Training and Practice of Teachers of Probability: An Epistemological Stance

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Abstract. Steinbring's proposition of the epistemological triangle (1997) *and Duval's work on semiotic registers of representation* (1996) *are the grounds of this research on stochastics in the elementary level of Mexican education. The training and practice of teachers of probability in secondary school classrooms* (12-15 *year old pupils) have been explored with a qualitative approach (Alquicira,* 1998). *Generally, the training of teachers does not seem to correspond to what the teaching of probability in classroom demands. A linear, formal conception of probability underlies a teaching using mainly simplified situations and neglecting semiotic registers other than the algebraic register, with a teacher centred exposition strategy. Whereas teachers' training referred to the epistemological triangle may result in teaching based on richer situations and on an interplay of semiotic registers of representation for the students to make sense of the activity, in which they become involved.*

Introduction

As a result of an investigation carried out on students' understanding of fundamental ideas of probability at pre-university level (Ojeda, 1994), a research project on stochastics in the Mexican system of education has been conducted over the last five years. Three factors are at the core of its investigations: the epistemological, the psychological and the social factors. Here we are concerned mainly with the first, and we exemplify the rationale underlying this aspect in the project with some results from research focused on teachers in the secondary level of Mexican education (12 - 15 year old children).

In general, mathematics teacher training for this level does not include Probability, nor does it include didactical aspects to teach the topic, as the documentary research carried out by Galvan (1996) attests, but there is also at this stage the case of some teaching of mathematics done by professionals in areas different from education (Alquicira, 1998). The results from research on mathematics education at this level are being implemented institutionally all over the country via the curriculum of mathematics, the syllabus and a guide for teachers. However, the staging of the corresponding proposals made for probability depends on the teacher's practice in the classroom. Firstly, his or her training may not fit the requirements for such an implementation to be successful, and secondly, very little research has been done about the actual interaction that takes place in the probability Mexican secondary school classroom.

The epistemological triangle and the role of semiotic registers of representation

The most common and formal account of what probability is refers to Bernoulli's theorem (the weak version of the law of large numbers). That is, with modem notation, given *n* independent trials of Bernoulli with parameter *p*, "i/ E >0, "i/ 11 >0, 3 Ne N 3 "i/ *n* > N, *P(lfn* - pi < E) > 1 -11, E being a measure of the deviation of the relative frequency *in* from the *a priori* probability *p*, and 11 a measure of our confidence on having this approximation (the probability P). Usually, this mathematical result is taken as the explanation of what probability is about: the gradually established statistical regularity is explained by the limiting *a priori* probability *p* which, in its turn, is explained by the tendency that the relative frequencies show "in the long run". According to Steinbring (1997), the concept of probability becomes gradually constituted from an interplay between context of reference, symbol and the concept itself from previous stages in its constitution. More generally, he refers to this basic scheme in the constitution of mathematical concepts as the "epistemological triangle" (1997; see Figure 1), but his proposition becomes particularly clear when referring to the concept of probability. The object, i.e. the context to which the individual refers his or her activity, must be distinguished from its aspect or aspects represented by the use of signs.

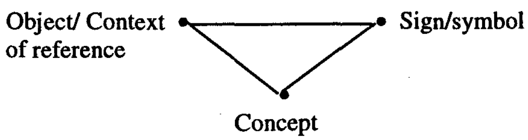

Figure 1. The epistemological triangle

From the cognitive point of view, for the students to be introduced to the basic stages of the subject of probability, the use in teaching of means to organise the relevant information about a particular situation (context of reference) and to treat their data, results in triggering their mathematical activity (Ojeda, 1994). In more general terms, for the subject to develop and to communicate a mathematical activity, a system of signs, a semiotic register support, is necessary (Duval, 1996). A semiotic register constitutes a system of representation if it allows three cognitive fundamental activities: its production, an inside treatment, and an in-between treatment or conversion between different semiotic registers. The semiotic registers used in the mathematical activity are the algebraic, the graphical, the figurative and the natural language. Different aspects of a mathematical concept and of its levels of sophistication (formal structure) demand the staging of particular semiotic registers (Duval, 1996): e. g. whereas the formal notation above for Bernoulli's theorem allows a more analytical (and formal) account of probability as a limit of relative frequencies, the diagram in Figure 2 prefigures this result as an account of the frequency approach to probability for a particular random situation (a sequence of tosses of a coin).

Inservice Teachers' training and classroom practice

The teaching of probability in secondary education is compulsory. Since there is no formal training to carry out this teaching, Alquicira (1998) was concerned, among other questions, with how teachers cope with this task, and how they could be trained to teach probability. In that respecting to this last question, a 20 hours training course on probability for inservice secondary school teachers was offered with the following structure: mathematical content (the measure of probability, sample space, the law of large numbers, independence, conditional probability, combinatorics, random variable and expectation), history of probability, probability in the syllabus for secondary school and didactical and psychological considerations for the teaching of probability. The mathematical content was worked out through solving problems, including the actual experimentation for several of the problems posed for the teachers to realise the interplay between the three elements of the epistemological triangle. Five hours along the course were devoted to discuss the teachers' proposals for didactical activities on probability. Seven teachers attended this course, and Alquicira explored their performances through participant observation along the sessions, and using the documents they provided with their written work.

