LEARNING MATHEMATICS IN GROUPS: A COORDINATION OF PSYCHOLOGICAL AND SOCIOLOGICAL PERSPECTIVES?

Anjum Halai

University of Oxford, UK & The Aga Khan University, Karachi, Pakistan

This study was part of my doctoral research and aimed to study the role of social interactions both, between students and between students and teacher in students' learning and understanding of mathematics as they did mathematics in small groups. The study was based in two classrooms in Pakistan where students in small groups (10-12 yr.) learnt mathematics.

The methodology was qualitative in nature. Data generation was through observations, which were recorded on videotapes and student interviews, which were conducted under, stimulated recall. Field notes were also maintained of all research activities. Preliminary analysis raised questions and issues about the nature of learning and how it takes place. Of particular significance were questions regarding the socio-cultural and individual psychological elements in learning?

INTRODUCTION

This study was part of my doctoral research. I studied over a period of nine months, two small groups of students in different schools in Karachi, Pakistan. The students were in the 10-12 years of age range. The focus of my study was the role of social interactions in students' learning and understanding of mathematics as they did mathematics in small groups (in the classrooms in Pakistan).

METHODOLOGY

The methodology was qualitative and interpretative in nature. Data generation was through observations recorded on videotapes. To follow up on questions emerging from ongoing observations, student interviews were conducted under stimulated recall. Fieldnotes were also maintained of all research activities. The data was in the form of fieldnotes, video tape recordings of students' small group work and audio tape recordings and audiotape transcripts from the interviews. The analysis of the data involved making inferences about:

The meaning the students in the group gave to their own activity, the group mates' activity and the task at hand

The learning opportunities that arose for each student in the group

The conceptual reorganisation that each student in the group made

THEORETICAL FRAMEWORK

Initially, I approached learning from a radical constructivist view of learning (Glasersfeld, 1987). A consequence of taking this view of learning is that individual learners construct unique and idiosyncratic personal knowledge when exposed to identical stimuli. I interpreted the role of the teacher to provide learning opportunities to enable students to construct sound mathematics knowledge.

However, like Cobb, Yackel & Wood (1995), I found that, as students in my research worked at mathematics tasks in small groups settings and constructed their own mathematical understandings, they did not do so in isolation. Interactions with both, other students and teacher gave rise to significant learning opportunities. Collaborative work involved developing explanations that were meaningful to another and trying to interpret and make sense of another's ideas and solution attempts as they evolved. During subsequent whole class discussion, students were expected to give coherent explanations of their problems, interpretations, and solutions, and to respond to questions and challenges by their peers. They were also expected to listen to and try to make sense of explanations given by others, to pose appropriate questions, and to ask for clarifications. When students engaged in such a discourse, the nature of their mathematical activity encompassed learning opportunities that do not usually arise in traditional instructional settings. These opportunities had their roots and beginnings in social interaction. I thus came to see that radical constructivism was not helpful in explaining classroom learning. I will highlight these arising issues and questions with the help of an example from the data.

Chand Raat[1]: A Vignette

The vignette comes from a lesson in one of the two schools. The lesson I observed followed the usual pattern of all lessons observed by me so far i.e. the class did some whole class work which was followed by work in groups on an activity sheet provided. The group work was followed by a plenary session where different groups shared their learning with others in the class. The first three groups to finish went up to the black board and put first, second, or third, rank in the order that they had finished along with the group members' names. The first three groups got a chance to contribute in the plenary. I observed the group comprising Mansoor, Salim and Faizullah. On the day of this particular lesson Salim was absent.

The episode titled Chand Raat was a task about a fair way of sharing the profit. The worksheet that was used in the class is given in the box below.

[1] Chand Raat is the night the new moon is sighted and heralds the beginning of the new month according to the Islamic calendar. The Chand Raat at the end of the month of Ramadhan heralds Eid the next day and is a night of great rejoicing. Girls go out to buy bangles and boys put up stalls to sell them.

Proportions

Ques 1: Suppose two of your friends decided to set up for Chand Raat Bangles Stall on Tariq road. One friend contributed Rs. 500 and the other contributed Rs. 1000. They made a profit of Rs. 1000.

What would be a fair way of sharing the profit?

What would be each friend's share in the profit?

Can you explain the method you used to share the profit?

It was related to the curriculum component of proportions and proportional reasoning. This was the fourth lesson in a series of five on ratios and proportions. The teacher had asked me to help her plan lessons on the topic of ratio and proportions. Hence, we planned this work on proportional reasoning. At the time of teaching it was the month of Ramadhan and so for the problem we provided a real life situation of Chand Raat that the boys could relate to. The amount of money involved was such that the students could be expected to have had experience of dealing with such sums of money.

