THE CHURCHILL CABRI PROJECT: BACKGROUND AND OVERVIEW

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Abstract

This paper provides an overview of a project[1] carried out last year with 5 Further Mathematics A-level students. The project centred around introducing the processes of conjecturing and proving in geometry, within the context of a dynamic geometry environment. The students worked on a sequence of designed activities for seven one and a half hour sessions and then worked independently for several weeks on a challenging project. In the paper we shall present some preliminary results which derive from the students' work in the classroom based sessions with both Cabri and paper and pencil.

Introduction

Many university mathematics students find it difficult to cope with the idea of proof at the beginning of their university course but they are expected to be able to read and construct proofs on their own. Many students may not have had previous experience with this aspect of mathematics in Secondary School. The role which proof should have in the mathematics curriculum is one of the issues discussed in the international research debate amongst mathematicians and mathematics educators around the world. In some countries proof is disappearing from the curriculum; for example in the UK, proof has become inaccessible to the majority, in that it is only at Level 7 or 8 of AT1[2], when students are expected to prove their conjectures in any formal sense (Hoyles, 1997). However a current reconsideration of proof in the curriculum seems to be taking place at some levels.

Historical and epistemological studies (for example, Barbin, 1988; Balacheff, 1998; Rav, 1999) have shown the centrality of the proving process within the discipline of mathematics. Consequently it seems important to develop learning environments aimed at immersing students in a mathematics proving culture, so that they can be supported to appreciate and understand the role of proof in mathematics.

The paper will describe how we planned and implemented a teaching and learning intervention drawing on this background and aimed at introducing students to conjecturing and proving in geometry. Preliminary observations about students' work are drawn and issues for further analysis are identified.

Theoretical ideas

The main ideas which constitute the basis for the project concern proofs in relation to conjectures and with respect to the introduction of a dynamic geometry software.

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[1] This project was carried out by: G. Moënne, C. Mogetta, F. Olivero & R. Sutherland, Graduate School of Education, University of Bristol.

2 AT1 is Attainment Target 1 of the National Curriculum. Pupils aged 11-14 years should be within the Levels 3 to 7.

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There are many different views of proof, depending on different contexts. We share the definition of proof given by (Rav, 1999). Every theorem is a statement B for which there is another statement A such that B is a logical consequence of A (A $\rightarrow$ B). The activity of proving which mathematicians undertake consists of entering into the relationship A $\rightarrow$ B. Instead of talking about proof as a result, we are going to talk about the *proving process*, defined as the process of entering the relationship 'B follows from A' until the agents involved are satisfied with the explanation for the truth of the statement.

Previous studies have shown the importance of the construction of proofs, pointing out not only their formal aspects of established products but especially the fact that, as processes, they are deeply rooted in the activity of producing conjectures as a whole. Students engaged in activities which require exploration of a situation and production of conjectures, are more likely to be able to organise a proof at the end, than if presented with an established statement and asked to prove it. (Boero et al, 1996; Mariotti et al, 1997; Arzarello et al, 1998).

The use of open problems (Arsac et al, 1988) has proved a way to allow this to happen. Open problems are characterised as follows: the statement does not suggest any particular solution method or the solution itself; the questions are expressed in the form “which configuration does…assume when…?”, which differs from traditional closed expressions such as "prove that…", which present students with an already established result. These characteristics can be observed in the following problem, which was used in the project and analysed later in the paper.

The angle bisectors of a quadrilateral

Let ABCD be a quadrilateral. Consider the bisectors of its internal angles and the intersection points H, K, L, M of pairs of consecutive bisectors.

Drag ABCD, considering different configurations.

1. What happens to the quadrilateral HKLM? What kind of figure does it become?
2. Can HKLM become any quadrilateral? Why?
3. Can HKLM become a point? Which hypothesis on ABCD do you need in order to have this situation?

Write down your conjectures and prove them.

If the production of proof is based on the generation of conditional statements, then suitable learning environments need to be developed in order to support students in this process. We identified dynamic geometry as a potential environment.

Description of the project

Drawing on this background, the project was an attempt to introduce A-Level students to processes of proving, exploiting the potentialities of open problems and of a dynamic geometry environment (namely Cabri). The aims of the project were twofold. The learning objectives were the following: conjecturing; validating conjectures; proving validated geometrical properties; moving between conjecturing and proving. The research aims were: to investigate the proving processes of pre-university mathematics students as they engaged with a dynamic geometry proof

