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PUPILS DOING ALGEBRA: INTERVIEWS WITH YEAR 7 PUPILS IN AN ESRC PROJECT

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Abstract: *In this ESRC small grant project*[1] *four teachers are working with their year 7 classes to develop algebraic activity. As one of the data collection activities I have interviewed 6 pupils in 3 of the classes (and, now, also the fourth class) and, in this session, presented data from these interviews for discussion of the pupils' approaches to algebra. A mathematical activity was presented to the pupils and their work on it discussed with them. The same pupils were then re-interviewed about two months later and the same activity offered to them so that their developing strategies could be considered. This pattern will be repeated each term during the year. Participants in the session were asked to consider what evidence of algebraic activity they could see in these interviews and samples of pupils' work.*

1. An outline of the project

This project has been reported on in a previous session at BSRLM (Laurinda Brown and Alf Coles, November 1999, Warwick) so I will only give a brief outline here. The project consists of 3 researchers (Laurinda Brown, Ros Sutherland, Jan Winter), one teacher/researcher (Alf Coles) and three teachers working together on the classroom practice in a year 7 class of each of the teachers. Each teacher's classroom is observed approximately once per fortnight and, additionally, videoed once each half term. Six pupils from each class are interviewed by a researcher once each half term. Work from these pupils is collected. Each teacher is interviewed once each half term. All of the data collected in these ways is shared with all the teachers and researchers. As a group of seven collaborators we form a 'community of inquirers'. We meet for a day each half term to share planning of activities and talk about classroom experiences, often watching parts of videoed lessons and working on what we see. In addition the four researchers meet on two additional occasions around each day meeting - once before and once afterwards. These meetings allow us to spend time looking at parts of the videoed lessons in great detail. Each of the researchers has a main interest in one strand of analysis and shared video viewing allows us to gain other researchers' perspectives on our own issue as well as experience the video in ways which may not have immediately been apparent to each of us. Our strands of analysis are Laurinda - metacommenting, Ros - algebraic activity, Jan - pupil perspectives and Alf - teaching strategies. This presentation therefore focused on pupil perspectives and a consideration of what we could learn about these from use of transcripts of taped

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interviews. For a more detailed consideration of the underlying methodologies employed, see Brown & Coles, 1999.

2. What is algebra?

> 'Can we develop a school algebra culture in which pupils find a need for algebraic symbolism to express and explore their mathematical ideas?' (Sutherland, 1991)

This quote symbolises what we felt was important in making an algebraic approach to mathematics successful for pupils - that without a **need** algebra was likely to hold little meaning for pupils and that this was likely to be generated through the development of a **culture** which promoted this approach.

We used a way of thinking about algebra developed by Ros Sutherland and others on the Royal Society / JMC Working Group on the Teaching and Learning of Algebra pre-19 (Sutherland 1997) which was adapted from the work of Kieran (1996). This identifies three components of 'algebraic activity': generational activities, transformational activities and global, meta-level activities. Generational activities involve pupil making generalisations and generating symbolic expressions. Transformational activities involve manipulating and simplifying expressions, solving equations and shifting between different representations. Global, meta-level activities involve awareness of mathematical structure and constraints, anticipation, problem solving, explaining and justifying. We have found that this last part of the definition of algebraic activity is present in many of the activities used by our teachers, and in the strategies they encourage pupils to use in thinking about their mathematics. The pupil interviews are intended to help provide insight into these strategies and approaches.

3. Pupil interviews

The half-termly interviews with pupils from the four classes have consisted of two components - some general questions about their experiences of mathematics, both now and in their Primary Schools and a mathematical activity to engage in with the interviewer. Pupils are interviewed in pairs, chosen by their teacher to represent a pair of 'higher achievers' a pair of 'middle achievers' and a pair of 'lower achievers'. These pairs were chosen early in the Autumn term when their teacher had relatively little knowledge of them and so have sometimes not proved to be quite the balance intended. In a couple of cases, a replacement has been added when a pupil has left the school.

