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BEING A TEACHER AND DOING RESEARCH: REFLECTIONS ON THE PRIMARY COGNITIVE ACCELERATION IN MATHEMATICS EDUCATION (CAME) PROJECT

Sally Dubben (Croydon LEA), Jeremy Hodgen (King's College London) and Ann Longfield (Croydon LEA)

In this paper we will explore the development of two Thinking Maths lessons as part of the primary Cognitive Acceleration in Mathematics Education (CAME) project. The focus of this paper is on the experiences of two of the authors, Sally Dubben and Ann Longfield, as teacher-researchers. We explore the lesson development as a collaborative process between university researchers and teacher researchers both in the classroom and in research team seminars. We believe this collaboration between academics and teachers to be a crucial element in linking theory and practice. In addition we seek to convey the excitement that teachers and children feel about the lessons.

Excitement and enthusiasm: doing mathematics and being a mathematician

"They loved it and I loved it."

(A teacher's reflection after teaching a CAME lesson.)

"I can't stop thinking about [Thinking Maths lessons] in other lessons."

(A child)

One of the outstanding features of Thinking Maths, or CAME, lessons is the enthusiasm with which children and teachers greet them. We have all experienced Thinking Maths lessons where children have spontaneously cheered or applauded at the end. For several children these lessons have changed the way in which they view maths, promoting a far more positive attitude to the subject. In many classes what has been noticeable is the way in which pupils respond to the challenges of the lesson; in some cases showing a desire to carry a lesson on through playtime or coming back to the teacher at a later date having figured out the solution to a part of the lesson. In one example, two children came running back after assembly to tell two of the team a number pattern they had discovered: *"We noticed that the first numbers always add up to ten!"*

This excitement reflects not just the content and structure of the lessons but also that dialogue is central to this way of teaching and learning, between teacher and pupil and more importantly between pupil and pupil. The lessons are about all members of the class, including the teacher, being mathematicians and exploring mathematical ideas and challenges together in a climate in which everyone's views are valued. Obviously the teacher has an additional role in managing the lesson and orchestrating the feedback but nonetheless is seen as a learner alongside others. Furthermore, the enjoyment felt by the children mirrors the enthusiasm with which teachers come to approach Thinking Maths lessons.

Taking this notion of lively dialogue and enthusiasm one step further back, these have certainly been characteristics of the primary CAME research team. The team

consisted of 4 researchers from King's College London, an LEA Maths Advisor and four Year 5 (Y5) and 6 (Y6) class teachers from two Croydon junior schools. Over the school year 1997/8, the project research team met in fortnightly seminars to develop lessons specifically for primary children. At these seminars all members of the research team participated in lesson simulations in which they tackled the mathematics children would tackle, whilst at the same time reflecting on the mathematics and the teaching. Members of the team then team-taught and observed lessons in the teacher-researchers' classrooms. This paper outlines the development of two lessons on fractions, "Share an Apple" (for Y5) and "Halving and Sharing" (for Y6), which began life as one lesson in Sally's classroom.

Cognitive Acceleration in Mathematics Education

The CAME project aims to promote children's land teachers' thinking through a series of Thinking Maths lessons linked to teacher professional development activities (Adhami, Johnson & Shayer, 1998a). The project has already developed a programme of lessons for use in years 7 and 8 (Adhami et al., 1998c). The primary CAME project aims to extend and develop this secondary teaching programme into Y5 and Y6 of the primary school. The lessons developed by the research team are now being trialled by a second group of fifteen Y5 and Y6 teachers.

CAME is not in itself a mathematics scheme of work. TM lessons are intended to replace just one mathematics lesson every two to three weeks. The emphasis of the TM lessons is on children grappling with the 'big ideas', or organising conceptual strands, in mathematics rather than on the mastery of specific skills. A key feature is the sharing and discussion of children's mathematical constructions as a whole class and in small groups.

The lesson development

The initial idea for a lesson based upon fractions stemmed from a brainstorming session as part of the research team meeting, where we identified areas of mathematical development that we felt it would be useful to address. Drawing on research into children's understandings and misconceptions of fractions (e.g. Jonhson, 1989; Kerslake, 1986), we knew that fractions, proportion and ratio are an area that children find particularly hard to fully understand. Sally suggested a problem that had been demonstrated in an earlier 20 day mathematics course.

> *The teacher holds two similar glasses. In the one glass is whisky; in the other water. If you pour half of the whisky into the water, mix it up, then pour half of that quantity back into the original whisky glass, which glass now has more whisky? More crucially, why, and can you prove it?*

Initial discussions at the research team focussed on identifying and comparing different methods for solving the problem. However, as the problem itself did not constitute a CAME lesson. Ann and Sally drafted a first version that also included the question, *"Is it possible for a third of someone's pocket money to be more than a half of someone else's?"* Other tasks were also included that involved constructing and

representing fractions in various everyday situations (e.g. halving an apple, a glass of water, a pound). The aim was for children to explore the meaning of these concepts that are often treated purely procedurally.