In order to explore into the teaching of probability in classroom, Alquicira carried out non participant observation in two cases: from two teachers who attended the training course (we refer here to only one of them, and quote his session as TC) after this course was finished, and from one teacher who did not attend the course (the NC session). The schools where the observations took place are official schools in the Mexican system of education, and the teachers who participated in the study are considered 'excellent' by the heads of their schools. They accepted voluntarily to be a part in this research and proposed the content and the way in which they taught it to their current students. The sessions were videotaped, and the corresponding transcriptions were analysed to identify the teachers' didactical strategies. The students' copies with their written work were also investigated. The analysis of the teachers' practices referred to the epistemological triangle and to the use of semiotic registers, among other aspects.

Results

The NC session. As a means to introduce Probability in secondary school, in the Guide for the Mathematics Teacher (SEP, 1993), which is an attempt to regulate the teaching of mathematics in secondary schools allover the country, didactical activities have been proposed. For instance, the notion of probability is studied from a frequency approach, by recording the outcomes from sequences of Bernoulli trials in a tree diagram, for which the scale is reduced to avoid the gradual opening of the external branches (see Figure 2). This graph is proposed for the students to record and keep track of the individual outcomes, as well as for having at sight a gradual regularity being attained, with a relatively small number of trials, at most 50. The teacher posed 'draws at random', with replacement, from an urn having four white marbles and four yellow marbles. Students did not know the proportions, but they had to propose it after registering the outcomes for series of trials on the tree diagram sheet with which they were provided. The teacher made the draws in front of the class and conducted the activity with the following:

> *We are going to carry out an experiment with the urn; you are going to mark on the graph* [he *shows* his own copy to the students] *with a line from the starting point, where the first point is in the first curve. Let's see; draw a line dividing at the middle* [he points at an imaginary vertical line going down from the starting point on the top of the graph] .
> *. .. On the left, if you agree, we are going to register the yellow marbles and on the other the white marbles .... Let's carry out the experiment as many times as possible, and you are going to repeat by watching at the graph.* [You are going to say] *if there are more of these or of these* [he shows a marble of each colour], *and then we'll try to agree on how many of each colour. The graph may say if there are more of these or of these. And then we'll measure how many in all.*

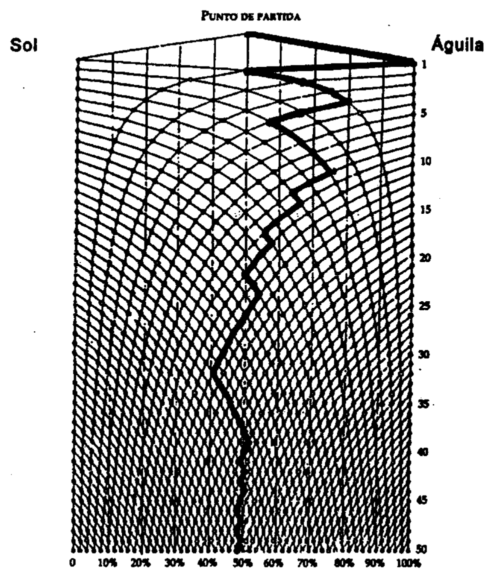

Figure 2. Tree diagram register sheet, produced correctly.

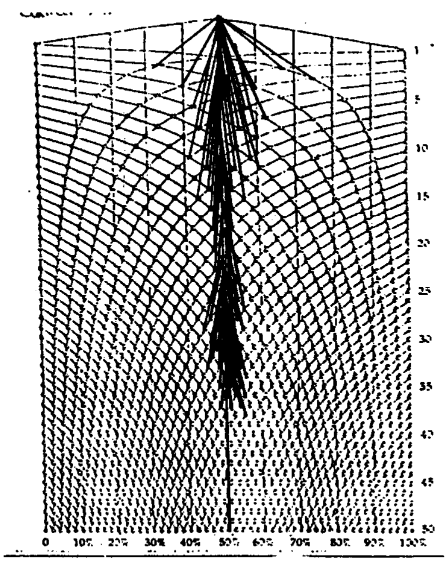

Figure 3. Pupil's graph after teacher's instructions. NC session.

