CHARTING KEY ELEMENTS OF CHILDREN'S MATHEMATICAL ARGUMENT IN DISCUSSION: A TEACHING TOOL

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We investigate how children reveal and develop their understanding of mathematics through collaborative argument in group discussion. Working with eleven-year-old children who had different responses to diagnostic test items, we describe how those children developed argument across conceptual locales. The analyses of the discussions led to a chart of the key elements of argument that arose, as well as general strategies for managing such discussions productively. Such devices and strategies are presented as planning tools for classroom teachers in the next stage of our study.

DISCUSSION AS A TEACHING STRATEGY

It has long been 'known' that children's errors and misconceptions can be the starting point for effective diagnostically-designed mathematics teaching. The seminal mathematical work on this in the UK was done in the 1980s by the ESRC Diagnostic Teaching Project (Bell *et al*, 1985), in which *cognitive conflict* was seen as the route to developing understanding.

Argument in discussion is seen as one important source of such conflict. The TIMSS video study reported that Japanese mathematics teaching typically makes use of a diagnostic approach: teachers are prepared with notes on a variety of *likely responses* to a key lead question, with guidance as to the thinking these responses indicate, and constructive teaching suggested related to each (Schmidt *et al*, 1996). This teaching method has been related to the success of Japanese children's mathematical learning and particularly their problem solving capabilities.

There can be no genuine discussion or argument without a '*problematic'*, ie. an unresolved or not trivially-resolvable problem. This induces some purpose and some tension that sustains a discussion. The problematic for a particular group of children can be established through prior testing which provides a range of student responses and methods of solution. The children are then set the task of *persuading* each other by clear explanation and reasonable argument of the answer. The giving of clarifications, reasons, justifications and informal 'proof' is the rationale for the discussion.

The teacher can establish rules for the children's argument – listening, clarifying, sustaining – in order to facilitate genuine participation and group progress in discussion and can, on occasion, act as devil's advocate for positions which may provoke improved reasoning. Mistakes and errors can then be recognised in the light of *alternative position statements* offered for consideration. Discussion can then provoke a demand to shift from procedural to conceptual knowledge – from the 'knowing-how' to the 'knowing-that'. It induces a need to develop a mathematical vocabulary. It locates the voice of 'understanding' with each individual who decides when they are persuaded. It promotes learning as a collaborative activity.

Dialogic methods involve the characteristics of conversation *and* the rigours of reason and persuasion: (a) sustained talk and listening; (b) statements of understanding; (c) thinking-in-progress; (d) the use and consideration of evidence; (e) cognitive conflict, and (f) the making of new connections. We use the term '*argumentation space'* to describe the collection of relevant arguments likely to be used productively in children's arguments about a particular problematic. In this paper we show how we are beginning to chart argumentation spaces in ways that may help teachers to plan classroom discussions to develop productive arguments. In addition we outline the main strategies that we found supported productive argument in group discussions.

METHODOLOGY

In this study a primary school cohort of 74 Year 6 children was screened with a test of some 30 items that was designed to reveal common errors that had already been identified as relevant to their mathematics curriculum and level (Ryan and Williams, 2000). We had previously identified the most interesting errors based on the criteria that they should be: (a) common enough to reward a teacher's attention, (b) relevant to a significant locale of the curriculum being taught at the given age level in focus and (c) significant in terms of the literature on the psychology of learning.

The test items were drawn from the whole primary school curriculum. By way of an example, we will cite the case of an item called 'Ordering' that asked children to sort the numbers 185, 73.5, 73.32, 57, 73.64 from smallest to largest.

There are two common errors expected for 'Ordering':

- 57, 73.5, 73.32, 73.64, 185 ('decimal point ignored'), and
- 57, 73.32, 73.64, 73.5, 185 ('longest is smallest').

These errors were identified in the APU study of the early 1980s (Assessment of Performance Unit, 1982) and are as prevalent today as then. The 'decimal point ignored' error is believed to have an important bearing on the development of children's number concept, and is typical of children's over-generalisation of whole number conceptions to the wider field of rational numbers.

From each of the three Year 6 classes, we selected 4 children for each discussion group on the basis that they had *provided a range of responses on the test items*. There were nine groups (36 children). The children from each group were from the same class and knew each other well, though were not necessarily from the same friendship group. Their teachers advised us on the likely successful dynamics for each group. They were mixed groups of boys and girls.

The children, in groups of four, recalled their test item response (an interval of a few days only) for selected items and were invited to present an argument for their response to the group. We, as researchers, adopted the teacher's role in discussion. All discussions were transcribed and analysed.

