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TENSIONS IN PLANNED MATHEMATICS LESSONS

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Lessons with written guidance are planned according to generalised empirical or theoretical typical expectations in the target age group. In practice the teacher must always make adjustments according to the particular responses of their own pupils.

*In **CAME** lessons success in topic work is not an objective in its own right but rather the context for reasoning activity. Hence the adjustments to lesson plans often involve resolving tensions, at various levels, between misconceptions in the topic agenda on the one hand and related unplanned deeper reasoning activity on the other.*

In this paper scenes of interactions of a group of Y5 pupils around a mathematical reasoning task involving tessellation concepts are discussed with the aim of eliciting possible teaching strategies that responsively accommodate pupils' difficulties through pupil-pupil and teacher-pupil interactions, while optimising their progress towards handling the main challenges.

Background

A current trends in Mathematics teaching is the increased availability of planned lessons with relatively detailed guidance to teachers on their agenda and conduct in the classroom. Examples are the National Numeracy Strategy materials (NNP 1999) in KS1 & 2, the Maths Direct (Adhami *et al* 1999) textbook series for low attaining KS3 classes, and the Mathematics Enhancement Project (Burghes 1999) for KS4 classes. Similar to these in providing guidance, but distinct in agenda and approach, is the programme of Thinking Maths (TM) lessons, an outcome of the Cognitive Acceleration in Mathematics Education (CAME) project (Adhami, Johnson & Shayer 1998a). Here, the thirty lesson plans and guidance are intended to challenge classes across the ability range in KS3 with the focus on reasoning rather than on knowledge. Teachers difficulties with such material is of a different type too.

All planned lessons address an idealised or generalised class of pupils, and necessarily offer guidance on a restricted set of expected pupils responses. But in an instruction mode a teacher with moderate experience with the curriculum across a few NC levels would normally be able to field a variety of responses that are not noted in the guidance given, correcting errors and providing answers, even maintaining a desirable interactive atmosphere in the process. That is more difficult in a CAME lesson where the agenda is on the logical reasoning underlying the topic rather than the formal mathematics itself. The range of pupils' responses provoked at key points are intended to be much wider than answering a closed question, are meant to be presented in pupils' own words, and all the contributions are to be valued and used to arrive at higher order shared concepts in informal or formal ways. So the teacher must add to their repertoire of skills a knowledge of the common misconceptions in main topics, as well as the ability to listen and engage with pupils' own formulations of ideas that may be conceptually valid but are not presented in formal ways.

Teacher's guidance on a TM lesson includes a plan of the desirable flow of the given lesson. This plan is based on the reasoning steps identified in cognitive demand analysis broadly based on a Piagetian framework for the range of cognitive levels both in the age cohort and in the task, validated in trials in mainstream classes. That cognitive agenda is also addressed in separate sections in the guidance. Additionally the guidance includes an account of a 'specimen lesson', which often shows how the teacher has deviated from the plan in response to the particular responses in the class, following other venues. That exemplifies the social aspect of the approach broadly based on the Vygotskyan framework, in which teaching is viewed as mediating pupil-pupil interactions in their zones of proximal development, and the move from the spontaneous concepts to the scientific or formal concepts. (Adhami, Johnson & Shayer 1998b)

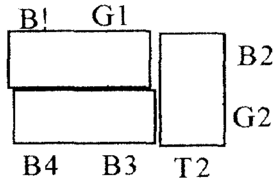
Observation methodology in CAME

To provide teachers with the two aspects of guidance, the cognitive and the social, the CAME research and development methodology relies on cycles of observed trials in mainstream classrooms. Two types of observations can be identified. One is focused on clarifying the cognitive agenda in the lesson development phase through attention to the potentials and the pitfalls in an activity in terms of pupils responses, regardless of the social environment and quality of teaching. The other is focused on ways of optimising, through classroom management and interactions, the quality of conducting a lesson that has been shown elsewhere to be cognitively rich. In both cases the observation involves conscious selective attention controlled by the theoretical frameworks of the approach. Seen in this light the published TM pack of guidance is a small but visible part of a much richer body of recorded empirical and theoretical work. The same processes, but with greater consciousness on our part, is current in our Primary CAME project. It is also the case that the cognitive agenda in Y5 and 6, remaining largely below the hypothesised Concrete Generalisation stage (Biggs & Collis 1982) is more sensitive to misconceptions than on Y7 and 8 where the emphasis is more on the move to early formal.

The following scenes of a group of six Y5 pupils working on an activity in its second round of trials may serve to exemplify how the reasoning steps and potentials for diversions are identified through attention to pupils' ideas. (Only a subset can be presented in these pages, together with comments written at the time to give the flavour of the selective attention noted above). This will be followed by a discussion on the implication of such observation for the agenda of the lesson.

