TOWARDS NUMERACY WITH PROGRESSION: BENCHMARKING WITHIN A PYRAMID PROJECT

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This paper is about a study of 48 pupils aged 4 to 14 years, within a 'pyramid' of 15 schools engaged in the early stages of a numeracy improvement project. The purpose of the study was to confirm or challenge judgements made by key teachers [primary heads /co-ordinators, middle school subject leaders, upper school KS3 co-ordinator]. The longer term objective was to create realistic benchmarks for pupil achievement throughout the pyramid. The paper describes the basis upon which the study was carried out, its findings, indicating the nature of the benchmarks that came from it.

THE PYRAMID PROJECT

The 15 schools had a history of co-operation in school improvement projects, the previous of which involved tracking the progress of individual children across the curriculum. The schools had identified a need for sustained help in the area of mathematics, partly from their Key Stage test results being well below national and county norms. The project was established to run over a three-year period from 1998-2001, and they had written success criteria; the second of these was:

> *Benchmarking of knowledge, understanding and skills will be established.*

Initial discussions with the project steering group and with the 16 'key teachers' made apparent the difficulties inherent in simply adopting and extrapolating the Benchmarks of the *Hamilton Mathematics Project*, or the Key Objectives of the *National Numeracy Project*. The central problem lay in the fact that these were not seen as achieveable within any reasonable timeframe, and so their application could be counterproductive, having negative effects on staff and pupil morale.

We suggested several possible ways forward, but the one that met with immediate approval was drawn from the work of the PrIME Project's Southern Consortium in which we had both participated. Consider a year group as split into 5 groups, each of 20% of the full ability range – the image offered was of the five digits of a hand. The variability at both extremes is too great to establish norms of achievement, but it is possible for the other three groupings. It was decided that we would all attempt a preliminary study to try to establish the levels currently being achieved by pupils who were typical of / centrally placed within each of these groups - this process was called 'Tidemarking'.

FOCUS POINTS AND THE BASIS FOR SELECTING THEM

The agreed broad focus was on number, but within this five focus points were selected. The basis for this choice was a consideration of the work of Gattegno,

Wittmann, Treffers, Vygotsky, and Askew. A theme running through Caleb Gattegno's work from *'The Commonsense of Teaching Mathematics'* to *'The Science of Education'* is a key idea: given that all learning exacts a price, the combining of known facts with 'educable awarenesses' will lead to the learner *deriving further knowledge* and facts at little further expense. Wittmann's work contains a similar vision, but also offers key images that can be placed *at the disposal* of the learner. Treffers introduced one of the most powerful of these images – *the empty number line*. Their ideas are extremely compatible with the theories of Vygotsky, in particular the idea of the teacher providing 'scaffolding' with which learners can extend beyond their unassisted achievement into assisted achievements within their current 'zone of proximal development'. Askew has recently contextualised and popularised some of these ideas in relation to current numeracy developments, providing practical examples.

Numerals

We decided that one focus would need to be the recognition and ability to read and relate numbers as numerals, whether these were whole number numerals to 10, or those for numbers up to and over 10 000, or decimals, such as 0.75 and 0.036.

It was important to identify *how* pupils recognised these, so our next focus was…

Locating

This involved deciding whether pupils could relate numbers to their 'neighbours', plus or minus 1, or 10, or 100, or 100, or 0.1, etc. and some negative numbers. Location was also looked at in relation to Gattegno cards and charts, Treffers' empty number line, and Wittmann's fields of 5, 10, 20, 50, 100, 1000 etc. as appropriate.

Counting

The third focus selected involved the freedom to move around in the number system through counting in various steps: counting on and back in 1's, 2's, 5's, 10's, 100's, 0.1's etc.

Complements

Strongly in evidence within both Gattegno's and Wittmann's work is the idea of complements in 5, 10, 20, 50, 100, 1000, or for decimals in 1, or 0.1. Again this was scaffolded through Wittmann's fields and Treffers' empty number line.

Known facts and deriving strategies.

We selected a set of 30 abstract calculations that had been used within the PrIME Project in the Southern Consortium, but needed to extend these with three simpler sets of calculations. These 60 number problems were to be applied appropriately to determine the limits of [a] factual recall, [b] ability to calculate mentally rapidly, or [c] ability to derive answers through strategic application of known facts, possibly by reference to the scaffolding imagery of Gattegno, Wittmann or Treffers.