The lesson began looking at several simple multiplications of whole numbers (e.g. 3x5) and the effect of the operation in each case. Children's misconceptions of multiplication as "making things bigger" were highlighted. The pupils were then asked to find and discuss ways of halving an imaginary pile of 24 apples. Sally produced an apple and asked the children how to halve it. Working in pairs the children produced a variety of written and spoken expressions to describe this process. The terminology of numerator and denominator arose and were explored to establish a natural language and meaning for these formal mathematical terms..

The lesson continued with a jug of water and a bowl. Sally poured half of half a pint of liquid from the jug into the bowl and asked the pupils to describe what they had seen happen. The children used a variety of number sentences (e.g. ½ of a half; ½ - ½; ½ ÷ 2; ½ - ¼ ; half of a half) as well as pictures of the process. Sally raised the question whether it was possible to use multiplication (or "lots of") or division.

Working in pairs, the pupils were given a range of similar situations to encourage them to construct as many different ways of expressing operations with fractions as possible. After sharing these, the class discussed the equivalence of the statements with a view to eliminating all except those containing the multiplication of fractions. Sally then focussed on one number sentence: ½ x ¼ = $^1/_8$ and asked: What do you notice about this calculation? Can you describe a method of calculation?

This initial part of the lesson had run reasonably quickly, with the pupils displaying a reasonable amount of understanding but not a great deal of enthusiasm. The problem of the whisky and water was then posed. There followed a brief silence and then uproar. Many of the children made intuitive guesses but the result was that of a roughly equal split between children arguing for the whisky glass, the water glass and both holding the same amount of liquid. Very quickly the pupils attempted to explain and justify their answers. The majority of them instinctively began to draw the various steps. Some children used these representations on their own; others annotated them using formal fraction notation; a few used a ratio. Reflecting back on the lesson at the time, Sally commented, *"They were really noisy. I had stand up arguments between children about the maths, shouting at each other. If anyone had come in, they'd have though it was chaos, but I loved it."*

The children's excitement and enthusiasm at such a difficult challenge was certainly encouraging and there was clear potential for the development of the lesson. However, it was also clear that the lesson needed more focus. A second trialling of the lesson was conducted in Ann's Y5 class with Ann and Mundher Adhami, a King's reseracher, team-teaching and observing each other. It quickly became apparent that the episode in which an apple was cut in half was more challenging for Year 5 pupils than was originally envisaged. Consequently this activity became a major focus of the lesson and was an opportunity for a rich exploration of children's misconceptions of fractions, formal fraction notation and more informal representations of fractions. The

experience of this Y5 lesson was a catalyst for the research team to clarify the mathematics underlying these activities in order to analyse the conceptual difficulty of the tasks. Thus, the research team decided that this work could form the basis for a complete Year 5 lesson whilst the latter part would be worked on with Year 6 in mind.

The difficulty of these apparently simple concepts has been reflected in the second group of teachers' lessons. Whilst many of these teachers initially felt that the content would not provide sufficient challenge, they have generally been convinced of its value by the way it has evolved with their own classes. It is clear that whilst quite young children have an intuitive feel for halving objects this is not necessarily easily translated into an understanding of the numerical symbols. Furthermore, whilst some children appear familiar with the terms 'numerator' and 'denominator', what becomes apparent through discussion, is that they are unsure about what these terms actually mean. For the team, it has proved to be a useful context for highlighting to this second group of teachers the kinds of misconceptions that children develop when more formalised mathematical concepts, particularly in terms of notation, are superimposed upon the children's intuitive understanding of fractions and the natural language they use to describe them.

Given the need to have Y5 lessons ready, a significant period of time elapsed before the lesson was picked up again for further development as a Y6 lesson. Sally and Mundher trialled the original lesson idea, using a slightly modified worksheet but still exploring concepts of operating on fractions and describing this using natural descriptions and language. The lesson began with children folding strips of paper and describing the fractions together with a quick revisiting of the formal mathematical language like denominator and numerator. The children were then given a notesheet problem concerning fractions of chocolate bars. The children were required to produced a grid of fractions of the chocolate bar working horizontally by progressively halving and vertically by progressively thirding. There was a huge amount of discussion within the class on fractions and fractions of fractions. Again the problem of the whisky and water was given, with a similar positive response to that described above. This lesson is still being trialled and it is expected that it will undergo further changes and modifications on the basis of these experiences.

The children's mathematics

In both lessons the children explore the distinction between the part-whole relationship as expressed in a fraction (e.g. $^1/_3$) and the part-part relationship within the whole (e.g. 1:2). In the Y5 lesson the focus is on the comparison of fractions whilst the Y6 lesson explores operations on fractions.