microworld and to evaluate the microworld as an effective environment for the learning of proof.
The project was developed in different stages: elaboration of a sequence of Cabri and paper&pencil activities (teacher & researchers working together); implementation of the sequence in the classroom and data collection (teacher teaching in the classroom and researchers collecting data: videos, tapes, field notes, students' work); data analysis (researchers, it is still ongoing).
The project consisted of: 7 classroom-based sessions, aimed at introducing the basic and essential ideas involved in the process of proving statements in mathematics, moving from the elaboration of conjectures to their justification and formalisation; project work, aimed at giving students the possibility of working on their own; 5 video tutorials and a final video presentation, aimed at giving students support in their project work and trying to exploit the affordances of video communication[3].
The classroom-based sessions involved three phases. First, the teacher introduced the problem. Second, students solved the problem in groups with Cabri and paper&pencil, so producing conjectures & proofs. Third, the teacher orchestrated a general discussion, in which students exposed, confronted and discussed their conjectures and proofs, co-ordinated by the teacher. The people involved in the classroom-based sessions were: the 5 students, the teacher in charge of the instruction, 4 researchers acting as participant observers.

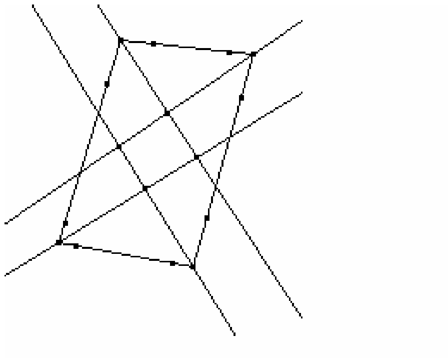
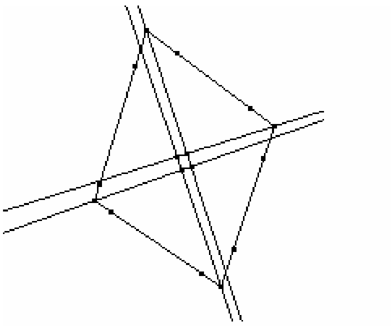
Patrick's proof: a preliminary analysis

This section will outline some issues which emerged from the observations and analysis of one student's protocol. The student was working on "the angle bisectors of a quadrilateral" problem (see previous page) in the second session. The student worked alone[4] and he was observed by one researcher.

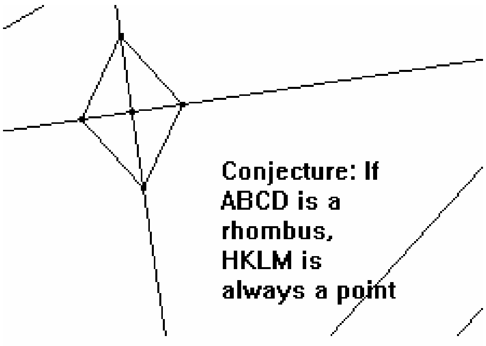
Patrick's work	**Observations & analysis**
EPISODE 1	
Patrick constructs any quadrilateral and angle bisectors in Cabri. He measures the angles. He drags the external quadrilateral until it becomes a rectangle.	At the beginning Patrick chooses to work in Cabri and constructs a figure for the problem. Then he engages in guided dragging, in that he drags the figure in order to obtain a rectangle. Actually the figure he has on the screen was not really a rectangle. Therefore he cannot work out what figure the quadrilateral inside is (he could not see it as a square). Patrick is in control over the Cabri figure, which may be used as a sketch on paper. The strategy of ordered exploration of particular cases is very common in the paper & pencil environment. His starting in Cabri is very similar to paper and pencil processes, in that he does not seem to explore the problem via dragging.

[3] A description and analysis of the video communication is in Moënne et al (2000).

[4] Since there were 5 students we decided to form two pairs and one student had to work alone.

 **Figure 1**	
EPISODE 2	
Patrick sketches on paper.	There is immediately a change in the medium used. Perhaps P, not being that confident with the new tool[5], prefers to work with paper and pencil.
EPISODE 3	
Patrick drags the external quadrilateral on the screen until it is a rhombus.  **Figure 2**	This episode shows the generation of a conjecture, which is based on perception and the use of Cabri.
Patrick “When the exterior is a rhombus or any parallelogram, the inside one has to be a rectangle or another parallelogram”.	He expresses (in spoken language) a first conjecture, based on a perception.
Teacher “When the quadrilateral is a parallelogram, HKLM is a parallelogram. Is that right?	Teacher intervention, making the conjecture more specific.
Patrick “If the exterior quadrilateral is a parallelogram, then the interior one will also be a parallelogram”	A spoken refinement of the first conjecture.
Patrick measures the interior angles of the interior parallelogram and finds they are all 90degrees.	Use of Cabri for testing the conjecture. He uses measures but not dragging.
Patrick “If exterior is a parallelogram then internal is a rectangle”.	Another refinement of the conjecture, that is based on a perception again.
EPISODE 4	
Patrick writes on paper “The bisectors of adjacent angles are always perpendicular in a parallelogram?”	Conjecture refined, generalised ("always") and written on paper.
EPISODE 5	

[5] This is only the second time he has used Cabri.