ANALYSIS OF TRANSCRIPTS

The analysis of argument follows Toulmin's scheme in general (developed by Cobb and Bauersfeld, 1995 and Cobb *et al*, 2000; Krumhauer, 1997, and others). Propositions relevant to the issue are 'backed' by arguments that are then subject to testing. In general children find it unnecessary to argue propositions which are believed to be shared, (i.e. taken-as-shared) so any particular discourse reflects the presumed shared points of departure, including the rules of argument in such situations. In this the Researcher as a quasi-teacher assumes the authority and seeks to ensure reasonableness, the need for the inquiry to persuade by good thinking and argument, and so on (Costello *et al*, 1995).

An important role in productive argument may be played by *tools in practice*, which may provoke the formulation of connections between components of mathematical knowledge, new constructions and hence productive backing. The number line has been shown to play a significant role in many such contexts, and does so in the following example. Here we present part of one transcript for an argument about the ordering of decimals. Some commentary and analytical categories used are shown in bold to the right, these relate to the argumentation, the conceptions/language and the tools/referents.

Kim:	OK. I put 57 there – …. Then I put 73.5, … Then I put 73 point thirty-two, then I put 73.64 point sixty-four, then I put 185.	**Everyday language**
Natalie:	Well, I got 57 at the beginning too. And then I got 73.5. Then I got 73 point three- two Then I got 73 point six- four. Then I got 185.	**Mathematical language**
RES:	Could you explain why you put 73.5 before 73.32 (three, two)?	**Focus**
Natalie:	Because 73.32 (three, two) has got two digits after the decimal point and 73.5 has only got one.	**Backing: separating decimal as wholes**
Elise:	I'm not so sure, because 73.5 is basically 73 and a half. 73.64 (six, four) is, I'm not sure if it would be over a half or under… Actually I think the same as Kim… because, like Natalie said, there are two digits there, and two digits there, and only one digit there.	**Intro fraction referent: conflict, backing**
RES:	If I had a number line… Are you used to seeing a number line? (*children nod*). And I had 72. 72 would be back there. 73 would be there. 74 would be there. Where would you put 73.5? Do you want to do that Richard?	**Introduce tool: number line**
Richard:	(*puts 73.5 half way between 73 and 74*)	**Number line product**

RES:	Can anybody put any other numbers in between 73 and 74?	**Check alternatives Press**
Kim:	Yeah (*puts 73.64 above 73.5*)	**Number line product**
RES:	Why have you put in bigger than 73.5?	**Check backing**
Kim:	Because it's over a half	**Backing: fraction equivalent**
RES:	Any other numbers you could put on that number line? Do you want to have a go Natalie?	**Press**
Natalie:	73 point two-five	**Number line produces new argument**
RES:	73.25 (?), where would that go? Could you tell us why you put 73.25 *just* there?	**Focus on 0.25**
Natalie:	It's a quarter of the number.	**Backing: fraction equivalent**
RES:	Do you agree with that? (*children nod*.) So, it's gone… why has it gone exactly there? Is that because it is halfway towards a half?	**Check backing**
Natalie:	Yeah.	
RES:	Could you put a number on that number line Richard?	**Develop number line**
Richard:	Erm, 73.45… (*places it between 73.25 and 73.5… places 73.75 between 73.5 and 74*).	
RES:	73.75, right? That's… ?	**More 2-place decimals**
Richard:	Three-quarters.	**Backing: fraction equivalence**
RES:	So you put that halfway between 73 and a half, and 74… Where do you think 73.32 should go?	
Kim:	Before 73.5	
RES:	Why?	
Kim:	Because 73.5 is a half and 73.32 is just after a quarter	**Resolution of referents**
RES:	Could you say *why* it's just after a quarter?	**Check backing**
Kim:	Because a quarter is 73.25 and 73.32 is bigger than 73.25 (*All agree*) I now think 73.32 is there, and 73.5 is there.	**Kim sees change of mind**
RES:	You all want to change your minds now? Now why	**Seeks reflection**

	did we go wrong in the first place?	
Kim:	Because we saw them as two-digit numbers, and we thought that the two-digit numbers were more than a one-digit number.	**Making 'new' explicit**
Elise:	I would say that 73.25 is a quarter, and it's less than 73.5 because that's a half, and 73.32 is just over a quarter, so it would be just under 73.5	**Fraction-decimal explicit**

Extracting the most productive and essential elements of this and other arguments about 'decimal point ignored' allow us to make a summary chart (Figure 1). This summarises the lines of argument that we found that we think teachers will find useful in preparing for a particular discussion about 'Ordering'. In addition, we have attempted to summarise effective strategies used by researchers in generating discussion (Ryan and Williams, 2000).

