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HOW DOES THE WAY IN WHICH INDIVIDUAL STUDENTS BEHAVE AFFECT THE SHARED CONSTRUCTION OF MEANING?

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Audio taped discussions between three students have been examined to shed light on the way in which the behaviour of individual students may affect the shared construction of meaning. These discussions revealed a complex pattern of interaction between the students. Each student was responsible for defining his or her own role within the discourse and these roles appeared to change as the discussion progressed. With reference to the framework offered by Winbourne and Watson (1998), it is proposed that local communities of practice have been established and that the individual student's positioning within the community of practice determines their success as a learner and contributes towards the creation of shared knowledge.

Introduction

This paper seeks to investigate whether three GCE Advanced level further mathematics students were able to develop a joint conception of the problems that they worked on together as part of a class discussion. Of particular interest was the part that each individual student played in creating shared meaning. The theoretical position adopted in this study is based on the Vygotskian idea that all learning is essentially social and that meaning is derived through interactions between students and with the teacher, and is mediated by tools. Each participant occupies a different role in the construction and negotiation of meaning and these roles are developed through participation in local communities of practice. These ideas which form the basis for this study are elaborated below.

Social Construction of Meaning

Lerman (1994) regards meaning as socio-cultural in nature, a product of discourse and discourse positions and he argues that individuals are thus acculturated into meanings. The individual student's input into meaning making changes and is changed by the discourse. In this way the student derives meaning from their *positioning* in social practices (Lerman, 1994). Meaning is seen to be *appropriated* by individual students, whereby each student forms his or her own something, from that which already belongs to others (ibid). Appropriation occurs through communication and tool use. Hershkowitz (1999) identifies a need for focusing on the individual student's development as he or she participates in the collective construction of shared cognition in small groups or in the whole class community. She claims (ibid) that socio-cultural studies focus mostly on the interaction or the interactional event itself and that the individual student is generally an anonymous participant in classroom episodes. This paper thus attempts to draw out the individual student's role in creating, maintaining and deriving meaning from the discourse.

Local Communities of Practice

Winbourne and Watson (1998) identify six key features of *local communities of practice*:

- Pupils see themselves as functioning mathematically within the lesson;
- There is a public recognition of competence;

- There are shared ways of behaving, language, habits, values and tool-use;
- The shape of the lesson is dependent upon the active participation of the students;
- Learners and teachers see themselves as engaged in the same activity;
- Learners see themselves as working together towards achieving a common understanding.

They propose that any classroom can be regarded as an intersection of a multiplicity of these practices and trajectories. They also argue, as does Lerman, that the individual student's *positioning* within a community of practice will determine their learning success. Ultimately, the students can come to operate masterfully, within the constraints of the social setting. The students fulfil their ultimate positions within the community of practice through smaller-scale "becomings" in which they join the practice and begin to assume their eventual position. The student's experiences at school are mediated by the images of themselves as learners that they bring with them.

The Role of the Teacher

Both the teacher and the students play a mutual and active part in creating the social environment. The teacher is seen as a mediator of student learning and assumes an active and necessary role in the learning process (Lerman, 1994). An important objective for the teacher is to apprentice students into the discourse of the mathematics classroom (Lerman, 1994). The teacher assists the students in "appropriating the culture of the community of mathematicians as a further social practice", so that the students will be able to operate masterfully in this setting. To establish *local communities of practice* the teacher must constrain the foci for attention, and recognise and work with pre-dispositions, rather than ignore them (Winbourne and Watson, 1998).

The Role of Technology

Borba (1996) proposes that the use of graphical calculators can enhance mathematical discussions and "reorganise" the way that knowledge is constructed. The graphical calculator is seen as a mediator of both the teacher-student relationships and the interactions between students. Pea (1987) argues that "social environments that establish an interactive social context for discussing, reflecting upon, and collaborating in the mathematical thinking necessary to solve a problem also motivate mathematical thinking" (p.104). He emphasises that technology can play a fundamental mediational role in promoting dialogue and collaboration in mathematical problem solving.

The Class Discussions

Robert, Martin and Julie were asked to identify the symbolic forms of six graphed functions from a list of twenty possibilities and discuss their ideas. The discussions surrounding three of the graphs are presented below. Terminology developed by Teasley and Rochelle (1993) was used to analyse the interaction. This involved identifying student 'initiation' of the discourse, student 'acceptance' of arguments and cases of students 'repairing' misunderstandings. There were also instances that appeared to involve 'collaborative completions' between students, where one partner's turn would begin a sentence or idea and the other partner would use their turn to complete it.

