y=x’ prototype: pupils’ tendency to believe that all the graphs have a slope of one;
4. The ‘Origin’ prototype: graphs are drawn through the origin;
5. Graph-as-picture: many pupils, unable to treat the graph as an abstract representation of relationships, appear to interpret it as a literal picture of the underlying situation (Clement, 1985);
6. Co-ordinates: pupils’ tendency to reverse the x and the y co-ordinates and their inability to adjust their knowledge in unfamiliar situations (Kerslake, 1993);
7. Scale: pupils prototypically read a scale to a unit of one, or more rarely ten (Williams and Ryan, 2000).

METHODOLOGY

The study sample (N=425) was of year 9/10 pupils from 7 schools in the North West of the UK. The pupils’ test results were subjected to a Rasch analysis. The result is a single difficulty estimate for each item and an ability estimate for each pupil (see Hadjidemetriou and Williams, under review for PME 2001).

The pupils’ teachers were interviewed (N=12) to check that the test was regarded as fair and valid. Our test was also given to the teachers but beyond answering all the questions they were asked to:

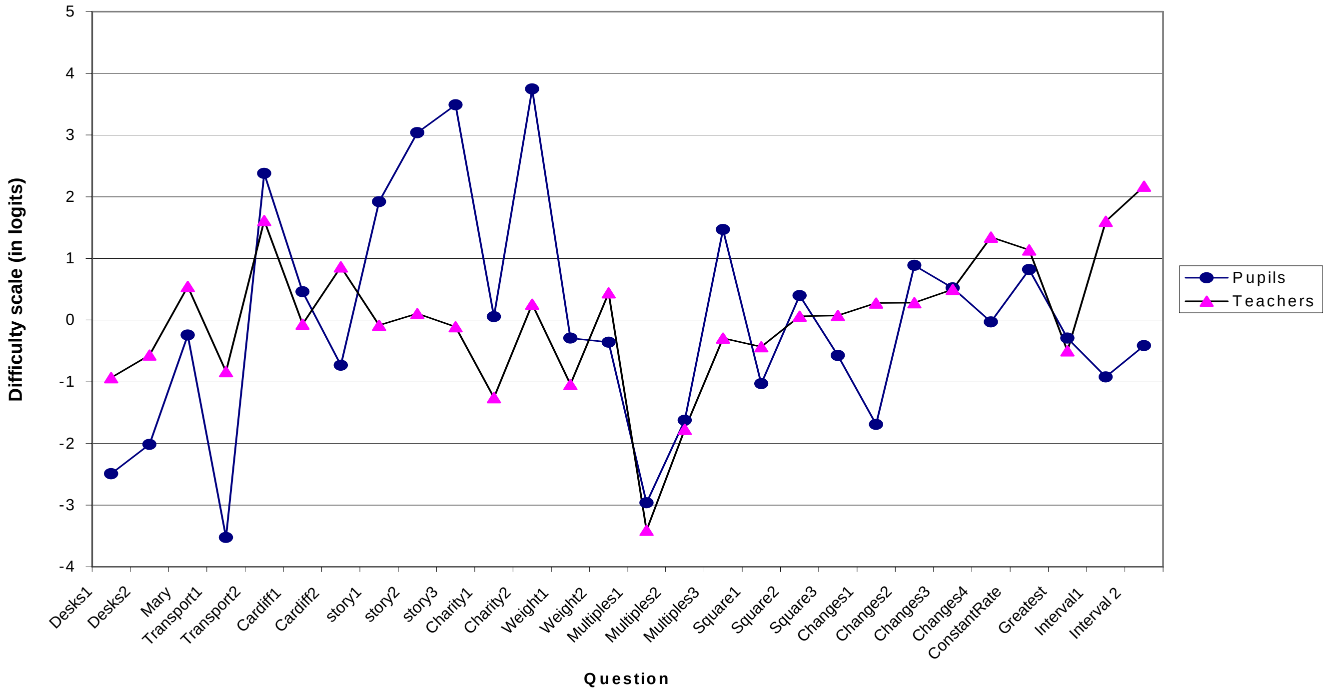
- predict how difficult their children would find the items (on a five-point scale starting form Very Easy, Easy, Moderate, Difficult, Very Difficult)
- suggest likely errors and misconceptions the children would make and
- suggest methods/ ideas they would use to help pupils overcome these difficulties

This data is used here to explore the validity of the research data on misconceptions and also the state of the subject matter and pedagogical content knowledge of this small group of teachers. Teachers' responses were also confirmed through informal interviews, where we began to explore their teaching practices.

