RESEARCHING RESOURCES FOR TEACHING AND LEARNING: THE COUNTING STICK

Christine Bold
EdD student, Open University

ABSTRACT

The purpose is to present a short anecdotal account of the authors experience using a resource recommended by the National Numeracy Strategy (NNS). The aim is to encourage research into the effects of particular resources on children's learning of mathematics. In primary classrooms, resources usually provide practical experience, along with language, that develops a concrete understanding in the early stages. Over time, through a range of activities children develop a more abstract understanding. The difficulty for the teacher lies in knowing the perceptions children have of practical situations, the mental images they might form and how these become translated into abstract mathematical concepts.

THE ROLE OF IMAGERY IN MATHEMATICAL LEARNING

One might observe children interpreting practical situations in a different way than intended by the teacher in almost any classroom. For example, when building a cuboid with construction materials a teacher spoke about the sides 'growing up' from the rectangular base. One six-year old child's interpretation was that 'cuboid' is a grown-up word for 'rectangle'. In this particular case, the teacher deliberately set out to provide a relevant practical experience in order to develop the children's understanding of the mathematical concept and the associated language. Unfortunately, one child developed a misconception.

The power of the visual image is something that teachers tend to exploit, particularly with young children, but it seems that there need to be caution, when such images might not transfer easily into abstract situations. For example, teachers often use the region part-whole model to represent fractional parts. Children are more likely to recognise and be able to work mathematically with such visual representations than they are with symbolic forms (Dickson, et.al. 1993 reprint). Some researchers claim that particular representations of mathematical ideas obscure the development of other ideas (Ball, 1993; D'Ambrosio and Mewborn, 1994, Nunes and Bryant, 1996). For example, Ball suggests a number line might not necessarily be a good model for understanding that –5 is less than 5. He supports the idea that teachers should find ways of to help children develop their own models and representations of phenomena. However, others suggest that there is a place for specific models, for example in teaching fractions (English and Halford, 1995). English and Halford suggest that the region part-whole area model for teaching fractions is an important step enabling the child to 'map' these pictorial images onto new symbolic and linguistic representations. In contrast, the 'set' model requires children to visualise the set as a whole entity, even if the items in the set are different sizes. This can be potentially confusing for children. In my experience, it is useful to introduce both models as soon as children can work with the numbers involved. Year 2 children age 6-7 can calculate halves and quarters of small sets. In general, teachers use sets that are comprised of the same objects e.g. a set of counters. One might argue that a limited representation of a set is in use in such a situation.

Choosing a particular representation of an idea and presenting this as a single model, might eventually limit opportunity for the child to make sense of another situation. It is important that the mathematical knowledge gained is not context-bound i.e. it must transfer to other contexts (Tirosh, 1990). Transferability of mathematical learning to new and unfamiliar contexts is an area of research to develop further, particularly in relation to the transfer from 'concrete' situations to 'formal' situations (Askew and William, 1995). Developing imagery seems an important feature in developing mathematical ideas and children might develop various types of images that they use in different mathematical situations (Pendlington, 1999). Part of the teacher's role is to attempt to help the child build such images by providing a range of sensory experiences. Sometimes, pictorial images and physical models cause confusion because of their presentation, and the way the teachers and children interact with them. Language influences the visual perception and vice versa. A teacher might talk about 'higher' numbers on a hundred square when the larger numbers are physically lower down on the square. One might question whether either the language or the pictorial image ought to change, or alternatively whether teachers expose the children to a variety of number squares so that a flexible network of ideas develops.

A model of the recursive theory of mathematical understanding includes language at the level of 'image making' (Kieren, 1993). 'Image making' is the second level of effective action in which the child may work on some problems e.g. sharing items, recording the results and using fractional language. 'Image having' follows 'image making' in which a mental object develops i.e. the child has an idea about fractions. A mental image might not be a pictorial one, but might be words or numbers. Developing the mental image requires recursion to the 'image making' activities and the perception of a pattern within them usually expressed linguistically. Vygotsky's ideas have influenced Kieren's analysis in the sense of language helping the child form a mental image, as the mediator or 'inner voice' (Lee, 1985).

Sometimes children can interpret the same image in two different ways, and perceive no conflict as they use a different perception of the situation (Pa, 1991). Pa provides an example where a child shown an array of six yellow counters and six red counters describes it as 6/12 then 6/6. The child is able to provide different reasoning to support both answers. One applies the knowledge of parts and wholes, and the other applies a visual perception where the effect is one of matching six yellow for six red counters. The child is not concerned about internal consistency of images used. This might have implications for the relationship between the image they have and the descriptive language that derives from 'inner speech'.