In the first term, the following activity was presented to pupils (taken from Median materials on algebra (see reference)):

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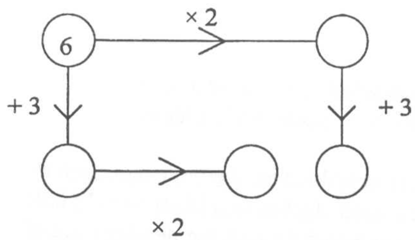

A number, say 6, is put into the top left circle and pupils asked to say what numbers they would get in the two bottom left circles. Will they be the same or different? What can they find out about the situation? Why do they get these results?

In the second term the activities were based around true/false statements:
At the first interview the interviewer stated that if you add together three consecutive numbers (this term was explained where necessary!) and then divide the answer by three, the final answer will be a whole number. Pupils were asked whether this was true or not and to explain their answer.
At the second interview the statement was that if you add two even numbers together, the answer will be even. Again, pupils were asked if this is true or false and how they would explain their answer.

Pupils are asked to work on the problems, collaborating with their partner if they choose to, and the interviewer talks about their work with them and draws out their thinking, encouraging them to try to express themselves either verbally or on paper. Some examples follow of pupils' work and the associated interview transcript. These examples are taken from the first three interviews (i.e. they do not include samples of work on 'even + even' task). I is the interviewer.

Excerpt from Susie:
I Now what I'm saying is that if I take three consecutive numbers and add them up, and when I've added them up, divide the answer by three then the final answer's going to be a whole number. Is that true or not?
S Yeah.
I It is? Okay, do you want to tell me why?
S Because, say the first number's 1, then you've got 2 and 3 and 2 is 1 more than 1 and 3 is 2 more than 1 so you've got three spare ones then. So you just add it.
I You've got three spare ones? What do you mean spare ones?
S Well, you've got the 1 from the 2 and the 2 from the 3 and then divide those onto the other ones and you've got 2.
I Can you try to write that down do you think? Can you write down what you mean? Say you put down 1, 2, 3 how are you dividing them up then? *(Susie writes*

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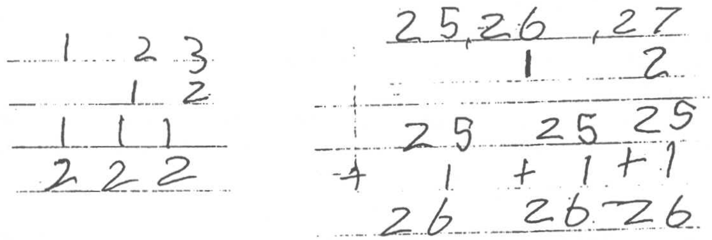
down her method) So you're saying that there's one spare one there and 2 spare ones there...
S So you've got 1, 1, 1 and then you just put those onto there to make 2.
I Okay, that's an interesting way of doing it. Could you see how that theory would work for a bigger number, other numbers. Suppose you started with say 25, 26, 27, could your theory still apply to something like that?
S Yes, cos it's still one more than 25 and 27's still 2 more.
I Okay, so you show me with your way of working it out what the answer's going to be.



Susie (a middle attainer) was able to access the structure of the numbers in order to generate a method which she could readily generalise . She went on to explain this method to the other pupil in the pair. She could also see how to extend her method to the case of four consecutive numbers divided by four. In this case she would have 6 'spare' which would mean adding 1.5 on to the first number.

Excerpt from Sally and David:
I Can you think of any way that we could convince other people that it was going to happen always. You described it quite nicely David, why you were convinced about the difference of three. How might we do something to convince someone else that it was always going to work? Prove it if you like.
S We could use algebra, instead of add 2, put in a letter.
S You could put like 2 times 2...leave a gap there and put times 2, leave a gap there and put add n and then put it there and then the difference would be n.
I Do you want to try it? Do you understand what she means David?
D Not really!
I Why don't you show us Sally. So we'll put a number in to see what happens?
I So you're expecting the difference is going to be n. What number are you going to put in to start off with?
S I'll do 9. How do we know what n is?
I Well, we don't do we! So you've both got the same problem, haven't you? What might you write in that last circle down in the corner?
S 18 add n
S 9 add n

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I That's right. So you can write that at the bottom there. But now we've got the problem of doubling it.

S Could you just put 9 add n times 2?

I You could, yes. You could put it in brackets - do you know about brackets? It would be more helpful if we could compare it to the other answer that we've got. By doubling each bit - can you double each part.

