CHANGES IN YOUNG CHILDREN'S STRATEGIES WHEN SOLVING ADDITION TASKS

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The aim of this study is to determine the pathway of changes that occur in the problem solving strategies of 5-6 year old children when they are engaged in solving a specific form of addition task. Karmiloff-Smith's model of Representational Redescription (RR) suggests that higher conceptualisation and control of the employed strategy develops both before and after the achievement of an efficient solution. Evidence from data reported in this paper tends to support this hypothesis.

RATIONALE AND AIM OF THE STUDY

Very often in mathematics lessons, students who produce a correct answer or solution to a problem are not asked to justify their solution. Correct answers are usually "ticked" while justifications, explanations and further work are often only asked for when errors occur. Karmiloff-Smith's model of Representational Redescription (RR) (1984, 1992) suggests that higher conceptualisation and control of employed strategies develops both before and after the achievement of an efficient solution. The aim of this study is to determine the move 5-6 year old children make from procedural success to higher conceptualisation and understanding of the procedures they employ in the successful solution of an arithmetical task.

THEORETICAL FRAMEWORK OF THIS STUDY

The distinction between *procedural* knowledge (the ability to perform a task) and *conceptual* knowledge (the ability to understand the task) has been the subject of long debate in mathematics education (Skemp, 1971; Baroody & Ginsburg, 1986; Silver, 1987; Hiebert & Lefevre 1986; Bisanz & Lefevre, 1990). In the field of developmental psychology, Karmiloff-Smith (1992) has developed a theoretical model to account for development and learning based on the assumption that inflexible procedural behaviour lies on knowledge which is *implicit*; i.e. knowledge which is not available as manipulable data. Karmiloff-Smith argues that after procedural success certain types of cognitive change may take place. In this process of change, implicit information embedded in an efficient problem-solving procedure progressively becomes more explicit, manipulable and flexible. In Karmiloff-Smith's developmental model, the issue of cognitive change is addressed in terms of knowledge explicitation that applies in a variety of domains, including mathematics. Within the context of problem solving however, the idea of knowledge explicitation has been studied in physics, spatial, linguistic and notational, but not in mathematical tasks (Karmiloff-Smith, 1984). In the framework of this study, children's developed strategies are seen as a product of the combination of different pieces and forms of arithmetical knowledge (conceptual, factual, procedural). Conceptual understanding as it is built in the "micro-context" of an arithmetical task is approached and studied on the basis of the idea of *knowledge redescription* (Karmiloff-Smith, 1992).

THE MODEL OF REPRESENTATIONAL REDESCRIPTION (RR)

Karmiloff-Smith argues that implicit information which is already stored in the mind, in a certain form of internal representations and is embedded in special-purpose procedures, is subject to an iterative process of redescription. The RR model is a *recurrent 3-phase model* with the following main characteristics:

Phase 1: "The procedural" phase
During this phase children's behaviour is considered to be "success-oriented". Separate units of behaviour are not brought in relation one to another. At the end point of this phase consistent successful performance is achieved, and this is what Karmiloff-Smith calls "behavioural mastery" (*ibid* p. 19).

Phase 2: The "meta-procedural" phase
In this phase, an overall organisation of the internal knowledge representations takes place. As a result, children generate "organisation-oriented" behaviour. They move beyond procedural success to a phase of internal representational organisation and the generation of a unified, single approach for all the parts of the problem.

Phase 3: The "conceptual" phase
During this phase the interaction between external data and internal representations is regulated and balanced as a result of the search for both internal and external control. Representations that sustain children's behaviour in the third phase are considered to be richer and more coherent, even though children's behaviour in this phase can seem identical to the behavioural output at phase 1.

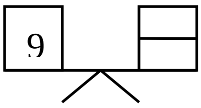
METHODOLOGY

The aim of the study is to describe and analyse changes in children's strategies on the basis of the premises and explanations that the RR model introduces. The study focuses on a number of cases. Changes in children's successful strategies are studied at a micro-developmental level. This means that the focus is on changes that occur within the context of a specific form of arithmetical task and within the boundaries of a sequence of limited-in number-sessions. For the data collection, the micro-developmental method is combined with the clinical method of interviewing. Children are interviewed individually while working upon specially designed tasks.

The "card" task asks children to find all the possible number bonds that result in a "target" number (for example, find all the possible number bonds to make 9, or 10 etc.). A pile of identical cards with incomplete number sentences, such as the one on the right, is at children's disposal.

$$\boxed{\square + \triangle = 9}$$

Children have to pick up one card at a time, put a number in the square and another one in the triangle in order to complete the number bond until there are no more possible ways to do so. The task is repeated with different "target" numbers. Each number bond that a child produces is considered as a step within the solution process. The hypothesis is that, in the context of this particular task, children initially approach each step of the task separately, employing different methods, and calling upon different pieces and types of knowledge. This kind of approach may well be successful. However, if appropriate motivation is given to children to keep working on the task, eventually, the different pieces of knowledge will be organised in a strategy applied consistently for every step, to the whole of the task. Similar-in goal tasks are presented to children to test the flexibility of the new strategy, and its transferability to similar goals. One such task is the "balance" task. On a piece of paper balances, such as the one shown below are drawn.



Children have to write one number on each of the blocks at the right side of the balance. The sum of these numbers must be equal to the number written on the block at the left side.

SAMPLE

Five children from a year-1 class of a South England infant school participated in the pilot study. Because the study focuses on children's evolving strategies after success and during a relatively limited number of sessions, children most competent in addition were selected to participate so that less time would be devoted to consider arithmetical misunderstandings and errors.

