CONJECTURING IN OPEN GEOMETRIC SITUATIONS IN CABRI-GEOMETRE: AN EXPLORATORY CLASSROOM EXPERIMENT

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Abstract

In this paper I present a description of a classroom experience which I carried out with a class of Year]0 students]. It consisted of a sequence of activities aimed at introducing students to conjecturing and justifying in geometry, within the Cabri-Geometre environment. In particular I shall focus on how the intervention was planned in collaboration with the teacher and how students reacted. An evaluation questionnaire was given to both students and teacher at the end of the experiment. The issues raised, mainly concerning working in groups, working with Cabri and open problems, need to be taken into account when designing future teaching and learning sequences.

Introduction

The classroom experience discussed in this paper is the first phase of a major project for my PhD, aimed at studying how students can be supported to accept and understand the role of proof in school mathematics, through engaging with a computer-based microworld, namely the software Cabri-Geometre. Computers are now available in almost every school, however it is not easy to find a way to integrate them into the normal classroom practice, given constraints in terms of time, schedules and the need for readjustments to the ways of teaching. My project falls within 'research for innovation' (Arzarello & Bartolini Bussi, 1998): given the fact that we want to innovate, how can we do this? There are many models addressing this issue. We can mention the 'long-term model' in Italy. In this, researchers and teachers work collaboratively in the whole project (Arzarello & Bartolini Bussi, 1998). In this way the research and analysis is grounded in practice, which improves its validity and the usefulness of its insights to other practitioners. Earlier research on learning with computers tended to look at children out of the regular classroom (Lawler et aI., 1986). More recent research has worked with teachers in the classroom in a more directive way (Hoyles & Sutherland, 1989).

One of the aims of the study presented in this paper was to work collaboratively with teachers in order to plan and carry out an intervention in the classroom.

Theoretical Background

The work described here sits within the current discussion in Mathematics Education about the teaching and learning of proof in school mathematics and the use of dynamic geometry software in the maths class. I will give a brief summary of this discussion.

Currently there is a debate around proof amongst researchers in mathematics and mathematics education (Duval, 1991; Hanna, 1996; Mariotti et aI, 1997) and one characteristic of the debate concerns the role of explorations and conjectures with respect to proving and proof. My project shares the perspective that exploration and conjecture is a key phase in the overall cognitive process of proving. The arguments

[1] I wish to acknowledge Mr. Jim Baker and his students for participating in this experience, and John Rogers for carrying out the project with me.

needed to prove a statement develop within this initial phase, before being re-organised in a deductive way in order to form conditional sentences. A considerable body of research has pointed out the problems and difficulties in the teaching and learning of proof in school mathematics (e.g. Balacheff, 1998). Usually students do not understand the meaning of the act when they are asked to prove a theorem (Chazan, 1993; Harel & Sowder, 1996; Healy & Hoyles, 1998). Moreover most of the time students are presented with ready-made proofs, which they do not understand.

New issues about the role of proof arise when new technologies are taken into account and dynamic geometry software such as Cabri-Geometre is introduced into school practice (Hanna, 1996). Cabri is not a tutorial system, therefore it cannot be autonomous from the learning point of view, that is it cannot be considered responsible on its own for the devolution and acquisition of mathematical content. As a consequence, educators must avoid the "fingertip effect" (Perkins, 1993), that is simply making a support system available and expecting people will more or less automatically take advantage of the opportunities that it affords. On the contrary Cabri should be seen as a "conceptual reorganiser" (Pea, 1987), in that it provides users with new tools that were not available when carrying out paper and pencil geometry. For instance, introducing Cabri provides the learner with two worlds (Sutherland & Balacheff, 1999): a theoretical world, which is that of geometry, and a mechanical, manipulative world, which is the phenomenological domain of Cabri. The shift between these two worlds is not always obvious and this makes careful planning of the whole learning situation necessary.

Classroom Experiment

Aims of the intervention. When planning an intervention in the classroom both research aims and learning outcomes for the students need to be taken into account. In terms of my project this study allowed me to experience an English classroom situation and to try out activities that might be feasible, methods of data collection and forms of analysis. The learning objectives addressed by the intervention were: using Cabri as a tool for discovering and conjecturing; making explorations and conjectures; validating conjectures; justifying conjectures.

The planned activities meet the following requirements of the National Curriculum, Key stage 3 and 4, Shape, Space and Measures: use computers to generate and transform graphic images and to solve problems; knowing and using properties of triangles and other shapes; using geometrical language with increasing precision

Sequence in the classroom. With these objectives in mind I planned the activities in the classroom following different stages: choosing a topic to teach; developing a sequence of activities in Cabri in the classroom; developing worksheets; carrying out the activity in the classroom; analysing students' productions. The topic chosen in negotiation with the teacher was circle theorems, to be introduced to a class of Year 10, mid-top set students. Since the classroom intervention involved an innovative mode of working in the classroom, both for the teacher and for the students, the planning of the whole sequence was important. The experience lasted two months.