WHAT HAPPENED

At the start of the Chand Raat activity, Faizullah informed Mansoor that they should divide the profit into three shares because one person had put in double the amount of the other. Mansoor did not agree and suggested that the teacher had said to divide the profit between two friends, which meant each friend would get Rs.500. Hearing this Faizullah left his line of thought and divided the 1000 into two. He wrote this work in the notebook. It appeared that, none of the two were sure of their reasoning. Having written the solutions they sat apparently unsure of what to do, moved onto some other work, talked about other things. During this time, Mansoor said repeatedly that they were in trouble because Salim was absent and so they had to do the work. In the meantime I happened to come along and they beckoned me for assistance. They asked me if they were right in thinking that the profit should be equally divided between the two. I invited them to put themselves in the situation and think, if they had put in unequal amounts of money would they share the profit equally? However, immediately after I left them, the two beckoned the teacher and showed her both the solution procedures, that of dividing the profit in two shares and of dividing it in three shares. By then the time allotted for group work was almost to an end and being in a rush the teacher accepted the three share division and asked them to be ready to share their work with the whole class. At the teacher showing her approval Faizullah gave a shout of joy while Mansoor asked Faizullah repeatedly, how was his answer correct? Faizullah however could not answer him because he was called to the blackboard. He went up to the blackboard and gave a clear explanation of what he had done and why

ANALYSIS

The analysis raised a number of issues. What was the interpretation of the task that each of the two made? It appeared that Mansoor saw the task as that of equal sharing. The idea of 'fair share' for him seemed synonymous with equal share. While Faizullah interpreted it as sharing but an equitable sharing with the proportion of investment kept in mind. Even though in their conversation no explicit mention was made to work done in earlier lesson on ratios and proportions Faizullah had managed to get to the mathematical ideas behind the context.

The learning setting provided the two students with opportunities to learn. First, the situation created by both Mansoor and Faizullah interpreting the sharing differently created an opportunity where had they challenged each other and justified their work; both might have modified their interpretations of the task. For Faizullah, however, another opportunity was created when he was put in a position to explain his work to the class. When Faizullah explained his work it was clear that he knew that fair share meant looking at sharing as equitable. In the stimulated recall interview he said that he became convinced of his reasoning being sound as a result of the explanation he had to give in the class.

However, a number of issues arise when I think of what Mansoor learnt. It appeared from what happened in the above lesson that Mansoor was perplexed as to how Faizullah's answer was correct. In the stimulated recall interview, in response to the question, did he agree with Faizullah's answer to the Chand Raat question? He said that he would have divided the profit in two and seen what was the right answer. When asked how he would know which was the right answer he said that he had no idea. Discussing the psychological development of the concept of proportions Nunes et al (1993) state that Piaget tried to form a general picture of the development of concept of proportions and found that to begin with students either focus on one variable or take no account of the relationship between variables. It is possible that Mansoor was at that initial stage of development of proportional thinking.

It was clear that Mansoor had not modified his interpretation of fair share from an equal share to a more mathematically appropriate one of equitable share. But, in a real life situation does a fair share always mean a share proportional to the investment? Perhaps, Mansoor was looking at other variables in the problem setting such as the time and effort put in by each. Perhaps, the issue was not the interpretation of 'fair share', instead it was the problem setting itself. The context of the problem was acting as an obstruction so that he stopped at the relatively simple mathematics of halving the profit but had not moved beyond it to extrapolate the more sophisticated mathematical idea of proportional division. Discussing the lessons for education that were learnt from their study of Street Mathematics Nunes etal (1993, pg.152), believe that the value of the problem situation lies in providing

meaning to the mathematics being done and Ideally the problem solving task is carried out under the constraints of social, empirical, and logical rules that would be present in a real life situation. In this case the teacher could not leave the students thinking that Mansoor 's answer was acceptable because she would be doing injustice to the students of not letting them learn the mathematics i.e. the general concept of proportions and proportional reasoning. However, ignoring Mansoor's response might have meant that the mathematics did not make sense for Mansoor as it did not agree with his real life experiences. In that case issues regarding equity in real life setting come into the problem setting. This raises the issue, does one stay in the mathematical realm and ignore the issues of equity and ethics that are part of the social setting of the problem? If so then what is the role of the social setting of the problem?

The analysis could take a look at the teacher's role, and the norms of the classroom to understand why the two students were so concerned with the 'right answer'.

For my discussion here, I will move on to something that Mansoor learnt in the lesson. As noted earlier, Mansoor had mentioned a number of times in the lesson that they were in trouble because Salim was absent and so they had had this difficulty in doing the work. However, once the teacher accepted the work of the pair and Faizullah successfully explained his thinking to the class the two were very happy. In the following lesson when Salim was present Mansoor told him that he was pleased that they could do the work and the whole class was also surprised that Mansoor and Faizullah could do the work when Salim was absent. I probed this issue in the stimulated recall interview and found that there were interpersonal tensions in the group, because of the fact that the group wanted to finish early and be in the first three ranks. The result was that Salim was taking over the group work and the other two followed his instructions.

The question arose for me where these issues fit in my scheme of analysis. The beliefs about one's own ability to do mathematics and the self image within the group was part of each individual's psychological make up. But, the emotive issues that arose were at the cross section of how those beliefs came into play in the social. Once again I came up with this question of what is primary to learning the individual or the social context ? Both have a role to play but the nature of the role is very complex and not easy to establish, because social interactions are very difficult to define. While, learning and understanding cannot be seen, only inferred from what one hears and sees.

REFERENCES

Cobb,P., Yackel, E., & Wood, T. (1995). The teaching experiment classroom. In P. Cobb & H. Bauersfeld.(Eds.) *The emergence of mathematical meaning: Interaction in classroom cultures.* New Jersey: Lawerance Erlbaum.

Nunes, T. Scheilmann, A. D., Carraher, W. D. (1993) *Street mathematics and school mathematics.* New York: Cambridge University Press

von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R. B. Davies, C. A. Maher and N. Noddings (Eds.), *Constructivist views on the learning and teaching of mathematics.* Journal for research in mathematics education. Monograph no. 4. Reston, Virginia: National council of teachers of mathematics