The 4 boys and 2 girls were seated at joined-up tables to the front of the class to the left of the board. The working NC levels are as assessed by the class teacher.

Working	NC Level
B1	4c
G1	4c
B2	3a
G2	2a
B3	3c
B4	4b

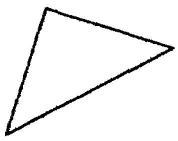


T1 Alexandra, whole class
T2 Mundher, group only
Present also, but not mentioned in these ,notes were the regular class teacher and two other colleagues.

The activity is to explore, in two cycles, the relationship between shape and space, focusing on triangles. First pupils are to explore the patterns that can be generated on the page by tessellating different triangles using templates. Then pupils are to focus on and describe the possibility of scalene triangles producing continuous bands, or strips, and the stacking of these strips to fully cover space.

Isosceles triangle

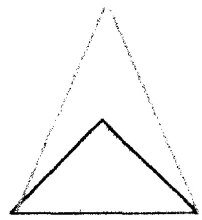
About ten minutes into the lesson, and following whole class introduction.
T1 gave the group some blue plastic triangles enough for each of the members to have one saying 'Your group has this isosceles triangle. What else you notice about it? See what patterns you can make with it.'



G1 (With agitation): This is not isosceles. It is scalene. (She was turning the shape in her hand, touching the sides, and repeating her indignant phrase. Others in the group are also turning the shapes in their hands. No one responds.)

T2 Why do you think it is scalene, not isosceles?

G1 (Holding the shape with the long side horizontally in left hand running thumb and forefinger of the right hand from either end of the base up in the air meeting higher up) Isosceles will be there.



B4 (Running a finger over the two equal sides of the plastic triangle.) These two sides are the same.

G1 (Suddenly changes her mind.) Ah, yes it is an isosceles.

T2 Can you tell me why you changed your mind? That is a useful thing to know.

G1 I have always seen isosceles with equal longer sides upwards.

G1 seems confident, alert and has a well developed language. The demonstration by the teacher T1 at the start of lesson (that the isosceles triangle can be stood upside down and remain such) dealt with orientation of shapes but did not deal with the issue that you can have long OR short equal sides

Partly however, G1's change of mind may be due to B4's remark. The group in general seem to look up to B4.

Right angle triangle

About 5 minutes later. Pupils have been drawing around the triangle on the blank sheets using it as a template.

T2 What is special about the angles in this triangle?

G1 These two the same. This one is bigger.

T2 How big?

B4 (Having drawn two triangles with the longer sides aligned -  .) Half a square.

G1 (Looking over to the B4's drawing.) Yes. Half a square.

T2 What does that mean to the angle?

B4 Square. 90 degrees.

G1 (Indignant.) No! No! (She runs with her finger around the right angle sides, with a puzzled expression. Turns the triangle around so that there are horizontal and vertical sides.)

B4 They must add up to something. (Asking B1 opposite him.) How much do they add up to? 360?

B1 Yeah. They must add to 360. or is it 180?

B4 Yeah. 180. So this is 90 and these are 60 each.

B1 Can't be 60 can they? That is equilateral. (they both add up in their heads and aloud: 60 and 60, 40 and 40)

B4 45 each.

G1 is attentive, but not involved in this discussion.

T2 (Pointing G1 to what B2 and G2 have started doing which is to draw around the triangle starting exactly from the corner of their A3 sheet.) The angle fitted the corner of the sheet. What angle is the corner?

G1 Oh yeah. It is 90 degrees.

It is likely that G1's initial rejection of the 90 degrees was due to her not recognising the right angle due to the way she held the triangle, even though she recognised it as bigger than the other two. While B1 and B4 were happily guessing the measurements of angles to fit a formal mathematical piece of knowledge they have, G1 was not involved. She did not readily see that when two such triangles make up a square the angle must be right angle. But she accepted the 90 degrees after seeing the angle fitting the corner of the page. Perhaps she needed more than one instance, or more time, to connect the square, the right angle and the 90 degrees. Is this again an issue of prior experience of orientation in the examples used in earlier teaching?

Logic of inclusion

About 15 Minutes later. Whole class sharing

In the discussion following other groups' contributions - some of these groups had actually begun putting shapes together to fill the space - which led to parallelograms and parallel lines, T1 asked each of the groups to say what type triangle they had, and what other shapes and patterns they have discovered.

For this group:

B1 We have an isosceles but also turned out to be right angle.

B4 (Holding up his sheet and pointing to the square made-up of 4 triangles.) You can build bigger squares.

G2 Diamonds (shows it).

T1 Is that the same as a square if you turn it around? A pupil from another group says 'It is a rhombus'. T1 Yes it is a special kind of rhombus.