The resource under discussion in the paper is the *counting stick*. The counting stick is of interest, as it seems to have the potential to prompt development of a mental image of a number line. It also seems to be a stage between a number line, and an 'empty number line' that researchers in the Netherlands identified as having the potential to help children understand whole number place value and developing mental strategies (Beishuizen 1995, cited in SCAA, 1997). One might ask whether the counting stick helps children to develop mental images of the relationships between numbers. The difficulty in researching children's images is that one can only access what the child can express verbally or on paper. One researcher found that children's descriptions of mental number lines grew more elaborate as children took turns to describe them (Allen, 1997). It seems that children built on their peers descriptions. Researching children's thoughts is problematical and the researcher has to try to remain objective, yet interpret the spoken word that 'describes' thoughts. In my own EdD research, I attempt to find out the children's ideas about particular mathematical words. Some probing is required and there is conflict between accepting the child's response, and the possibility of causing modification of the response through questioning.

A BRIEF CASE STUDY

A demonstration lesson provides the situation for my observations. In most cases, the demonstration lesson is the only time I have contact with a particular class i.e. there is no pre-visit to the class, nor follow-up. Planning occurs with the teacher during the previous week. The example in year 3, during the autumn term is from the short mental and oral whole class activity. The content of the lesson aims to meet the teacher's needs i.e. to observe a particular resource in use, and to demonstrate the teaching of a particular concept. I relied on the teacher's description of children's previous attainment, and response, to pitch activities at an appropriate level for the class. The children had experienced counting forwards and backwards in two's fives and tens, but the teacher had not used the counting stick. In fact, their counting had been unsupported by any resource. When I introduced the stick, children had little difficulty in identifying points on the stick as numbers and easily counted forwards, but less easily backwards. We played 'follow my finger' in which the children had to remember number order as I jumped forwards or backwards to points on the stick in order to develop the backwards counting skill. Using the stick to represent different values did not appear to confuse the children. We practised counting in multiples of twos, fives and tens as well as exploring the relationship between the multiples. The count did not always start at zero, and the children used their imagination to extend the count beyond the end of the stick, but not into negative numbers during this session. As a teaching aid, the stick served a very useful purpose in fulfilling the objective for the mental and oral part of the lesson. It was a focus to provide interest and motivation in the whole class situation. The children appeared to improve their backward counting skill as they gained confidence and focused on following my finger. They counted backwards with much more confidence at the end of the ten minute session, and teacher comment supported my view that the children had

Jaworski, B. (Ed.) Proceedings of the British Society for Research into Learning Mathematics 20(1&2) February/May 2000

improved their backward counting skill. The teacher also commented on its relationship to the number line and expressed a positive interest in developing further work with number lines as a useful resource to aid understanding of the number system. The versatility of the stick and the ease with which it represented different parts of the number system the teacher considered a benefit for the children.

DISCUSSION

Counting sticks exist in various shapes and sizes, but the most common style derives from the metre stick sectioned into tenths of a metre in two colours as in Figure 1. My own immediate perception of this style of stick is that it represents a number track in which each section is worth 10 if the whole stick is worth 100. I also perceive the point at which the colours meet to mark the 'tens'. I can also imagine the coloured sections split into smaller and smaller sections, but my imagination usually places lines along them as markers on a ruler signify where numbers lie. I can also imagine the stick representing any numerical amount, not beginning at zero, in a different orientation, and representing measures. Other people might have a different perception than mine.

Figure 1. A typical counting stick

Other sticks are a single colour with tape around them at equal intervals. My own perception of this stick is a number line with the tape representing points that mark particular numbers e.g. tens. Again, one might imagine other points on the line, and determine the value of the space between markings. The tape enables clarity for the whole class to see, but is often wider than a 'point' at which one might consider placement of a number.

Figure 2. A variation

The stick used in my described demonstration is a hybrid of the two sticks. The purpose of this paper is not to discuss the relative merits of the different styles of stick in their classroom application, but to consider how children and teachers *might* perceive them. The counting stick is one of several aids to promoting whole class counting in an enjoyable and purposeful way. The fact that it is a flexible model, and may represent measures, enhances its qualities as a resource that enables the connection of ideas from one area of mathematics to another. In a practical sense, it is the ideal teaching resource for use throughout school. However, it also has the potential to create imagery that is different from other resources to develop counting skills. Other resources include number squares, fingers and body movements, rhymes, swinging toys, counting wheels, various number cards, arrays of dots and physical resources such as Cuisenaire.