PREDICTIONS AND THE INTERVIEW PROCESS

The key teacher group worked together to make predictions about how the typical pupils in each group and year would perform in relation to the first four of these focuses. Their resulting tables contained a number of gaps and hotly debated predictions. The agreement was that none of these areas would be focused upon deliberately by teachers prior to the interviews, but that subsequently they would have access to our interview materials and could continue and extend the study.

Over little more than a two week period we visited all of the 16 teachers in their schools and interviewed 3 pupils they had selected as typical of the 3 central 20% achievement groupings. Our 'scripts' varied as is natural between 4 year olds and 14 year olds, but we retained sufficient commonality of presentation and expression to remain convincing to the key teachers and their pyramid steering group.

To illustrate this, for **numerals,** a large set of A5 graduated numeral cards was available to us within each interview. *In practice,* the Year 2 pupils typically attempted mainly whole number cards to around 100, while the Year 5 pupils often attempted whole numbers to 10 000 and some common decimal value cards, such as 0.5, and the Year 9 pupils also attempted more demanding decimal cards. These same cards were used for both the **locating** and the **complements** parts of the interview.

The scaffolding materials available to us also varied with age and mathematical maturity of the pupils. They included [a] a 0-10 floor number track, foldable to look at smaller ranges such as 0-5, [b] Wittmann's Fields of 5, 10, 20, 50, 100 and 1000, [c] Gattegno boards and Gattegno cards, to be overlapped to compose and decompose numerals, [d] and some *almost* empty number lines.

Interviews lasted approximately 20 minutes per pupil, and were conducted away from the main class from which pupils were drawn. We filled out our notes on a carefully designed recording sheet that worked well, then followed this up shortly afterwards with a fuller transcription of these typed into the same kind of sheet. Once all 48 sheets had been compiled and collated, we analysed them to draw out any major variances with the predicted outcomes, and to synthesise the main 'messages' from our part of the study.

SIGNIFICANT FINDINGS

There were a number of significant factors identified, including the extent of variation between the 3 ability groups, which was greater than teachers predicted.

Numerals

We identified a need to extend pupils' numeral recognition well beyond current levels. Indications were that Gattegno boards and cards could be used effectively to scaffold higher levels of achievement, particularly if used in combination. With older pupils there was a great need to create an effective transition between whole number and decimal recognition, and so enhance recognition of the value of digits within decimal numbers. Errors and misconceptions were seen arising from [a] an

overemphasis on 'teens' within the early number curriculum, [b] place-holding uses of zero, and [c] limited understanding of decimals arising from overemphasis on money contexts as a bridge.

Locating

There was a clear need with younger pupils to enhance achievement in locating numbers and finding one more or less. Scaffolding with images of fields and empty number lines enabled pupils to find *e.g. 10 more and less*, that was not possible unassisted. Where 'more / less' challenges involved crossing round number boundaries, *such as 1000*, pupils were more hesitant and achievement was reduced. Outcomes relating to decimals showed a real need for extending such work, supported by use of Gattegno boards. Empty number lines similarly were indicated as supporting work needed on positive / negative number location.

Counting

One of the strongest findings indicated a need to balance 'counting on' with more 'counting back' where achievement lagged well behind. This seemed to be linked to later findings where subtraction lagged well behind addition. The sustaining and extending of counting in varying steps from a range of starting points was a need indicated, as was the absence of any significant level of achievement in counting within decimals, fractions or positive / negative contexts.

Complements

This tended to be almost a new idea for many pupils. Few had experienced activities where they were expected to *say* give the amount needed to make numbers up to 50 or 100 or [with decimals] to 1. This kind of activity was only familiar to younger pupils in relation to complements of 5, 10 or possibly 20. There was strong evidence in pupils' responses to suggest that more use of Gattegno's boards and cards, Wittmann's fields and Treffers' empty number line would enhance achievement.

Known facts and deriving strategies.

Findings from this focus were more variable and mainly used to lay down a baseline for the pupils involved against which later checks on progress could be made. Some pupils had a wider range of known facts, but these were less available to them as tools in deriving new knowledge, while other pupils made more flexible use of a smaller range of known facts. The latter group tended to be underrated in relation to the former group in teachers' indications of achievement levels.

EXAMPLES OF BENCHMARKS SUBSEQUENTLY AGREED FOR 2000-01

We could have used categories labelled as B, C and D for the three ability groups [where A would be the highest achieving 20%, and E the lowest achieving 20%.] However in practice we chose the headings: Betty, Chris and Dawn, so these are used in the examples presented below: Data is presented as table extracts, with related NNS Key Objectives indicated alongside, for reference.