RESULTS

As mentioned before, the test was given as a questionnaire to the teachers with instructions that they should record their perception of the difficulties of the items on a five point Likert scale. These data were subjected to a rating scale analysis and the item-perception-difficulty measures that resulted were correlated with the children's actual difficulty as estimated by the test analysis (rho = 0.395).

However, the teachers' estimates were significantly incorrect on a number of items



(see above figure, in which teachers' ratings of difficulty were scaled on a rating scale analysis, and plotted against 'actual' scaled values of the pupils difficulties). In this paper we will only discuss two items: the 'Transport 1' item where teachers overestimated its difficulty and the 'Story 3' item whose difficulty was underestimated.

'Transport 1' (shown below) was an easy question according to pupils' answers but some teachers seem to have given quite high difficulty ratings. These teachers believed that pupils had to be aware that the slope of the distance-time graph represents the speed of each transport. However, pupils' transcripts verify that they could find the answer by looking at the time taken for each transport to travel to school:

Interviewer: how can you see that it (A) is quick and that D is slower?

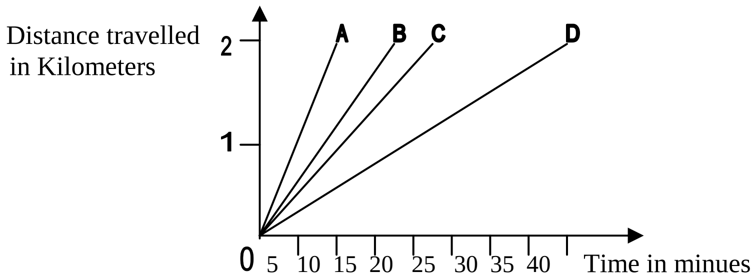
Sara: Because …

Andrew: It takes more time.

Sara: Yes it takes more time. It takes more time to get to the same part. It takes 40 minutes to get to school and the others it takes 15, 10…

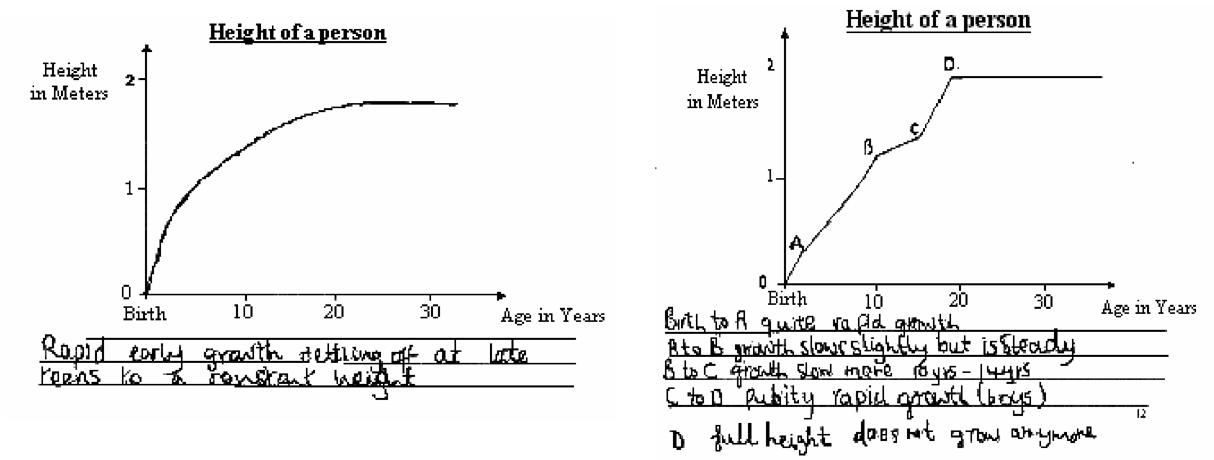
The graph shows journeys by four different means of transport from home to school, a distance of two kilometers: Bus, Car, Walking, Bicycle. Match each line with the appropriate transport

Distance traveled from home



Some pupils identified (after the interviewer's intervention) that the lines differ as far as steepness is concerned but none of them mentioned that the slope of the graph represents the speed. So, sometimes teachers may overestimate the difficulty of an exercise because they overestimate the level of knowledge needed to answer it correctly.

On the other hand, teachers underestimated the difficulty of some other questions such as 'Story 3'. This item requires pupils to draw the 'Height of a person' from Birth up to late thirties. A closer look at teachers' graphs illustrates the problem.