B4 You can make a big triangle.

B1 verbalised the insight he arrived at with B4 accurately as a logic of inclusion. The group has mostly interpreted the task as finding the different shapes that can be made from fitting the triangle together, not how to tessellate to fill the space.

Meaning of 'straight'

About 10 minutes later.

In the discussion following other groups' contributions which led to parallelograms and parallel lines, T1 asked **what pattern of lines** they can see?

In this group:

B4 Some are diagonal and some are straight.

T2 Can't a diagonal be straight?

G1 No!. A straight is just not wonky. Doesn't matter which way it is.

T2 (Turns B4's page.)

B4 The same but turned around.

G1's 'No' is non-sequitur to T2's question, and seems a response to B4's statement which indicated that he didn't see that the diagonal line can be straight while she did. G1 seems to have assimilated the idea that the description of an object is independent of its orientation, perhaps from T1's demonstration on the triangle orientation. B4 readily accepted the straight-ness of the diagonal.

A common confusion between the daily language meaning of straight being straight across or straight up and the mathematical meaning.

Logic of inclusion (2)

About 10 minutes later

T1 asks the class to draw and cut out scalene triangles from card paper to use later in tessellation as templates, showing them how to do this.

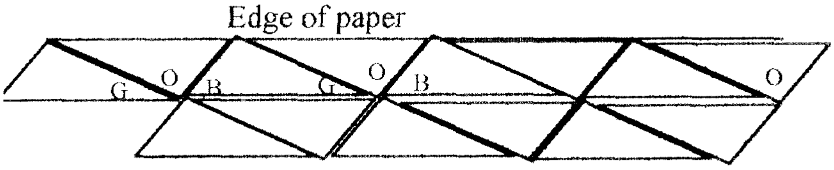
B1 We need a scalene triangle which is not a right angle.

This is the third time B1 shows logic of inclusion. It follows: "Can't be 60 can they? That is equilateral" and "isosceles but also turned out to be right angle"

Producing continuous strips

About 10 minutes later (one hour into the lesson!).
Following a period of pupils using scalene triangles, T1 asks the class to colour-code the three angles of the scalene triangle then throughout their patterns.

T2 starts G2 in the colouring of her triangle and the pattern. She turns the triangle to fit it into the shape and marks the angles with the correct colours in accordance with the colour of the angle in the template. Then she dispenses with the template and, going along the line, starts colouring the angles without it.



T2 How do you know this is orange?
G2 (Pointing to a corresponding angle.) It is the same as this one. They go Brown then Orange then Green.
T2 (Pointing to a 'middle' angle at a distance but on the line.) So what colour should this one be?
G2 Orange. It is the middle.

G2 was helped earlier by T2 to use the edge of the paper, and seems to have understood the idea of making a straight line and labelling of angles. She then seems able to recognise the correspondence of angles in a grid of parallel lines directly, after first handling the template with angles coloured.

Inferences for the lesson design

Analysing these and other observations in light of pupils estimated working NC levels leads to conclusions that:
a) The assumed prerequisite prior knowledge is suited for a Y5 age-group with a normal range of ability, since the difficulties and misconceptions encountered are well within the zone of proximal development estimated through the interaction with the group. Strategies for fuller coverage of these issues in a pre-lesson or in a well designed preparation phase can be devised, i.e. addressing the vocabulary of the cross-classification of triangles by angles and sides, and the inclusion relationships involved.
b) The reasoning agenda of the move towards continuous bands, hence infinite lines and full coverage of space, is realistic. This is an extrapolation from the fact that the pupil with the lowest estimated working level was approaching it at the end of the lesson. The issue for the teacher is how to reach this focus without too many diversions. One option is to follow on or to start from filled space with three sets of

regularly intersecting parallel lines, concentrating on the scalene triangle as a generic example.
The observation confirmed the viability of the activity as a Y5 TM lesson and a third round is planned with modified guidance.

Whole class teaching as an optimisation process

The accumulated experience in the CAME programme of research and development confirms the feasibility of planning lessons with a reasoning agenda that challenges pupils across a wide range of ability, each at their own level. On the other hand it also confirms that the teacher must find the balance between responsiveness to the particular class of pupils and adherence to the planned agenda. Communicating this balance is extremely difficult except through several actual examples of full lessons. Teaching a planned lesson focused on reasoning can be defined as an optimisation process of both the extent of pupils' engagement and the cognitive levels of their ideas, including misconceptions. That is a tension that no lesson plans can fully cater for and which the teachers must resolve 'on the hoof' in each particular class. What would help most is a clearer identification of what is good about 'good practice' seen from this perspective, and the describing and honing of the professional teaching skills involved in the light of coherent and practical theoretical frameworks.

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