Numerals Benchmarks Extract

Betty	Chris	Dawn	Year & NNS KO
• Recognises reads and says numerals up to 1000 • Uses Gattegno boards and cards to construct and read/say numerals to 1000 and some up to 9999 • Deals successfully with placeholder zero in numerals to 1000	• Recognises reads and says numerals up to 100 and exact hundreds to 1000 • Uses Gattegno boards and cards to construct and read/say numerals to 1000 and some up to 9999 • Deals successfully with placeholder zero in numerals to 200	• Recognises reads and says numerals up to 100 and exact hundreds to 1000 • Uses Gattegno boards and cards to construct and read/say most numerals to 1000	**YEAR TWO:** Read write… numbers from 0 to at least 100. Know what each digit represents [including 0 as placeholder]

Points to notice here include the increased expectation at year 2 in this aspect. In relation to the NNS Key Objectives, the tidemarking study had shown the pupils to be underachieving, with only 'Betty' meeting the targets. Now, in the second cycle of pyramid benchmarking, all three categories are achieving at least the equivalent level and some are moving considerably beyond this. Another point is the different emphases: e.g. the pyramid benchmarks refer to 'recognise', 'read', and 'say' but not 'write', while the NNS objective does not use 'recognise' or 'say'. We presume that 'read' in the NNS conflates 'read to oneself' and 'say out loud'. Therefore the significant difference here is what lies behind 'recognise' on the one hand and the requirement to 'write' on the other. Pyramid approaches have moved to valuing pupils finding *meaning* in the numerals, and being able to find them within fields, lines, Gattegno boards, and to compose them from Gattegno cards. The ability to write them is less central, even though in practice it has grown concomitantly.

Locating Benchmarks Extract

Betty	Chris	Dawn	Year & NNS KO
• Finds up to 10 more/less & 20,50,100 more/less up to 1000 • Locates numerals to 2000 on a line, square or field • Locates and corrects errors in a part-filled 100-square	• Finds up to 10 more/less & 100 more/less up to 1000 • Locates numerals to 1000 on a line, square or field • Locates and corrects errors in a part-filled 100-square	• Finds up to 10 more/less & 100 more/less up to 500 • Locates numerals to 500 on a line, square or field • Locates and corrects errors in a part-filled 100-	**YEARS 4/5:** Use less than and greater than signs Order a given set of positive and negative integers

for any century to 1000 • Places positive / negative numbers on a no line to ± 100	for any century to 1000 • Places positive / negative numbers on a no line to ± 50	square for 1-100 or 101-200 • Places positive / negative numbers on a no line to ± 10	

It was an early decision to combine benchmarks for pairs of years from Year 4 onwards, due to the wave effect caused by significant changes of approach earlier on. It would have been unrealistic to expect more of the Year 5 pupil in these focus areas than of the Year 4 pupil of the same category, since the Year 5 pupil would have had no longer 'on the programme'. Notice again here the increased expectations in this aspect: the tidemarking study had shown the pupils to be greatly underachieving, but now they were able to achieve much more substantially. Again there are different emphases in the NNS Key Objectives, which refer to the use of the signs < and > and putting integers in order. The pyramid benchmarks reflect pyramid approaches that value pupils locating numbers and moving freely from these on and back, extending this with the support of fields, lines, squares and boards. Incidentally, almost, pupils have acquired knowledge of the relevant signs.

THE STATE OF PLAY

The 3-year project concludes by July 2001. The tidemarking / benchmarking study was only one [significant] part of the project, and we have only touched on that study. How this relates to other factors in the progress effected by the pyramid is intriguing. A crude, but effective, indication of that progress lies in the End of KS2 Test results. A project success criterion was to increase the %age of Level 4's by 4 to 5 % annually. Over the first 2 years, all 3 middle schools' results improved by over 20% and one by over 30%. We hope to find the time to write more fully about all three years of the project and its major features after next July.

REFERENCES

Gattegno, C. (1987,89) *The Science of Education,* (Educational Solutions, Reading)

Treffers, A. (1991) Didactical Background of a Mathematics Program for Primary Education: In L. Streefland (ed.) *Realistic Mathematics Education in Primary School*, (Freudenthal Institute, Utrecht)

Vygotsky, L.S. (1978) *Mind in Society: The Development of Higher Psychological Processes*, ed. M. Cole et al (Cambridge, MA: Harvard University Press)

Wittmann, E.Ch, (1998) Standard Number Representations in the Teaching of Arithmetic, *Journal fuer Mathematik-Didaktik* **19**, No.2/3, 149-178.