One might question whether the counting stick actually helps children to develop mental images of number lines, and aid the development of understanding of the number system. The style of the stick might be relatively unimportant if it generates different mental images depending on previous experiences. Critics of my stick comment on the bands of colour making the point at which a number sits less clear, but children of all ages, from Reception to Year 6, tell me that the purpose of the bands of colour is to mark clearly where the numbers are. They can also tell me the numbers that lie between the bands, and extend the count beyond the ends of the stick either way. If such extension occurs then it seems that the stick promotes a useful image. Children generally seem to have little difficulty in the stick changing its value and purpose. With any style of stick, one cannot be certain that the child's focus is the same as the teachers. For example the most common stick in Figure 1, allows one to focus on the blocks of colour as representing an amount, or on the joins representing a point on a line. Children might focus on either if these. In the mental and oral starter described, there existed an apparent success of both teaching and learning. The question such an event raises is whether the stick actively promotes understanding and knowledge of the number system, and if it does this through promoting mental imagery of a number line. If such mental imagery is promoted by the use of the counting stick will the image reflect the continuous nature of the number line, and does it link with the idea of the empty number line?

CONCLUSION

Research into effective teaching of numeracy identifies that teachers who promote mathematics as a set of interrelated ideas are most effective (Askew, Brown, Rhodes, Johnson and William 1997). Research into mathematics education often focuses on curriculum innovation and the evaluation of new teaching approaches. The National Numeracy Project was initially such an initiative, but with the period of research shortened, introduction of the NNS occurred before teachers and researchers had opportunity to reflect on particular resources and methods more fully. Judgements of success in the current political climate rely on test results rather than on objective evaluation of the pragmatic aspects of teaching and learning.

The aim of this paper is to promote thought about the nature of the various resources we use, and to encourage more classroom-based research on what teachers actually do. The effectiveness in building connections between mathematical ideas depends on the teacher's ability to appropriately question and challenge the children through using a range of resources. The counting stick is one such resource. Through classroom-based research, involving teachers, we can identify the strengths and weaknesses of the counting stick in relation to children developing a secure understanding of the number system. Now is the ideal time to focus on looking forward to developing practitioner-based research while the national numeracy strategy is developing. Ideally, research has to be on a large scale, representing all school situations. Teachers are interested in research findings that have a direct relationship with their own current practice e.g. they want to know how to use a counting stick and how it helps the children learn. Researching resources such as the counting stick is an opportunity to unify researchers and classroom practitioners in pursuing the common goal of improving educational opportunities for all our children.

REFERENCES

Allen, B. (1997) 'Number lines revisited'in *Primary Mathematics* Summer 1997, MA

Askew, M. and William (1995) *Recent Research in Mathematics Education 5-16*, HMSO, London

Askew, M. Brown, M. Rhodes, V. Johnson, D. William, D. (1997) *Effective teachers of Numeracy,* London, Kings College

Ball, D.L.(1993) 'Halves, Pieces and Twoths' in Carpenter, T.P., Fennema, E., Romberg, T.A., (Ed's) Rational *Numbers and Integration of Research,* Lawrence Erlbaum Associates, Hillsdale, New Jersey.

D'Ambrosio, B. S and Mewborn, D.S. (1994) 'Children's constructions of Fractions and their Implications for Classroom Instruction' in *Journal of Research in Childhood Education*, Vol. 8, No. 2 pp150-161

Dickson,L , Brown,M. and Gibson,O. (1984, reprinted 1993) *Children Learning Mathematics - A teachers guide to recent research.*, London, Cassell.

English, L. and Halford, G. (1995) *Mathematics Education – models and processes*, Hove, UK, Laurence Erlbaum.

Kieren, T.E. (1993) Rational and Fractional Numbers: From Quotient Fields to Recursive Understanding in Carpenter, T.P., Fennema, E., Romberg, T.A., (Ed's) (1993) Rational *Numbers and Integration of Research,* Lawrence Erlbaum Associates, Hillsdale, New Jersey.

Lee, B. (1985) Origin's of Vygotsky's Semiotic Analysis, in Wertsch, J.V. (ED.) (1985) *Culture, Communication and Cognition – Vygotskian Perspectives,* Cambridge, Cambridge University Press.

Nunes, T and Bryant, P.(1996) *Children Doing Mathematics*, Blackwell, Oxford, UK.

Pa, N.A.N. (1991) 'Primary school pupils' knowledge of fractions', in *Research in Education,* No 45, May, pp23-39, Manchester University Press.

Pendlington, S. (1999) 'Young Children's Mathematical Imagery: Methodological Issues' in *Proceedings of the Day Conference held at the Open University* Feb 1999, British Society for Research into Learning Mathematics (BSRLM)

Tirosh, D. (1990) 'Inconsistencies in Students' Mathematical Constructs', in *Focus on Learning Problems in Mathematics* Vol. 12, Nos. 3and4, pp 111-129 Center for teaching/Learning of Mathematics, USA.

Schools Curriculum and Assessment Council (SCAA) 1997, *The Teaching and Assessment of Number at Key Stages 1-3*, SCAA.