Session]2 gave students the possibility of exploring what Cabri is without any precise instruction. *Session* 2 consisted of a test on proof (Healy & Hoyles, 1998) aimed at developing an idea of students' conceptions and knowledge about proof. *Session* 3 was devoted to a more structured exploration of Cabri, with the purpose of making students learn the meaning of constructions in Cabri and a way to check the correctness of their constructions, i.e. the dragging test. In *Session* 4 students had to make conjectures in Cabri by using dragging in order to explore and discover properties. *Session* 5 consisted of a classroom discussion in which students had the possibility of sharing their discoveries with their classmates. *Session* 6 was the first activity aimed at introducing one of the circle theorems. *Session* 7 was a follow-up lesson, in which a general discussion about what students found and proved was orchestrated by the researcher. *Session* 8 *and* 9 had the same structure as session 6. *Session*] *0* was devoted to constructing a proof of one theorem and to obtain a feedback of the whole intervention from both the students and the teacher.

Discussion & Analysis of Some Students' Productions

In this section I will capture a selection of issues concerning the development of the sequence in the classroom setting.

The activities. The teaching sequence was aimed at supporting students to discover circle theorems for themselves. Reconstructing the process of discovery of a theorem might be a way towards the construction of its justification (Arzarello et aI., 1998). We prepared activities constructed as open problems, in which students had to explore a situation, make conjectures, validate them and try to justify them. No precise questions were asked. The structure of the activities was changed during the development of the sequence, according to students' responses. Some of them found the activities too open and the wording not clear: "they were very vague, I couldn't understand some"3; at the beginning they found it difficult to get started, they had to be guided through more specific questions. Some others liked the open structure of the problems: "the way they required you to think and design on the computer, gave a lot of freedom", "we rarely do problems, just questions", "make you think and give a lot of answers".

In the computer lab students worked in pairs at the computer, following a particular activity presented on paper[4]• Students had never worked in groups before, as they are normally taught with the teacher teaching the whole class from the front. Working in groups turned out to be very powerful with respect to students' production. The students themselves stressed the importance of the interaction with other people in such an open activity: "one doesn't know, the other can help", "express our ideas and hear others' ideas". Working in groups encourages students to make explicit what they think through talking; in this sense it can help the formalisation of what they see in Cabri, in that they need to make explicit the relationships between the objects which move in Cabri.

Teacher-students interaction evolved during the sequence. At first students were completely dependent on the teacher's instructions and validation, as part of the didactic contract (Brousseau, 1988). They expected us to give them very precise

2 Each session lasted 40 minutes. We had nearly one session per week. Sessions 1, 3, 4, 6, 8, 9 took place in the computer lab, while sessions 2,5, 7, 10 took place in the maths classroom.

3 Quotations are taken from the evaluation questionnaire, which was given to students at the end of the intervention.

4 Two researchers, the author and John Rogers, were participating in the activity in the classroom with the teacher.

instructions on how to work. For example when required to *construct a right-angled triangle in Cabri*, they immediately asked for help: "how can we do this? We don't know how to construct a right-angled triangle". Initially they did not explore the situation. Moreover they were reluctant to write down anything which had not been 'approved' by the teacher: "but we don't know whether it is right or wrong, we can't write it down ... Mr Baker!!". As the sequence continued, students gained more confidence in their own work in Cabri and they started to write conjectures, such as "ideas that might be correct or incorrect that need to be proved" or "a theory or idea that you have come up with yourself and you believe to be true. It is not proved and may be found to be false if tested through". They searched for the validation of their conjectures in Cabri, and not for the teachers' approval."It leaves it open to you, you can't get a wrong or right conjecture just one that can be proved to not be true". The devolution of the problem (Brousseau, 1988) had taken place. However it is important to notice that the role of the teacher is still fundamental, as students' work is richer when they interact with the teacher (Bartolini Bussi, 1998).

Use of Cabri. Regarding the use of Cabri, the students' work showed an evolution towards exploiting all the potentialities of the dragging function, which seems to be the most powerful feature form the didactical point of view. They moved from drawing only to constructing figures and exploring the richness of the situation proposed.

In the first session students produced colourful pictures, which for them had the simple status of drawings. The second session was more productive in terms of students' appropriation of Cabri features. In fact, we did not want students to learn Cabri, but to use Cabri as a tool for discovering mathematics. "In the classroom the problems are shut; in Cabri everything is more open-ended". They all understood the use of the dragging test. However they found it rather more difficult to use dragging in order to explore a situation. At the beginning they did not move many things, but just looked at the figure on the screen. After some time they started to exploit all the potentialities of dragging to make their conjectures.

