Bills, L. (Ed.) Proceedings of the British Society for Research into Learning Mathematics 19(1) February 1999

UNTAUGHT MENTAL METHODS OF 11-YEAR-OLDS: DATA FROM THE 1987 APU SURVEY

Derek Foxman

Institute of Education, University of London

I have been collaborating with Meindert Beishuizen of Leiden University in a reanalysis of the 1987 APU survey data on methods used by 11-year-olds to obtain answers to mental arithmetic questions. The methods used by the children would almost certainly have been untaught at that time. The categories used in this analysis have been derived from the work of Beishuizen in the Netherlands and in England. The three main methods noted are: manipulating digits (paper and pencil algorithms); splitting numbers into tens and units; and operating on unsplit or complete numbers. So far five of the 12 APU questions have been analysed in these terms, with complete number methods generally proving most efficient.

The APU surveys

The Assessment of Performance Unit (APU) was set up in the mid-1970s by the then DES in order to develop new methods of assessment and to monitor performance in schools in the core curriculum areas. The Unit commissioned independent research agencies to conduct the surveys and report the results. The NFER was the agency which carried out the mathematics surveys of 11- and 15-year-olds (Years 6 and 11) in the schools of England, Wales and Northern Ireland between 1978 and 1987. Six surveys of each age group took place during this period, annually from 1978 to 1982, and then an interval of five years before the final ones in 1987.

The APU practical tests

All the surveys included practical tests which were administered orally by experienced teacher-assessors in one-to-one sessions with pupils. The teachers acting as assessors were nominated by their LEAs and trained to administer the tests by the NFER research team. Usually six age 11 and five age 15 pupils from each of about 200 schools in were administered practical tests. The practical test schools and pupils for each age group were selected from the main probability sample of about 13 000 pupils who took the more standard written tests, and so were reasonably representative of their populations.

The mental skills test

The original intention of the practical tests was to assess mathematics carried out with apparatus or materials and many of the tests did have that

feature. But the NFER team saw the one-to-one situation with its oral delivery as providing opportunities for controlled interaction between assessor and pupil which did not necessarily imply the use of concrete materials. This interaction, especially the 'probe' question, 'How did you get your answer', provided rich information on pupils' methods. An article by Jones (1975), 'Don't just mark the answer - have a look at the method', was influential in the APU team's adoption of this approach.

An attempt was made in the first survey of 11-year-olds to include some calculations which children had the option of carrying out mentally. The questions produced a range of calculation methods, but the recording technique was then insufficiently developed to cope with the data. However, some of the responses given to two subtraction questions were described in the first APU report (Foxman et al 1980). A test of mental calculation skills was not attempted again until the final surveys in 1987 when there was a particular incentive to include them: the recommendation of the Cockcroft Report, published five years before, that mental calculation, which had been neglected for some years, should return to the classroom. The APU could not report changes in mental calculation performance since 1982 as there had not been a test of those skills in the surveys held in that year. However, there was a clear pattern of changes in the results of the written tests. These showed an improvement in Geometry and Probability/statistics, but a decline in performance in calculation at both ages in the five years since the publication of the Cockcroft Report. It seems unlikely, therefore, that there had been any return of mental calculation by 1987, and it is reasonable to assume that the mental calculation methods used by pupils in the 1987 surveys were untaught, and probably unknown to many teachers.

The mental skills test was one of about a dozen tests used in 1987, all of which were untimed. Many of the results of the tests of both age groups were reported in Foxman et al (1991), but there was little research on mental calculation at the time to relate them to. A lot of the responses to questions were unclassified and I thought that some of these might now fit into the classificatory schemes developed by researchers in this decade (eg Beishuizen, Fuson, Thompson). I was contacted at this time by Meindert Beishuizen of Leiden University who had seen the APU report on the mental skills practical tests and we decided to reanalyse the data. The NFER archives yielded the interview data on the 256 age 11 pupils who took the test, and we have so far looked at five of the 12 questions which are listed here. The five analysed are in bold in the list.

APU Mental Skills Questions 1987 Age 11

1. **26 + 7**
2. **64 – 27**
3. Start at 13 and count on in fours
4. Start at 28 and count down in threes
5. I buy fish and chips for £1.46. How much change should I get from £5?
6. **How much would I pay for 4 tapes costing £1.99 each?**
7. **How many 18p stamps can you buy for £1?**
8. I catch a bus at 9.43 am and arrive at my stop at 10.12 am. How long does the journey take?
9. I took my pulse. I counted 21 beats in 15 seconds. How many beats per minute is this?
10. If December the 9^{th} is on a Tuesday, on which day of the week is December 25^{th}?
11. **16 × 25**
12. 238 + 143

The questions were printed in a booklet, one question per page, which was placed in front of the child who was asked to read it out. The recording sheets included instructions to the assessor about what to say to the pupil and how to present materials. The response section of a sheet was divided into four parts: the left hand part to record anything the assessor did or said, the right hand section for what the pupil did or said. The upper part was for recording the answer given and the lower part for the response to 'How did you work out the answer'. Sometimes pupils changed their initial answer when describing their method. In response to a probe from the assessor pupils could elaborate or explain their method further. The initial response and method were recorded as well as any changes in either resulting from a probe.