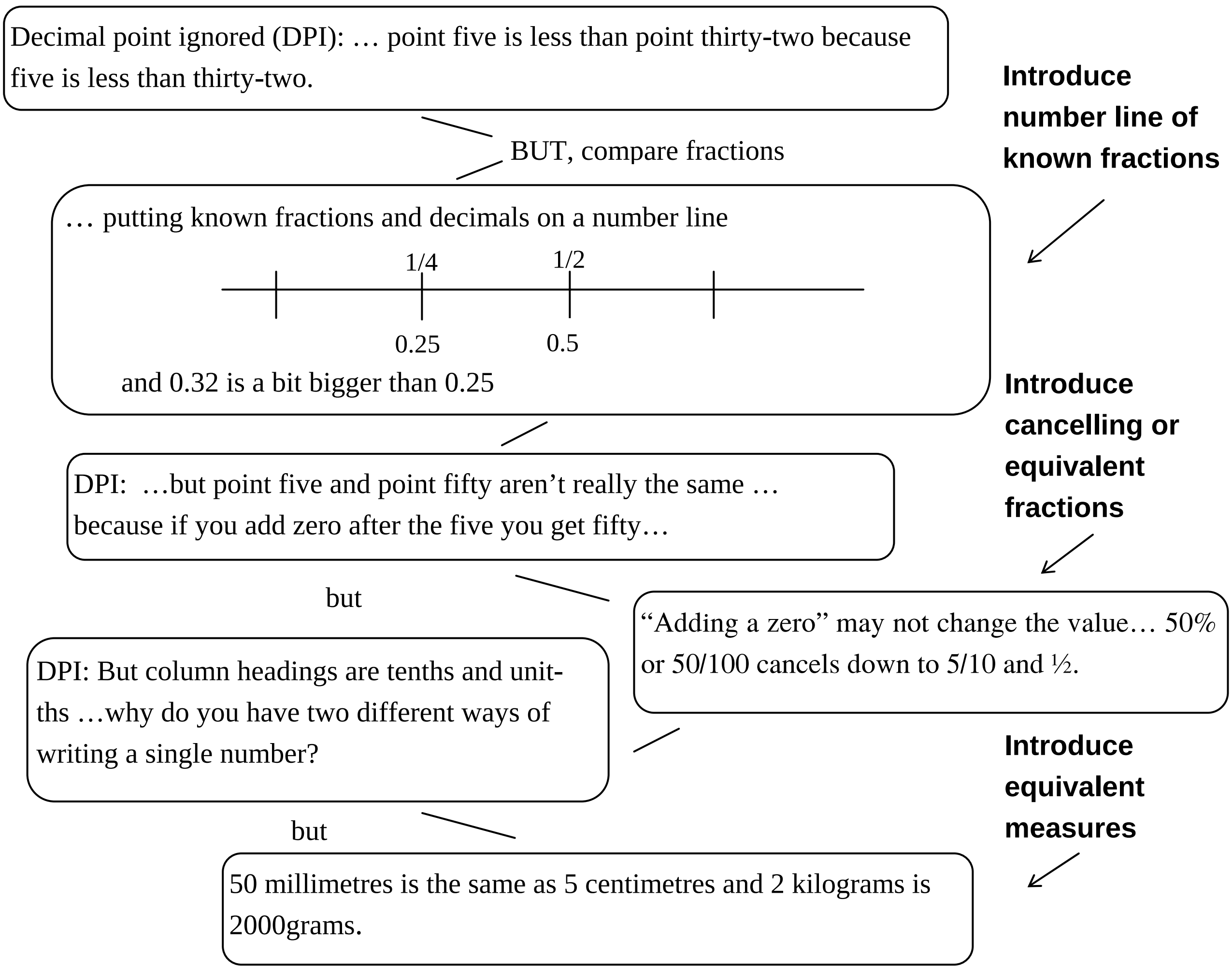

Figure 1. A chart of argumentation space for 'decimal point ignored'

CONCLUSION

We have shown that it is possible to use dialogue generated in research to chart an argumentation space that describes the children's arguments in response to a provocative diagnostic item in a conceptual locale. The concept of an argumentation space located around a diagnostic item is designed to be helpful in supporting teachers' pedagogical content knowledge. We interpret these spaces as providing potential classroom discourses structuring potential zones of proximal development of individuals within a class. The dialogues teachers might generate in replication of the research setting might thereby provide opportunities for individuals to learn by testing their responses against those of their peers, and being given an opportunity to evaluate and shift their position accordingly.

We are currently investigating and evaluating how helpful these 'charts' can be to teachers in practice, and whether the resulting dialogues will be successful in helping children learn. The next step in the project involves a study with teachers delivering, marking and interpreting the diagnostic test and observation of their subsequent teaching through discussions based on these argumentation spaces.

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