After some draws were recorded, the teacher stated:

> *... More we draw, more it's going to approach the corresponding probability, isn't it?*

Having completed the sequence of 50 draws, the teacher said to the class:

> *This graph is pointing at 50%. Of course, that's probable, isn't it? It's not exact, it can't be exact ... We see there is a fluctuation, but it tends to 50%. The more repetitions we do, the more it's approaching 50%; that's a personal appreciation.*

The aim of the activity is misguided since the initial drawing of a line through the middle that the teacher proposed: using his knowledge of the proportions of the marbles in the urn (the context), he favoured the sign corner over the context of reference corner in the epistemological triangle. In addition, the way in which the graph was drawn by the students prevented them from following the sequence in which the outcomes were occurring, as for each draw, the corresponding line started at the top of the graph; then, instead of one branch (path), there were as many branches as trials were carried out (see Figure 3). Thus, no treatment of the register was possible. The question that promoted the activity was not answered at the end; the class did not interpret the resulting 50% in terms of the context of reference (the composition in the urn). Finally, the teacher's remark at the end of the session regarding the tendency towards 50% as to be taken as personal opinion, deprives the activity from the aim of the search for objectivity when facing chance.

The TC session. The teacher posed to the students the following situation to work on the frequency approach to probability: *In the football championship, if a team wins it gets 3 points, if it draws, it scores 1 point, and if it loses it scores O. What are the different possible scores for a team after two games? Determine the experimental and the theoretical probability of each score.*

The students were organised in six groups, each of which was supposed to be the football team in the problem. Each group simulated the situation using an urn with three balls labelled 3, *1* and *0,* from which 18 draws were made at random, with replacement, and the outcomes were registered in a table (Table 1 shows the results from group 1).

Table 1. **Frequency of sums from the simulation activity in the TC session.**

Possible Sums	6	5	4	3	2	1	0	Total
Frequency	2	0	4	3	2	6	1	18
Relative Frequency	2/18 = 0.11 11.1%	0/18 = 0 0%	4/18 = 0.22 22.2%	3/18 = 0.16 16.6%	2/18 = 0.11 11.1%	6/18 = 0.33 33.3%	1/18 = 0.05 5.5%	18/18 100%

Then, the a priori probability of each possible sum was calculated by using a tree diagram, supposing independent games and equiprobable outcomes for each game (see Figure 4, where w stands for "win", *d* for "draw" and *l* for "loose"). The result was compared with the corresponding relative frequency from the simulation (rounding figures) as it follows *(T* stands for "teacher" and *Ps* for "pupils"):

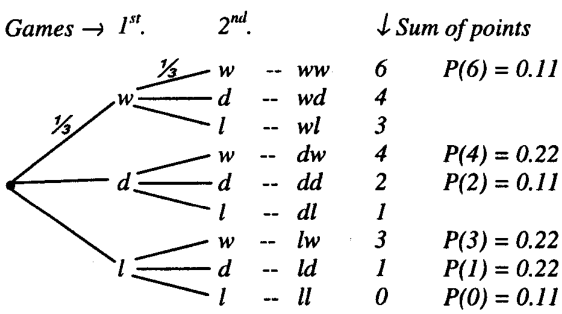

Figure 4. Tree diagram produced in the TC session.

T. Why don't we obtain the same results?
Ps. Because the procedures are different.....
T. But could we say that the results are valid? How close are the two results? **[He refers to the relative frequency and the a priori probability].**
Ps. Yes **[they are close].**
T. We had already said why we obtain different results.
Ps. Because it's at random.
T. Because that's chance.
Ps. Because it's ... is probability.

Afterwards, the teacher addressed again to the activity of the draws (as the context of reference) to summarise the idea of the frequency approach to probability, by adding up the sequences of trials:

T .... Eighteen [trials] *times the five teams. How many in all?*

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Ps. Ninety. ,
T. We did ninety [trials], *did we? Should we have done 200 trials ...*
Ps. They [the relative frequency and the a priori probability] *had become closer! T. What would happen? We would have more results ...*
Ps. More alike.
T. More even, closer. The bigger the number of trials is, the closer the experimental probability would be to the classical [a priori] *probability.*

Remarks

The NC session was centred on the teacher exposition. In addition to the misunderstanding of the instructions to record the results of the different trials in the diagram, since the basic lines were already provided, the activity for its production was just partially due to the students' work. The difficulty for tracing the outcomes of particular trials and to follow closer the gradual statistical regularity restricted severely the linking of context, diagram and concept.

In the TC session, the pupils became involved in the activity. Even if it seems that the situation posed to the students was oversimplified, several fundamental ideas were put into scene in the analysis of the situation (random variable, probability, independence, addition of probabilities, combinatorics), and the students worked out the production and treatment of the semiotic registers they used (numerical and figurative registers). The figures they obtained were interpreted for the situation they were initially posed, so that the interplay between sign, context of reference and concept promoted the activity.

Although the results that Alquicira obtained cannot be taken as representative of what actually occurs in the secondary school classroom of probability, they do provide some insight at least in two aspects. Firstly, the study we presented here points out to the way in which the reference to the epistemological triangle promotes designs of didactical situations without lost of sense for the students of the concepts taught. Secondly, in the constitution of the concept, the context of reference is as important as the system of signs used to represent the relevant aspects in that context to answer the question posed, in order to organise the corresponding information and to work it out.

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