Discussion of Graph B [y = sin 3x]

1. SE: Can anybody think of a function for B?
2. M: I reckon its sin3x.
3. SE: Sin3x.
4. All: Yes.
5. SE: You seem to agree on that one. So how did you come up with that conclusion?
6. M: It's a sine wave and it's been er…
7. R: Three times x would condense it.
8. M: It's got a stretch parallel to the x-axis of a third, because it got closer together.
9. SE: Yes, you're all right it's sin3x.

Martin initiated the discussion by asserting that this was the graph of sin3x. The other two students immediately accepted that this was the correct form of the function. When asked to give reasons why, Martin and Robert took turns to give an explanation, each building and elaborating on the previous utterances (lines 6, 7, 8), thereby producing a collaborative completion. When Martin paused to think (line 6), Robert anticipated what he may have intended to say and completed his statement. Together they provide a convincing argument for their choice of function. Although, Julie did not participate verbally in this part of the discussion, she did make gestures that indicated her agreement with the arguments being put forward. The knowledge constructed by the students in this example appears to be shared between the students, especially Martin and Robert. Instead of concentrating on developing their own arguments separately in the discussion (which had occurred during the discussion of graph A), they produced a joint explanation of why sin3x was the correct function. In this case each of the students appeared to be able to clearly picture the effects of the transformation, without using the technology.

Discussion of Graph E [$y = e^{x-1} + 4$]

1. R: It could involve an exponential this time.
2. SE: Yes this is an exponential.
3. R: It's got +4 on the end, so it's either $y = e^{-(x+1)} + 4$, $y = -e^{x+1} + 4$, or $y = e^{x-1} + 4$.
4. J: It hasn't been reflected, so it's not $y = -e^{x+1} + 4$.
5. R: It's probably $y = e^{x-1} + 4$ actually.
6. SE: Why do you say that one?
7. R: Because the negative sign somehow has to fit that [the graph], although I can't explain how the minus sign affects it.
8. J: That's some sort of reflection, isn't it? [referring to $y = e^{x-1} + 4$].
9. R: $y = e^{-(x-1)} + 4$ would be a reflection.
10. J: Why?
11. R: It would be a reflection in x, wouldn't it?
12. J: I don't know.
13. R: $y = -e^{x-1} + 4$ would be a reflection in y. This is like ignoring the transformation of +4 which I'd say is $y = e^{x-1} + 4$.
14. SE: Yes you are correct. If you two are not sure you can always draw their graphs.

The fifth graph to be considered was of a type unfamiliar to the students and resulted in Robert assuming the role of peer tutor. This discussion also provided Julie with an opportunity to share her thoughts with Robert, and was the first example of her engaging fully in the discourse. Robert was the first to comment on the possible forms of the graphed function. Julie then voluntarily presented her argument to eliminate $y = -e^{x-1} + 4$ (line 4). Following this Robert guessed the correct function. At this point Robert and Julie began to conjecture incorrectly, about the effects of the functions on the shape of their graphs. They were unsure about their ideas, turned to each other for help and steered the conversation accordingly. Julie proposed that one of the functions was a reflection and invited acceptance or repair from Robert (line 8). Robert responded by suggesting that another of the functions would be a reflection, thus dismissing Julie's proposal (line 9). Julie could not see why this would be a reflection and sought an explanation from Robert (line 10). In response Robert merely restated that this would be a reflection, adding that it would be in the x-axis, inviting acceptance or repair from Julie (line 11). Julie was still unsure and Robert's utterances did not make things clearer (line 12). Robert finished by proposing that another of the functions would be a reflection in the y-axis and re-emphasising his choice of function (line 13).

Whilst Robert and Julie turned to one another for support, they were unable to answer each other's questions satisfactorily. Julie was confused about which of the functions are reflections (line 8) and Robert was confusing a reflection in the x-axis with a reflection in the y-axis and vice versa (lines 11, 13). In this way they were able to develop a shared, albeit flawed understanding of the problem. Martin on the other hand does not offer any comments, although he appeared to be considering the arguments posed by Julie and Robert. The evidence suggests that these students would need additional support to enable them to visualise the effects of certain transformations on exponential functions correctly. This is an occasion where technology and the teacher could be particularly effective in mediating the students' visualisation powers. The students needed to test their conjectures and investigate the visual connections between the various exponential functions.

