Jaworski, B. (Ed.) Proceedings of the British Society for Research into Learning Mathematics 20(1&2) February/May 2000

# STUDENTS' CONCEPT IMAGES FOR PERIOD DOUBLINGS USING COMPUTER EXPERIMENTS IN CHAOS THEORY

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*This research was conducted using computers and oscillators at the University of Warwick, UK. Several kinds of concept images are found, including those related to: 1. supervision, 2. an experiment using computers, 3. an experiment using oscillators, 4. past learning.*

*Empirical evidence is presented to support the hypothesis that graphic representations play an important role in conceptualising the notion of period doubling in chaos theory. That is, graphic representations generated by computers and oscillators are not only visual but also conceptual.*

## 1. Introduction

According to Tall and Vinner (1981), a concept image is the total cognitive structure connected in our mind with the concept, i.e. all the mental pictures and associated properties and processes. In particular, the authors introduced the term 'evoked concept image' to denote the part of the memory evoked by a given context. Later, Vinner (1991) suggested that a concept image is something non-verbal associated with the concept name by means of visual representations, mental pictures and a set of impressions or expressions which can subsequently be transformed into something verbal. He also pointed out the crucial role of definitions in technical contexts with respect to shaping the concept image. Based on this view, we have investigated students' concept image of period doubling in chaos theory via post-tests after experiments using visualisation generated by computers and oscillators, using a metholodogy described in the following section. During the experiment, supervisor helped students to understand the phenomenon of period doubling using the bifurcation tree, i.e. Feigenbaum diagram. The results suggest that graphic representations play an important role in forming a concept image.

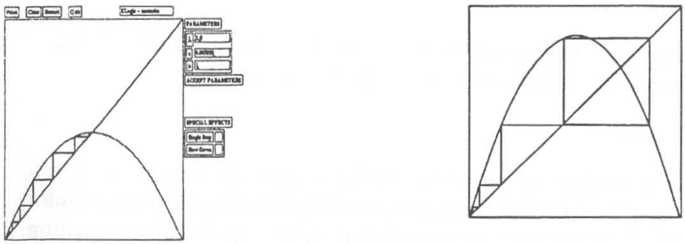
## 2. Methodology

### 2.1 Subjects

The subjects were students enrolled on the Experimental Mathematics course at the University of Warwick in 1999. Most were the first-year mathematics students.

### 2.2 Software

The software used was an updated version of *xlogis* which was written in C and run under Xwindows, Graphic User Interface at Sun terminals operating Sun Solaris. The software *xlogis* is designed to enable students to control either several steps or the iteration as a whole by inputting the number of iterations. What is more, students can observe the remaining part of iteration by removing the previous part of the iteration. This is intended to empower students to develop conceptual knowledge of period doubling in chaos theory. The students were given tasks to experiment with the logistic map $f(x) = \_x(1-x)$ (see Figure 1)



(a) convergence to one root of $f(x) = x$ (b) period doubling

Figure 1. Graphic representations using *xlogis*

### 2.3 The experiment using computers and oscillators

Students explored the phenomenon of period doublings through the computer screen and oscilloscope, using instructions written on hand-outs which were distributed before the class. These described the theory of period doubling bifurcations and the tasks to be performed, and so students could observe the phenomenon of period doubling through the computer screen by inputting parameters, $\lambda$ in $f(x) = \lambda x(1 - x)$ and a starting point. Through the oscilloscope, students can look at a projection onto the X-Y plane of a limit cycle. Here, X and Y are two voltages at two different positions in the circuit which produces a closed curve on the oscilloscope by oscillating periodically in time. During the experiment, the supervisor led the students to pursue the problems by giving directing questions in response to student-initiated questions. The computer experiment was followed by an oscillator experiment in which students could observe the phenomenon matching the phenomenon of Period Doubling by counting the number of intersection points on either the negative x-axis or positive x-axis which was drawn by crossing the limit cycle and called as cross-section. Poincare return map is called by defining a map from this cross-section to itself.

**2.4 Post-test**

After the experiment, we distributed post-test to students, asking them to answer the questions and return the tests within three days. The test comprised several questions designed to reveal the students' concept definitions and concept image. Examples relating to students' concept image for period doublings are as follows:

What first comes into your mind when you think about 'period doubling'?

Please draw/make an example of a period doubling by using your mind's eye.

**2.5 Semi-structured interviews**

These were held several days after experiments in general, to identify the reasoning behind students' responses. Five students were voluntarily interviewed.

**3. Results and Discussion**

As we can see from the table below, most students' concept image for period doubling is based on graphic representations generated by the computer or oscillator. Among 18 respondents, only one have an image based on the supervisor's instruction, using the bifurcation tree,i.e. Feigenbaum diagram. If we analyze the image from the computers, the most interesting fact is that students are more likely to regard a period doubling as the second stage of period doubling, that is, period doubling with four orbits rather than the first stage of period doubling, that is, period doubling with two orbits outformed as a rectangle. By contrast, all but one of their images from oscillators are correctly formed by virtue of the Poincare return map metaphor, scaffolding metaphor for the concept of period doubling. The student having an image of a circle might be a long way from Poincare return map metaphor. Instead, he uses his memory of a circle which was presented as the initial picture occurred on the oscilloscope without understanding the meaning of Poincare map.

