Primary Children's Understanding of Probability

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The National Curriculum for England and Wales introduced in 1989 brought probability into the mainstream primary curriculum for the first time, but just over ten years later in the curriculum review 2000 it has to a large extent been taken out again. This paper examines some evidence from the analysis of children's performance in national assessments to try to decide what children have learned from extensive teaching of probability in primary schools in the intervening years, but concludes that the assessment of probability in the primary school is unavoidably ambiguous.

Until the 1980's it seems to have been generally accepted that probability was too difficult for most primary children. For example probability is not listed among the "main objectives for the majority of children at the age of 11 years" in the HMI (1979) document Mathematics 5-11 (page 77). With the introduction of the National Curriculum in England and Wales in 1989, however, the primary curriculum expanded to include much more probability, suggesting that pupils should throughout primary school be able to understand, estimate and calculate probabilities. As a specific example, the probability "attainment targets" at Level 3, intended as appropriate for the 'average' 9-year-old, were that they be able to:

- "place events in order of 'likelihood' and use appropriate words to identify the chance";
- "understand and use the idea of 'evens' and say whether events are more or less likely than this";
- "distinguish between 'fair' and 'unfair'" (DES 1989, page 43)

In the subsequent ten years, despite extensive work on probability in the primary school, there has been a progressive scaling back of ambitions, and the English curriculum from 2000 has contracted to such an extent that the only expectations for performance are set at Level 5 and above.

In this paper I shall examine the apparent understanding of probability manifested by primary aged children who have been taught within the context of the English National Curriculum between 1989 and 1999, where it might be supposed that they have had extensive probability related experience in school, to see whether the decision to cut back the expectations is justified. This will be done partially through a 'performance analysis' of the outcomes of some of the probability assessment items used in national assessments during the later part of the period.

Understanding probability

The National Curriculum itself has not specified what constitutes an appropriate understanding of probability for primary children, just that it "develop …. through experience as well as experiment and theory" and that it involve the use of "a vocabulary that includes the words 'evens', 'fair', 'unfair', 'certain', 'likely', 'probably' and 'equally likely'." (DFE 1995 page 10). Given that this is said in the context of a mathematics curriculum, however, one must suppose that what is intended is not merely a casual discussion of 'chance' that uses these words, but something that is rooted in mathematics.

The mathematics of probability quantifies the structures and relationships of uncertainty, offering an account of events within chance governed situations. However, mathematical probability is hypothetical, rather than about actual events. It does not try to predict what will occur, but offers the means to obtain a proportional expectation of each of the alternative possibilities. This can then be used to inform predictions about actual events, but that step is not part of the mathematics. The relative likelihood of each possibility in a situation may be called the 'collective distribution', and according to Piaget and Inhelder (1975) "the discovery of the collective distribution is psychologically the true and single basis for probability" (page 234). The ability to understand probability that is presumed in the Key Stage 2 curriculum from 1989 to 2000 can be thought of in these terms. If children have learned to think of likelihood as distributed among possibilities, the scaling back of probability work was probably not justified. On the other hand, if they have not, then the decision was probably a reasonable one.

One indicator of inadequate thinking about probability may be the preponderance of what Shaughnessy (1992) calls 'naive' mistakes. For example, a number of children seem to have a distorted view of "more likely" based on "biggest category". They seem to estimate probabilities on the basis of cardinality, regardless of the population proportion. Another is an expectation that likelihood is affected by recent events - for example that after a sequence of heads when tossing a coin, a tail is then more likely. Fischbein and Gazit (1984) among others have referred to this as the 'negative recency effect' or 'gambler's fallacy'. It seems to take too far the thought that a 'balance' among the options is to be expected, and is an example of what has been called the "representativeness heuristic" (Amir and Williams 1994; Nickson 2000; Shaughnessy 1992). Piaget and Inhelder (1975) see such an "each gets their turn" perception as an example of children's typical interpretation of 'chance' situations as "an apparent disorder veiling an underlying order" (page 215).

A different kind of evidence of limited understanding of probability is when a non-mathematical option is adopted, for example making a judgement about likelihood from an analysis of causal relationships in the situation, howsoever this is understood (Konold 1991; Amir and Williams 1994).

Evidence about understanding

To what extent do children display the required understanding as opposed to the described misunderstandings? To pursue this in practice, evidence about understanding was sought by considering children's actual responses to probability assessment questions that required children to explain something about their thinking, which can be taken to be the most revealing of available information.

On Question 21 of the 1999 Key Stage 2 mathematics test A, children were asked to give a reason for saying which number on a 12-section circular spinner the pointer is "most likely to stop on". The test responses of 335 children were examined for clues to their understanding. In terms of the kind of thinking that might be called a misunderstanding, there was some focus on the mechanics of spinning, seeing the picture as a moment in time in a particular imagined event.

- "It is nearer to the 5 so I think it is more likely to stop on 5."
- "There is a quarter of a space and if it starts to slow down it won't move to another number."
- "When it has had its full spin it goes back to the middle then drops a little."