Discussion of Graph F [y = tan(x/3)]

1. R: It's a tangent.
2. SE: Think about the scale the TI-92 uses.
3. R: To see if it was increasing, I could just draw the normal graph.
4. SE: Ok, if it helps you can draw the - you can all draw the tanx graph and see what happens on your machine and then from there you can hopefully deduce what the function is.
5. R: It's a stretch of factor 3.
6. M: It's tan of x over 3.
7. R: Yes.
8. SE: Is that y = tan(x/3) or y = tanx/3 because there are two of them?
9. M: y = tan(x/3).
10. SE: y = tan(x/3) and what do you think? Have you managed to get the tan?
11. J: Yes. That's the whole thing. [Julie pointed to the tanx in tanx/3].
12. SE: That's tan of x all divided by 3.

13. J: So yes y = tan(x/3).
14. SE: y = tan(x/3), yes well done you are right.

Robert was the first to state that this graph belonged to the tangent family of functions. There was however some uncertainty amongst the students as to what the graph of y = tanx would look like in relation to graph F. Recognising this problem, I asked the students to think about the scale that the graphical calculator uses to draw trigonometric functions (line 2). Robert then suggested that he could draw the graph of tanx using the TI-92 and compare this with graph F to deduce the relationship (line 4). I then advised all three students to try this approach. Robert compared the graphs and deduced that graph F was obtained using a stretch of factor three. To complete Robert's statement, Martin added that the correct function was 'tan of x over three', again producing a collaborative completion and Robert immediately agreed. As there were two functions which could be verbalised as 'tan of x over three', I sought confirmation that Martin had identified the function correctly and was quickly satisfied that he had. Up until this point Julie had not contributed to the discussion and I drew her into the conversation again to see if she was following the arguments being presented. Julie accepted the choice of function offered by Martin and provided some evidence that she had understood why this was the correct function (line 12). The students had thus been able to develop some shared understanding of the transformations used in this example.

Reflections

During these discussions *local communities of practice* appear to have been established. The students each showed willingness to explore and explain, and they began actively working together towards achieving a common sense of each problem through the sharing of ideas and by questioning one another. Each student created their own role in the practice, which varied accordingly and they shared behavioural traits, language, and technology use. I tried to ensure that they received public recognition of their competence and we saw ourselves as being involved in the same activity. Finally, the students considered themselves to be functioning mathematically within the lesson, as they were each offering suggestions as to which functions represented the given graphs, based on mathematical reasoning, which enabled them to obtain the correct form of the function in each case.

The patterns of interactions between the students changed as each new graph was considered. Throughout the discussions the individual students appeared to occupy different positions within the discourse, modifying their roles depending on their needs. Martin initiated the discussion around the first two graphs, and Robert took over this role for the discussions concerning the remaining four graphs. Robert began to act as a peer tutor (Graphs E, F). He continually made verbal contributions to the discussions and at times took control of the discussion, whilst the Julie and Martin spent some time actively listening and thinking rather than speaking. Robert, in particular, adopted the role of steering the discussions, whilst reacting to the arguments presented by the other students. So as the discussion developed, Robert's positioning within the discourse evolved and he proceeded to occupy a central role. Martin was initially quite instrumental in moving the group towards the correct solutions (eg Graph B). However, as Robert took over initiation and steering of the discourse, Martin seemed to fade into the background. Martin indicated that he was unsure about the symbolic forms of some of the graphs and he made fewer contributions when discussing these (eg graph E) and appeared to be listening to the arguments being presented by the others and thinking about their validity. He needed time to take into full consideration the arguments offered, to enable him to form his own ideas and to convince himself of their meaning. In this way Martin was attempting to derive his 'own something' from that which already belonged to Robert and Julie.

Julie operated as an active listener during the majority of the discussion, offering her suggestions in the main when specifically asked to do so. She had to be drawn into the discourse. However, during the discussion of graph E this pattern changed and her contributions were more spontaneous. She seemed to be particularly unsure about this question and appeared keen to further her understanding. She actively questioned Robert about his arguments, whilst offering her own for acceptance or repair. In this instance Julie's needs appeared to change. Her difficulties with this graph encouraged her to share her ideas more freely, in an attempt to derive meaning from the interaction.

None of the students appeared to be self-reliant. They learned from each other, my comments, using the technology, by participating in the community of practice and through their discourse. Robert was an eager and very active participant. His position developed into that of a peer tutor and he began to initiate and steer the discussion. His thought processes were more apparent as he more often verbalised his ideas. Martin's role in steering and initiating the discussions was superseded by Robert, and during the questions that he found difficult he became more of an active listener. Julie's position as a reluctant participant, was changed into that of active contributor when the need arose. The positions that the students occupied within the discourse were their ways of appropriating meaning and contributed towards their success as learners. Each made their own contribution to the construction of shared meaning, yet in cases when students were acting more as active listeners it is difficult to determine whether they have developed a joint conception of the problem's solution.

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