AN EXAMPLE OF EMPIRICAL DATA

Key:
C: the experimenter
( ): movements, actions
[ ]: writing

An example of changes observed while working with a child is given below. Chris was 5 years 8 months old and participated in four sessions.

First session

The target number was 7. Chris produced the following number combinations in the order shown in the inset below:

[6+1]
[5+2]
[4+3]

After writing down the first number, Chris counted on using his fingers to figure out the second. The interviewer asked Chris how he chose which number to write first. Chris replied:

Chr: Cause 6 it's just next to 7.
C: And why did you choose 5 after that?
Chr: Cause it's 1 more.
C: And how did you choose 4 afterwards?
Chr: It's 3 more.

The first number that Chris chose was the one that was closest to the target number. This choice allowed him to count less. It was an economical in counting method. Chris completed the task and produced the following number bonds:

[3+4]
[1+6]
[2+5]
[0+7]

To produce these number combinations Chris "swapped around" the previously produced number bonds, which is how he changed their addend order. After writing the last number bond Chris seemed to keep thinking. The interviewer asked:

C: So are these all?

Chr: There are more but I don't know them.

In this session Chris approached the task by focusing on the production of each number bond separately. The mixture of the two methods (economical in counting method and 'swapping') for the production of each number bond allowed Chris to be successful. Each one of these steps in the solution process was a separate unit of behaviour which was elaborated enough to allow success. However, Chris was not aware of his success. In terms of the RR model a first level of procedural success had been achieved.

Second session

The target number was 8. Chris produced the following number combinations:

[7+1]
[0+8]
[8+0]
[**1**+7]
[**2**+6]
[**3**+5]

After writing down the last number bond Chris took some time to look at the completed number sentences and said:

Chr: Then it gets higher and higher and higher…

C: Which one is getting higher?

Chr: (shows first numbers in the last three number bonds).

Chr: It goes 7,…6, 5 (shows from the top to the bottom). Oh, 5, 6, 7 (shows from the bottom to the top). The lowest numbers go down there (shows 1, 2, 3 from the top to the bottom) and then like that (shows 5, 6, 7, from the bottom to the top)…like a zigzag. 4 and 4! **[4 + 4]**

C: How did you think of that now?

Chr: I don't know.

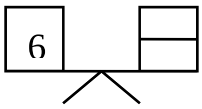
Chris noticed a pattern: a regularity of the numbers in the number bonds he had produced up to that point. He produced a new number bond following this pattern. However, this seemed to be a discovery which was not explicit enough yet to allow the formulation of verbal explanations.

Third session

The target number was 6. Chris completed the task producing the following number bonds:

[0+**6**]	[6+0]	[**5+1**]	[1+5]	[2+**4**]	[4+**2**]	[3+**3**]

For the production of these number combination Chris applied his initial methods. At the end of the task however, he uttered the numbers in order, to check if he had finished. The numbers that he was showing while uttering them in order appear in bold. This was the first time that Chris realised the need to put the numbers in order so that he could check. However, it was clear that he had not yet fully grasped the rationale behind this strategy because he checked the numbers by moving from one column to the other. This would not allow one to know if all the possible numbers had been used either as first or second addends and thus whether all the possible number bonds had been produced. However, at the next run with the same task Chris employed the idea of "ordering" not only to check but to solve the task. This is considered as an important change and indication of Chris' movement away from the initial mixture of isolated though successful methods to an organisation-oriented "meta-phase" marked by the discovery of a new strategy. He applied this strategy consistently for all the steps in the task. Furthermore, he transferred the strategy to the balance task:



Chr:

5	4	3	2	1	0	6
1	2	3	4	5	6	0

Chr: No more (he said right away after finishing).
C: How do you know?
Chr: 1, 2, 3, 4, 5, 6, 0 (shows second numbers from the top).
That's all the different that you can make.

Chris produced these combinations in a few minutes by simply putting the numbers in the upper and lower boxes in descending and ascending order, correspondingly. Chris did not only transfer the "ordering" strategy to the balance-task but most importantly, this was the first time that he appeared to be certain of the completion of the task.

Fourth session

In this session there were indications that Chris' approach to the task had been subjected to a process that made it more explicit. Chris applied the "ordering" strategy in the card-task for bigger target numbers such as 19 and 25. He justified the use of the strategy by saying: "It's easier… I know when I finish". Moreover, a violation of constraints imposed by Chris' initial theory of "economy in counting" was observed: Initially, in order to count less, Chris always used as first addend in the first number bond produced, the number which was closest to the "target" number. As a result, in the first number bonds the first addend was always bigger than the second. In the last session the first addend of the first number bonds produced was the smaller number. For example, when the target number was 19, Chris started the solution process producing these number bonds:

[0+19]
[1+18]
[2+17]

This was in opposition with Chris' initial theory of economy and probably happened because he realised that the use of the new strategy allowed him to avoid counting and any type of calculation all together. It allowed him to be successful and even more economic in effort.

PRELIMINARY CONCLUSIONS

Overall, this was an example of changes that occurred while working with a particular child in the context of a small-scale pilot study which was part of a broader research project. At this initial stage, the data shown suggest that children introduce qualitative changes and modifications to their successful strategies. These changes indicate the passage from initial success-oriented behaviour to an organisation-oriented phase in the RR framework during which the problem solver acquired a better control over the features of the task. If this evidence is replicated in the main study, then one theoretical implication is that the form and type of changes that the RR model accounts for, seem to pertain to the changes observed in children's strategies within the setting of arithmetical problem solving situation. A practical implication of this for teaching is that if the process of Representational Redescription constitutes another way of constructing knowledge, then it is worthwhile giving children the time and space they need to work upon the knowledge that supports their own successes.

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