Two teachers' graphs for the 'Story 3' Item

Prototypes such as the 'Origin' and 'Linearity' are dominant in these graphs. These graphs not only underestimate the difficulty of the item but also show that these teachers have low expectations from their pupils since none of those two graphs would receive full credit, if given by a pupil.

In the questionnaire and interviews, the teachers were encouraged to list the misconceptions that children might exhibit. Here we summarise the misconceptions mentioned by the 12 teachers we worked with:

TEACHER MISCONCEPTION	1*	2*	3_I	4_I	5_I	6_Q	7*	8*	9*	10*	11*	12*
Slope-height						✓	✓			✓	✓	✓
Linearity prototype				✓								
‘y = x’ prototype											✓	✓
The Origin prototype											✓	✓
Picture-as-graph			✓		✓		✓	✓	✓	✓	✓	✓
Co-ordinates					✓			✓	✓	✓	✓	✓
Scale	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓

*=Interview and Questionnaire, Q = Questionnaire only, I = Interview only

Teacher knowledge of the seven different misconceptions varies dramatically, with half the teachers mentioning only one or two of them, and two of the teachers mentioning all but one of them.

These indications of teachers’ knowledge seem highly sensitive to whether the data comes from Questionnaire or Interview data: we suspect the different data sources may relate to whether the teachers’ knowledge is tacit (elicited when provoked by an example question) versus explicit (suggested spontaneously in the interview without the questionnaire prompt). We believe this aspect of the research has interesting potential.

CONCLUSION

The ‘discrepant’ items were examined for face validity and found perfectly acceptable as test items. However, the teachers’ mis-estimation of their (relative) difficulty could be explained by one of two reasons:

- in at least three items the teachers underestimated the difficulty for the children because they apparently misunderstood the actual question themselves, i.e. they had the misconception the item was designed to elicit, or they had a limited understanding that did not receive full credit; or
- on two items the teachers' overestimated the difficulty because they did not realise the children could answer the question without a sophisticated understanding of a cocept.

This paper presents the part of a research project that deals with evaluating teachers’ subject matter and pedagogical content knowledge. Another part of the research evaluates pupils’ graphical literacy by identifying their common errors and misconceptions and by group discussions as a way to get an insight into their mathematical thinking process. However, the aim of the project as a whole is to bring all the findings together and to help inform teachers about their pupils' actual difficulties and pupils' actual arguments in order to encourage them to use these in their practices as a starting point for more effective teaching.

However, the initial aim of this paper is not to generalise about teachers' pedagogical content knowledge but to suggest a methodology for evaluating and maybe developing this knowledge. There seems to be a gap between pupils' difficulties and teachers' perception of these difficulties. Our concern is to provide research findings and propose a methodology that will help to bridge this gap.

BIBLIOGRAPHY

Bell, A.., Brekke, G. and Swan, M.: 1987, 'Diagnostic teaching: 5, graphical interpretation teaching styles and their effect.' *Mathematics Teaching*, 120, 50-57.

Clement, J.: 1985, 'Misconceptions in graphing.' *Proceedings of the 9th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 369 - 375.

Cornu, B.: 1991, 'Limits.' In D. Tall (ed.), *Advanced Mathematical Thinking* (pp. 153-166). London: Kluwer Academic Publishers

Even, R.: 1998, 'Factors Involved in Linking Representations of Functions'. *Journal of Mathematical Behavior*, 17(1), 105-121.

Hadjidemetriou C. and Williams J.S.: (2001, under review) 'Children's graphical conceptions: assessment of learning for teaching.' *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education.*

Janvier, C.: 1981, 'Use of Situations in Mathematics Education'. *Educational Studies in Mathematics*, 12, 113-122.

Kerslake, D.: 1981, 'Graphs.' In K. M. Hart (ed.), *Children's understanding of mathematics 11-16* (pp. 120-136) London: John Murray

Leinhardt, G., Zaslavsky, O. & Stein, M. S.: 1990, 'Functions, Graphs and Graphing: Tasks, Learning, and Teaching.' *Review of Educational Research*, 60, 1, 1-64.

Sharma, S. S.: 1993, '*Graphs: Children's strategies and errors.'* London: Kings College.

Shulman, L. S.: 1986, 'Those who understand: Knowledge growth in teaching.' *Educational Researcher*, 15 (2), 4-14

Swan, M.: 1985, *The language of functions and Graphs, An examination Module for Secondary Schools*. Joint Matriculation Board, Shell Centre for Mathematical Education.

Williams, J. S & Ryan, J. T.: 2000, 'National Testing and the improvement of Classroom Teaching: can they coexist?' *British Educational Research Journal*, 26(1), 49-73.