Only one student has an image in terms of a trigonometric function, the sine function, based on his past experience, that is, his learning from the past. He interpretes period doubling as a doubling of period and illustrates this by means of a sine function having a function which occurs for twice as long.

This result shows that they have conjured up concept images via graphic representations generated by computers to construct the concept of period doubling in chaos theory.

### 3.1. Concept image related to the supervision

Only one among eighteen students drew the Feigenbaum Diagram, which was described by the supervisor following the context in the text written by Robert Devaney during the experiment in order to help them understand the concept of period doubling, that is, bifurcation. In fact, this student was the best in our class and demonstrated what we explained rather than the graphic representation generated by the software during the experiment.

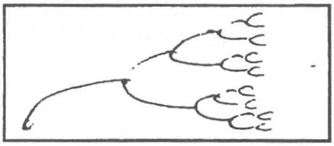


Figure 1. Concept image related to supervision using the Feigenbaum diagram

### 3.2. Concept image related to the experiment using an oscillator

To improve students' understanding of the concept of period doubling, we used electronic equipment, i.e., an oscillator which demonstrated the connected circular in order to define the Poincare return map. Six among 18 students demonstrated concept images related to the graphic representation using the oscillator. This kind of metaphor was clearly vivid to them. Five out of six demonstrated a corresponding alternating picture of the period doubling of orbit two. Only one among six students demonstrated a single circle which was an initial status before bifurcation.

Figure 2. Concept image related to an experiment using an oscillator

### 3.3. Concept image related to the computer experiment using graphic software

Ten (56%) out of 18 students demonstrated the picture generated by the graphic software which was developed by the Department of Mathematics, University of Warwick. Seven out of 10 students illustrated the graphics (picture) of the period doubling of orbit 4, that is, the orbit generated by the second bifurcation . This implies that students seem to be struck by the picture of the period doubling of orbit four, which is generated when the λ-value is $1+\sqrt{6}$ for the logistic functon $f(x) = \lambda x (1-x)$.

One of the reasons for this is that students usually consider the period as a result of two steps such that $f(x_1)= x_2$, $f(x_2)= x_1$. So, they demonstrate the graphic representation of the second bifurcation, i.e., the period doubling of orbit four, as in the following picture.

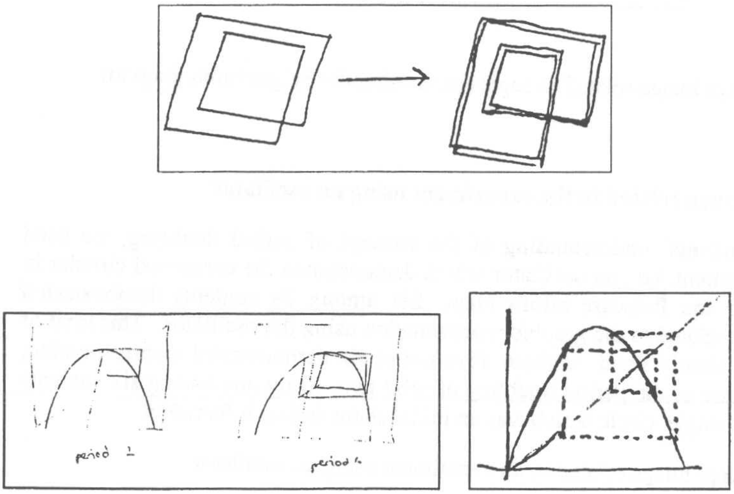


Figure 3. Concept image related to an experiment using computers

### 3.4. Concept image related to past learning

Generally, lecturers use the word 'period' frequently during trigonometry classes and represent it using the sine or cosine function. As a result of past learning, one student demonstrated the period twice in the same interval, which implies that he is confused the concept of period doubling with the meaning of period. In this case his concept image is quite primitive, and not as refined as other images.

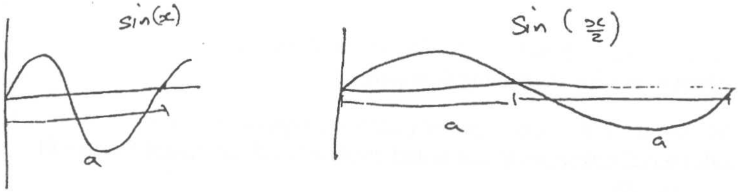



Figure 4. Concept image related to past learning

### 3.5 The distribution of students

| Type of concept images related to | | Number of students (%) | |
|---|---|---|---|
| Supervision (Feigenbaum Diagram) | | 1 | |
| Computers | the period doubling of orbit four | 10 | 7 |
| | the period doubling of orbit two | | 3 |
| Oscillators | alternating picture of the period doubling of orbit two | 6 | 5 |
| | circle | | 1 |
| Past learning | | 1 | |

Table 1. Students' concept images for period doubling

### 4. Conclusion

The data above show that 16 out of 18 students constructed a concept image from graphic representations generated by computers or oscillators. This finding supports the hypothesis that graphic representations play an important role in forming a concept image. Furthermore, it tells us that graphic representations generated by computers or oscillators are conceptual as well as visual. Therefore, we can enhance students' conceptual understanding through computer-aided learning.

## References

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