However only 6 children in the sample (2%) answered in this way. The vast majority of the reasons given by the children for their answers were in one way or another to do with the amount of the circle occupied by the number chosen (with likelihood remaining implicit), such as:

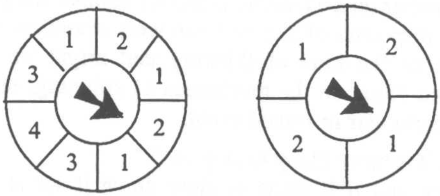
- "One is the biggest section on the spinner."
- "Number 1 takes up 3 sections but there are two 3s and if you add them up they take up 4 sections."
- "Three takes up more of the circle."
- "One has a large area of the circle and the others cover small areas."
- "I measured the angles with a protractor and the number threes have the largest angle between them."
- The number 1 is 90 degrees."
- "4/12 of the circle is 3"

Such reasons were given for both the 'correct' answer "3" and other answers (most commonly "1"). Is there evidence here of the understanding of probability, or the lack of it? Certainly giving the correct answer is not a strong indicator of understanding, as the reasoning behind the wrong answers seems to be largely the same as that behind the correct answers.

One could suppose that a child with a mathematical approach would see the probability as a function of a division of the spinner into equal, and therefore equally likely sections, which operate additively, regardless of the behaviour of a spinner on any particular spin - but what would evidence of that actually be? Mostly the children say only that the number they have chosen is the biggest section. Is that enough? A generous interpretation is that children do not need to be completely explicit, and that a limited account is sufficient to show that the children are thinking about the situation in the right way. If so, however, then the children who were wrong were also thinking about probability in the right way, and should have been rewarded (a case of false negatives in the assessment). If not, then given they offered no other evidence, the children were getting rewarded for too limited thinking (a case of false positives in the assessment).

Similar problems apply to other examples, such as Question 17(b) on 1998 test A.

Lee has two spinners.



A B

On which spinner is he more likely to get a 1? Give a reason for your answer.

Here the reasons for the incorrect answer are commonly in terms of the number of the numbers on each spinner, an example of the 'naïve' mistake of presuming that the biggest category is more likely, e.g.

- "Because there are more ones on spinner A than on B"

Some of the reasons for "B" are also in terms of the number of numbers, but in this case with an implicit proportionality:

- "Because there are less [i.e. fewer] numbers on B"

There are also reasons for "B" in terms of the amount of space taken up by the segments of the spinners:

- "The spaces on B are bigger."

and comparisons on the basis of numerical proportions:

- "Because 2/4 on the B spinner is equal to 4/8 of spinner A and on spinner A only 3/8 of the spinner is 1."

Which of these kinds of responses reveal some consideration of the collective distribution? If the comparison of proportions seems the most likely candidate, does that mean that the children who do not think proportionally do not understand probability? Is it enough to say that again just that the size of the section is bigger? Piaget and Inhelder (1975) caution about "the illusion of the implicit which consists of attributing the most complex notions to elementary levels as if they were already contained in the initial intuitions" (page 217) and this seems to be a risk here.

As a result this item, like the previous one and all the other test items on probability used in the Key Stage 2 assessments from 1995-1999, offers very limited evidence about children's understanding of probability.

Assessment quality

Of course it could well be argued that these are just poor assessment questions, and better questions would not share these difficulties. Better questions are not that easily found, however. Given that this is for primary children, the probability situations cannot be complex, as this would involve other high level mathematical demands, such as combinatorics, proportional thinking, or fractional calculation. Hawkins and Kapadia (1984) argue that many of the difficulties children have with probability questions reflect difficulties in processing proportionality, rather than issues in understanding probability as such.

Yet simple questions always contain the risk that the correct answer may be arrived at by a more limited form of thinking about likelihood than that being assessed. A simple situation can usually be analysed descriptively, or anticipated as a particular event, rather than requiring the application of the 'collective distribution', and if the children get the right answer by these means there can be false positives in the assessment. Furthermore, simple probability questions, since they are set in a context, can all too readily tempt children to think about them in non-mathematical terms. For example in a question from the Year 4 optional test, children are asked to identify which kind of marble is most likely to be pulled out of a pictured bag, and to explain their choice. In the sample of responses examined for this item (n = 332) most children gave reasons that were reflections on the mechanics of choosing, such as:

- "The grey one is at the top"
- "He might go to the bottom of the bag"
- "If you shake the bag, the one below the top will be first"
- "If you put your hand in you would feel one and leave it and pull another one out."

It is possible that many of these children were capable of analysing the situation as a collective distribution, but the immediacy of the context meant that they supposed a non-mathematical approach to be perfectly appropriate - giving in this case false negatives in the assessment. Cooper and Dunne (2000) argue that despite teachers' best efforts to train children to respond to mathematics items in mathematical ways, many children at Key Stage 2 persist in responding 'realistically' - using their everyday experience - to questions set in a context, and as a result:

> "responses often failed to indicate their actual capacity to carry out the 'mathematical' operation demanded by the item" (page 200)

Unfortunately, however, it is hard to imagine how a probability question could be asked at primary level without involving the description of some specific situation to which a consideration of likelihood is to be applied. The degree of formality that avoids that ("In an experiment the probability of outcome A is .....") would not be understood. The risk of 'realistic' interpretations is therefore unavoidable.

Was the decision to reduce probability in the curriculum justified?

The available evidence does not show that children do not understand probability, it shows how difficult it is to say whether children do understand probability. It is revealing the limitations of assessment of primary children in this area.

Perhaps that was the reasoning behind the reduction of probability in the primary curriculum for 2000. Demonstrable evidence of achievement is too elusive. This of course begs the question of whether its reduction in the curriculum will prevent the experiences that are necessary foundations for later development - and risk leading to a reduced performance among older children.

Despite that, one can see why the decision may have been made to prioritise on areas where there is less doubt about benefits, those where any achievement can be identified and celebrated.

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