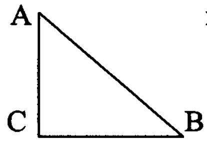
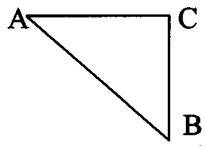
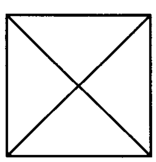
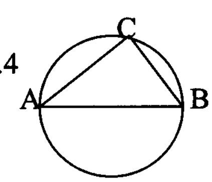
Conjecturing and proving in Cabri. As far as the conjecturing and proving process is concerned, the main issue to be considered is the fact that students actually engaged with dragging in Cabri and produced rich conjectures, using still a Cabri language. A certain appropriation of the dragging can be observed, while at the beginning students' conjectures were more static, visual and pictorial. Students seemed to do experiments by moving different points and observing the related effect of dragging on the figure.

Conjectures students wrote on the worksheet (problem 1: Angles in circles 1)[5]
If A is moved up or down the line AC then in effect you can make unlimited triangles, with C being the right angle. **If B is moved along the line BC then C will remain a right angle but the values of C and A will change.**
If a circle is drawn, with its centre on the midpoint of AB. Then if C is moved to the perimeter of the circle then the angle ACB will be 90°.
In order for the triangle to be a right angle (where C is the right angle) C must be on the circle with

[5] Problem 1: Draw any triangle ABC. Make it a right-angled triangle in C. How many right-angles triangles can you make? How can you characterise them? Make conjectures.

diameter AB.
Conjectures students wrote on the worksheet (problem 2: Angles in circles 2)[6]
If you have a triangle of 60° then the intersect of the perpendicular bisector of each side will be the centre point around which point C can be moved.
If you keep A and B fixed there are infinite 60° angles either side of the line AB. There are two curves linking A to B where 60° angles can be found.
C=60° on major segment or C=120° if in minor segment. To construct the diagram draw a normal circle and construct a triangle with all its points on the circle. B can be moved up any point on the line BC; A can be moved anywhere along the line AC and C will still be 60°.

Let us analyse in more details the production of one group, regarding problem 1.

Students' work	**Observations**
They obtain a right-angled triangle with AC horizontal (fig.1). fig.1  fig. 2 	At first they draw a triangle in Cabri, then they use dragging in order to obtain a triangle with the required property. The only one they can find is the 'typical' one, that is with the sides parallel to the borders of the screen. In this first attempt they use Cabri in order to draw what they had in mind, like on paper. It is a kind of 'guided dragging'.
After some time. B: "Here's one…how many can you have?…Just one…no two". They move C across AB to get to the other side (fig.2).	The fact they can move C allows them to see another triangle with the required property. But again dragging follows what students had in mind beforehand. The kind of reasoning they do is: there is this one (fig.1), but since we can drag C, there will be also the other one (fig.2).
B: "There is two…no four, one for each vertex of the square" (fig.3)	The same reasoning leads them to say there are four.
C: "Try to move C…How many triangles?…You go on a circle". They move C trying to keep the angle 90. They say: "you can have infinite triangles!" And then they write "If a circle is *drawn*, with its centre on the midpoint of AB. Then if *C is moved* to the perimeter of the circle then the angle ACB will be 90°". fig.3  fig.4 	Now they exploit dragging in order to discover a conjecture. They move C looking at the measure of the angle. At first they do very small movements, not to lose 90 degrees. They drag C so to keep the property they want ('lieu muet dragging') and they realise they are going on a circle. Therefore now they read what happens on the screen. So they come to say there are infinite triangles. At this point they make their conjecture, translating what they see in Cabri in a logical form. Their conjecture is still linked to the Cabri-world, as they refer to the movement of points; however they are able to provide and read the logical dependence of dragged points.
They draw the circle with diameter AB and they say "Yes, it's true" (fig. 4).	After that, they use the 'dragging test' in order to check their conjecture. Probably for them this provides the required validation of the conjecture.

These notes provide an example of the use of different dragging modalities (Arzarello et al., 1988) in Cabri and of the continuous shift between the work in Cabri

[6] Problem 2: Draw any triangle ABC. Now we want the triangle ABC to have the angle C of 60 degrees. How many triangle with this characteristic can you find? Which property characterises them?

and students' thoughts. Everything reveals the use of Cabri-Ianguage in the Cabri-geometry. This is what we might expect from students. The important thing is that this can provide the basis for further development and evolution. Follow up questions might concern a way to start from what they did and provide a move towards justifying and towards the world of geometry. Further on, in which way can the use of dragging mediate the production of hypothetical thinking?

When students were asked to justify their conjectures, most of them found it very difficult to make sense of this request. Cabri seemed to be enough for convincing them of the truth of their statements. Here I tried to pursue the idea of proof for explaining why something is true (Polya, 1957). The main difficult point was the fact that this episode was isolated, in that students had neither experienced any kind of conjecturing or justifying in mathematics, nor had they experienced the idea of building a deductive chain of reasoning.

This small-scale experiment showed the importance and usefulness of establishing a collaboration teacher-researchers, when implementing experimental settings and new technologies in the classroom. The teacher said he wouldn't have been able to do such a thing on his own, therefore he found our help very valuable; next year he will probably repeat the same thing, and now on his own.

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