Patrick "If those two bisectors are always… can prove all angles are 90 degrees…so can prove a rectangle and a square is a type of rectangle. Patrick "Rhombus is a parallelogram with all sides equal, so if rhombus then inside will be a square".	The conjecture about the rhombus draws on the previous conjecture and on the theory. The reasoning is: since the rhombus is a parallelogram with equal sides (theory) and for a parallelogram you get a rectangle inside (previous conjecture) then it must be that for a rhombus outside you get a rectangle with all four sides equal (a square) inside.
EPISODE 6	
He makes the Cabri figure look like a rhombus measuring the sides. Patrick "Maybe when a rhombus always a point". Teacher "So when is it a square?" Patrick "So I don't know if it can be a square".	He checks this in Cabri and sees that for a rhombus the interior quadrilateral becomes a point, not validating his previous conjecture. He formulates a new general conjecture ("always") Then he formulates a conjecture about impossibility of internal quadrilateral being a square.
EPISODE 7	
In Cabri Patrick starts to construct a rhombus to test his conjecture but is unsure about how to do this.  **Figure 3** Patrick "They do definitely all cross…I'm just going to work out why" (**Figure 3**).	The fact that Cabri and the theory provide two different results is a motivation to search for an explanation ("why"), i.e. to construct a proof. First of all Patrick wants to be sure of what he saw in Cabri for only one figure, so he constructs a 'real' rhombus, i.e. using the geometrical construction in Cabri. Then he writes down the conjecture in a general form ("if ABCD is a rhombus, HKLM is always a point"), being convinced that this is always true. Finally he wants to understand why this happens. So once he is sure *that* the conjecture is true, he searches for an explanation *why* it is true.
EPISODE 8	
Patrick "I can see why it is" Teacher "So when you say you can see why it is…". Patrick "It's just symmetrical…angle bisectors of opposite angles are the same lines". Teacher "Go back to parallelogram idea…we've got a rectangle because the distance between parallel lines….	The perception of the symmetry of the rhombus is very strong and allows Patrick to see a proof (can we talk about visual proof?).
EPISODE 9	
Patrick "I've a feeling that it is impossible to be a square". Patrick "I think that with a parallelogram the closer you get to a rhombus the closer you get to a square…but get to a point…I suppose you could call a point an infinitely small square."	After seeing why a rhombus outside gives a point inside, Patrick wants to explain the impossibility of getting a square inside. His reasoning is 'transformational' and dynamic and involves theoretical considerations (a point seen as infinitely small square).

Teacher "I'm willing to be convinced, but I'm not sure I agree".	The teacher provokes reflection on 'impossibility of square' conjecture.
Patrick works on paper again and produces a written proof (**Figure 4**).	Patrick goes back to proving the conjecture which he made at the end of episode 3.

Some Concluding Remarks

In this paper we have presented an overview of the Churchill Cabri project and a beginning analysis of one student's proving processes. The student moved between exploration in Cabri, spoken conjectures, written conjectures and written proofs. The written proof for this episode seems to reflect the process of proving and emerged as a part of the whole process. Further work on the project will be presented at future BSRLM conferences.

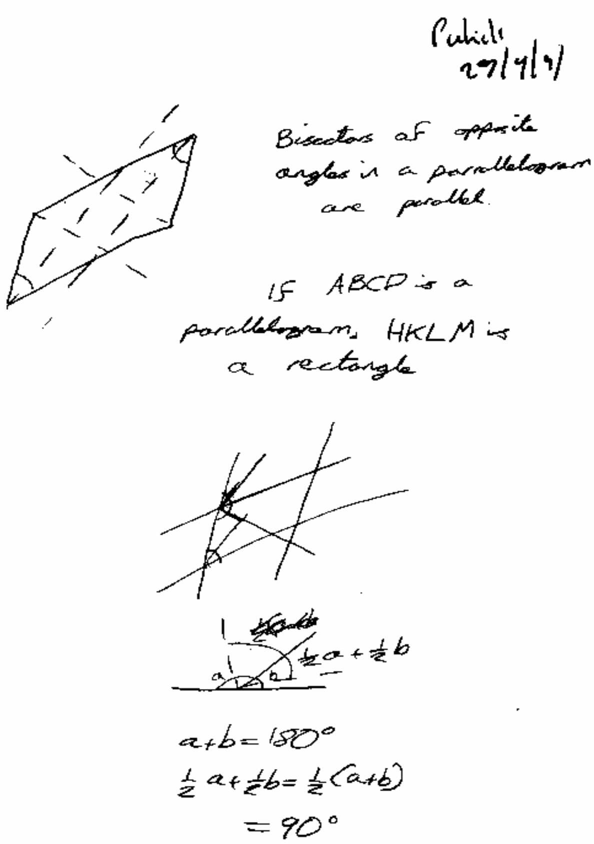


Figure 4

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