Some examples of the response categories used in the analysis

The categories used to classify the responses to questions were derived from Beishuizen's work in the Netherlands and in England (Beishuizen et al ; Beishuizen and Anghileri op. cit):

Algorithms: Pupils described a standard column manipulation of digits as they might do the sum on paper.

Split numbers: The tens and units are dealt with separately

Complete Numbers: At least one of the numbers in the calculation is left unsplit and operated on complete.

We have room here for just two of the analyses in detail and for the other three so far completed there is a summary table:

Examples for calculating 16 × 25	Number of pupils	% Success
Complete number methods	**85**	**75**
4 × 25 → 100, × 4	41	90
2 × 25 = 50, 50 × 8, etc	5	80
6 × 25 + 10 × 25 or 16 × 20 + 16 × 5	39	59
Split number methods	**21**	**10**
(10 × 20 + 6 × 20) + 16 × 5 *or 10 × 5 + 6 × 5*		
Separate digit methods	**35**	**0**
5 × 6 + 1 × 2 or 5 × 6 + 1 × 20 or 5 × 6 + 10 × 20		
Algorithm	**57**	**14**
Unclassified/No response	**58**	**0**
TOTALS	**256**	**29**

Examples for calculating 64 – 27	Number of pupils	% Success
Complete number methods	**85**	**80**
64 – 20 – 7	36	78
64 – 7 – 20	16	94
64 – 30 + 3	23	74
27 + 10 + 10 +10 + 7 27 + 30 + 7	10	80
Split number methods	**70**	**36**
60 – 20; 7 – 3 → 44 or 40 – 3 →37	45	31
60 – 20 + 4 – 7 60 – 20 – 7 + 4	25	44
Algorithm	**75**	**71**
Unclassified /No response	**26**	**19**
TOTALS	**256**	**56**

Summary table

	26 + 7		4 tapes @ £1.99 each		18p stamps for £1	
	No.	% success	No.	% success	No.	% success
Complete number	107	99	102	75	131	59
Split number	49	92	79	28	20	60
Other classified methods	57 (counting up)	81				
Algorithm	13	92	30	47	46	76
Unclassified/No response	18	69	45	9	59	14
TOTAL	**256**	**90**	**256**	**45**	**256**	**52**

In 4 of these 5 questions complete number methods were more successful than split number methods. The exception was the number of 18p stamps that could be bought for £1 where the split method was multiplying or counting 5 tens and 5 eights. Those visualisng the algorithm were even more successful in this case. The relative success of these methods in this particular case may well have been due to the multiplier being 5.

Dutch and British views on teaching calculation in primary education

It has been a tradition in several continental countries, such as the Netherlands, Belgium, Germany, Hungary and Switzerland, to focus more on mental arithmetic in early primary years than has been the case in British schools. Beishuizen, in his recent article with Julia Anghileri of Homerton College (December, 1998), outlines developments in the Netherlands since the 1980s in the teaching of calculation. Since the 1980s, textbook design and teaching practice have been influenced by the Realistic Mathematics Education theory (RME). Some main points of the theory are listed here – in references to different ways of carrying out calculations I tend to use the term 'method' as we mostly did in the APU work, rather than 'strategy' because there is an issue about what constitutes a 'strategy' in mental calculation (Beishuizen, 1997; Threlfall, 1998):

- the use of contextualised problems for developing children's informal methods of calculation;
- interactive, whole class, discussions of different methods;
- emphasis on the learning of basic number facts and basic skills up to 100;

- use of the Empty Number Line (ENL) to support both the proceduralisation and the development of mental arithmetic methods.

Among the issues which Beishuizen and Anghileri present in their article are:

- Should informal methods be taught, or should children develop them themselves?
  RME takes an intermediate position
- If methods should be taught, in what order?
  RME designs contextualised situations in order to lead children from informal to higher-level formal strategies ('progressive mathematisation')
- What should be the balance of mental methods and mental recall?
- ENL as a linear model of number representation in contrast to grouping models such arithmetic blocks

References

Beishuizen, M (1997) Development of mathematical strategies and procedures up to 100, in M.Beishuizen, K.P.E Gravemeijer and E.C.D.M. van Lieshout (Eds): *The Role of Contexts and Models in the Development of Mathematical Strategies and Procedures* (Utrecht, Freudenthal Institute)

Beishuizen,M and Anghileri, J (1998) Which Mental Strategies in the Early Number Curriculum? A Comparison of British Ideas and Dutch Views. *British Educational Research Journal*, 24,5 519-538

Foxman, D., Ruddock, G., McCallum, I. And Schagen,I (1991) *APU Mathematics Monitoring (Phase 2)* (London: School Examinations and Assessment Council)

Foxman, D.,Cresswell, M., Ward, M., Badger, M., Tuson, J and Bloomfield, B (1980) *Mathematical Development. Primary Survey Report No. 1* (London, HMSO)

Jones, D. A. (1975) Don't just mark the answer – have a look at the method! *Mathematics in School*, 4, 29 - 31

Thompson, I (Ed.) (1997) *Teaching and Learning Early Number* (Buckingham, Open University Press)

Threlfall, J (1998) Are Mental Calculation Strategies Really Strategies? *Proceedings of the BSRLM Day Conference held at School of Education, University of Leeds.* 14th November 1998