In the Y5 lesson children explore familiar two-dimensional representations of fractions starting with the halving of the apple. In the following vignette taken from a phase 2 lesson, the children were asked how to share a pound:

Child 1: Cut it.

Child draws: ϕ

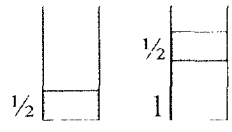
Teacher: How could we make sure it was equal?

Child 2: Go to the shop and ask for change.
Teacher: Change it for what?
Child 2: Two fifty p coins.
Children: Ah.

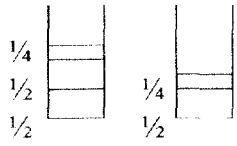
The first child's answer is a mathematically correct one in terms of halving a pound coin and was meaningful for halving an apple, although it is meaningless in terms of the pound as money. However, the teachers' response is not to simply correct her, but is rather to direct the children's attention to the issue of monetary value. Thus the children are encouraged to explore the meaning and usefulness of these representations. The children then move on to construct one-dimensional representations of fractions using paper strips. They explore equivalence and addition of simple fractions by directly comparing the strips thus beginning to construct a fraction number line.

In the Y6 lesson (titled Halving and Thirding) the context of Whisky and Water has been changed to mixing yellow and blue paint. Early on in the lesson the children construct ways of representing a half of half a jug of water being poured into a bowl. The children use their earlier representations to begin to justify their intuitive response, "You take the light green and put that back into the half of blue so that must be a darker green." We must emphasise that the aim is for children to use these representations to explore the multiplicative relationships underlying what many see as additive and not necessarily to reach an answer. However, the following vignette from a lesson trial demonstrates how children may use this to reach such a justification,

A child has drawn the following picture to represent pouring half of the blue (shaded) into the yellow (clear).



Child: So if we now put half of the mixture back into the pure blue then obviously we're going to get a darker green!
Teacher: Yes, very nice, but how much darker?
The child then performs the second step pouring half of the blue / yellow mixture back into the blue.



Child: So the one on the left is three quarter parts blue in five quarters of paint altogether, so it has to be three fifths blue.

It is worth noting that this form of representation is far from obvious. The paint have after all been thoroughly mixed. However, this form of representing the multiplication of fractions enables this child to produce an intuitive answer and then to justify this answer.

Discussion and Conclusion

The lesson story we have described above is typical of many primary CAME lessons. It is an exciting process for teachers, researchers and children. Initial ideas undergo considerable revision as a result of trialling and reflection. Specifically, the classroom experiences of teaching a lesson are often very different from its initial conception at a research team meeting. We want to stress the role of the teacher-researchers in this process of linking theory and practice. We do not wish to draw a simplistic distinction between teacher-researchers as practitioners and university researchers as theoreticians. However, the teacher-researchers' role in grounding the theory in practice together with the university researchers' role in theorising practice provides important lessons for other attempts to link developments in theory to developments in practice. Ann put this as follows when reflecting on the work of the research team:

> *"It's part to do about working as a team and actually seeing that different people have brought different things to the project. I mean there are people that have brought the academic knowledge and then there's been the classroom knowledge and I'm not saying that there's one group and there's another group because it's intertwined but there are variables within that in terms of what people have, in terms of knowledge of different things and that's been quite useful because you're bouncing off from different perspectives."*

As Wood (1999) argues, "From the point of practicality, knowledge of the dimensions involved in teaching is crucial to knowing how and why different forms of teaching are effective in supporting student learning" (p. 177). We believe that this can only be successful and effective in a framework which empowers teachers and which treats them as equals within the research process.

References

Adhami, M., Johnson, D. & Shayer, M. (1998a). Does CAME work? Summary report on phase 2 of the Cognitive Acceleration in Mathematics Education, CAME, project. In *Proceedings of the British Society for Research into Learning Mathematics (BSRLM) Day Conference, Bristol, November 15, 1997.* (pp. 26-31). London: BSRLM.

Adhami, M., Johnson, D. C. & Shayer, M. (1998b). *Thinking Maths: The programme for accelerated learning in mathematics.* Oxford: Heinemann Educational Books.

Johnson, D. C. (Ed.). (1989). *Children's Mathematical Frameworks 8-13: A study of classroom teaching.* Windsor: NFER-Nelson.

Kerslake, D. (1986). *Fractions: Children's strategies and errors.* Windsor: NFER-NELSON.

Wood, T. (1999). Approaching teacher development: Practice into theory. In B. Jaworski, T. Wood & S. Dawson (Eds.). *Mathematics teacher education: Critical international perspectives.* (pp. 163 - 170). London: Falmer.