S It would be 18 plus 2n.

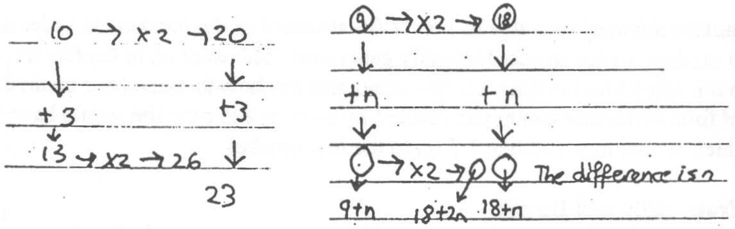
I Well, is that what you were expecting? If you look at your other answer. What you said was that you thought one would be n more than the other.

S But we don't know what n is.

I Write down what you both just said cos then it'll help if we can look at it. Now what we've got to do somehow is compare those two things. 24 add n and 24 add 2n. Is one of them n more than the other?

S Yes, it is because that's one n and that's two n.

(This excerpt is edited to show only Sally's contributions, so her thinking is clearer)



Sally, with support, was able to use algebra to show the general case. This interview took place early in the Autumn term.

Excerpt from Ryan:

(Considering the case of four consecutive numbers)

R Yeah, it would be in the middle of all those numbers. So say it was 1, 2, 3, 4 the middle of those numbers, between 2 and 3, that is what it would become, like 2.5. So it's still the same but it's actually in the middle between the 2 and 3. Second and third number.

I Because there isn't actually a middle number? Could you prove that to me? Could you find a way of writing that down to prove that it was always going to work? *(Pause)*

R Umm...

I Have you used any ways in your maths lessons of writing things down in a general way so you can show it's always true?

R We've used x as any number.

I Right, would that help you in writing this problem down?

R Are we still looking at the four numbers?

I I don't mind, we could look at three first if you like.

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R So that would be easier because you might have x, and x plus, no take away, 1 and then x plus 1 there.

I So that's the one that's one less and that's the one that's one more? And what happens when you add all those three together?

R That one would actually get converted over to there when you add them so it would be three equal xs. So then it would equal x.

$$x+1 \quad x \quad x+1$$

$$x-1\tfrac{1}{2} \quad x-\tfrac{1}{2} \;\big|\; x+\tfrac{1}{2} \quad x+1\tfrac{1}{2}$$

Ryan (a high attainer), whose 'pair' was absent from the interview, was able to use algebraic notation in quite a sophisticated way to describe and explain the situation. As can be seen from his work, he was able to go on to show the situation with four numbers, where the answer was *x* and the four starting numbers were *x-1½, x-½, x+½ and x+1½*. He recognised that x in fact was a mixed number and the others all whole numbers! There is not room to include the transcript here.

4. End note

These excerpts are illustrative of some aspects of pupils' approaches. The analysis of this data will continue throughout the year, through the frame of algebra described earlier. Participants in the session discussed these transcripts, and others, and offered valuable suggestions both about interpretation of the results and about refinements to make the interview process more accessible to others and the data richer.

References

Brown, L. and Coles, A. 1999 'The anatomy of a bid: From TTA to ESRC - looking at the developing algebraic activity in four Year 7 classrooms' *Proceedings of the British Society for Research into Learning Mathematics,* University of Warwick, 12 & 13 November 1999, pp.49-54

Kieran, C. 1996 *The changing face of school algebra,* invited lecture presented at the Eighth International Congress on Mathematics Education, Seville, Spain.

Median algebra materials c/o MEDIAN, Harlescott School, Shrewsbury, SY1 4LL

Sutherland, R. 1991 'Some unanswered research questions on the teaching and learning of algebra' *For the Learning of Mathematics* 11-3, pp.40-46

Sutherland, R. 1997 *Teaching and Learning Algebra pre-19,* London: RS/JMC

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[i] 'Developing algebraic activity in a 'community of inquirers'', Economic and Social Research Council (ESRC) project reference R000223044, Laurinda Brown, Ros Sutherland, Jan Winter, Alf Coles. Contact: Laurinda.Brown@bristol.ac.uk or Laurinda Brown, University of Bristol, Graduate School of Education, 35 Berkeley Square, Bristol, BS8 1JA, UK.

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