Bills, L. (Ed.) Proceedings of the British Society for Research into Learning Mathematics 19(1) February 1999

BSRLM Geometry Working Group

Convenor: Keith Jones, University of Southampton, UK

Language Use and Geometry Texts

A report based on the meeting at the Open University of Leeds, 27th February 1999

by
Anna Chronaki, Open University
(with contributions from Keith Jones, University of Southampton)

Recent research suggest that with classroom tasks that combine spatial experiences, mathematising, and communicating, pupils may reveal the nature of their own spatial images and personal language in describing these spatial contexts, and experience the use of formal terminology in making accurate descriptions of their observations and constructions. This report focuses on issues of language use involved in geometry activities when particular emphasis is placed on encouraging pupils' practice of informal and formal mathematical vocabulary.

Language Use in Geometry

The multifaceted impact of language on mathematics learning is particularly apparent in geometry. Although linguistic aspects are part of formulating and solving spatial problems, the ability to express spatial images and relations by means of verbal, graphical and symbolic language is not always an easy one. MacFarlane Smith (quoted in McLeay, O'Driscoll-Tole and Jones 1998, p87) argued that spatially gifted pupils can often appear to have poor ability to express themselves in linguistic terms. Love (1995, p125) also suggests that there is not a direct correspondence between the handling of mental images and physical objects, querying whether 'pictures in mind' can exist independently of thought and language. It seems that expressing spatial experiences may include and necessitate a variety of verbal, visual and tactile forms.

In school geometry, artefacts are used as a basis for exploring regularities and for developing the formation of geometrical concepts. Moreover, in classrooms, there is a tendency to make knowledge and thinking processes explicit and public. Yet despite there being well-developed arguments concerning the need to highlight the tacit and practical nature of mathematical knowledge (see Ernest, 1998), current forms of curriculum and assessment are not necessarily consistent with this perspective. As a result, teachers and curriculum writers have to consider the requirement that pupils master what is taken as conventional knowledge of school geometry (the use of 'formal' language and mastery of technical terms is just one expression of this). Teaching materials (often in the form of text based materials) become the major instruments for accessing and developing certain forms of knowledge valued in the institution of school. In addition, assessment procedures and curriculum conceptions of what is to be taught as geometry knowledge regulates the selection and the process of using such materials.

At present, in school practices, informal and formal uses of mathematical language become equally important for different reasons. On the one hand, expressions of thinking through *informal* language arguably enables pupils to participate in a learning task and to develop meaningful understandings by making links with their own prior experiences. On the other hand, pupils' awareness and use of 'formal' language to express their mathematical thinking provides them with access to and full participation in mathematician-like practices. For pupils, failure to understand vocabulary can be a barrier to completing a task, while lack of vocabulary mastery can mean an inability to communicate their thinking in verbal and written forms. Use of 'formal' vocabulary is not an easy task for pupils. O'Driscoll-Tole (1998) found in her study that although there can be evidence for pupils' understanding of a geometrical concept (e.g. by means of drawing), they cannot always name a shape due to their insufficient knowledge of technical vocabulary. However, overlooking the need for pupils to develop their capacity to use formal mathematical terminology may be as damaging as, for example, banning them from expressing themselves informally. Such practices may be especially damaging for pupils in disadvantaged circumstances, such as pupils with learning difficulties, of lower socio-cultural backgrounds, or whose mother tongue language is not English (see Adler, 1998).

The study by Chronaki (1997), alongside others, shares the view that spatial experiences of a practical nature are vital for pupils to develop geometry thinking. Yet the study also considers the importance of encouraging pupils' use and mastery of technical vocabulary that enables them to communicate their thinking. This linking of 'formal' and 'informal' expressions may be essential for pupils' learning and the key for their participation in current 'formal' geometry practices in school.

In Chronaki's research, the case of geometric transformations, and in particular the mastery of terminology to describe their features and regularities (e.g. period of translation, centre of rotation, axis of reflection) was taken as an example to illustrate aspects of the construction of curriculum materials which address the practice of formal and informal terminology, and language co-construction between pupils and teachers. For example, terms such as 'translation', 'rotation', 'reflection' can be considered as formal terminology when compared to the more informal terms of 'slide', 'turn', and 'flip'. Naming in a formal way is not necessarily an activity that leads to meaningful learning of a particular concept, but it can be conceived as an ability that facilitates participation in a mathematician-like practice. Below, we consider how explicit practice of informal and formal use of vocabulary can take place in the classroom.

Classroom praxis: formal and informal ways of speaking over geometry texts
Constructing 'text', in the form of worksheets, entails consideration of how the intended 'messages' concerning mathematical content should and can be communicated to potential readers. Such considerations can lead to debate over the

materials' transparency (see Lave and Wegner, 1991), and, in particular, over the degree of their explicitness or visibility (e.g. should the mathematical content embedded in tasks be addressed explicitly or implicitly). These are important considerations when tasks are designed for pupils' active interaction. The 'text' is not 'innocent' since the aim is to try to convey specific meanings for the content, but also the 'text' cannot be judged away from the context in which it is being used and the particular aims that shape its design. For example, by putting in a worksheet a question phrased like this: 'Find four translations of the Pelta shape' could mean that the very use of the word 'translation' may make the content over-explicit which may then harm the pupils' own active exploration of the concept itself. This can happen when pupils have no idea about what a 'translation' may be and the aim is to get them explore its meaning. But when pupils know already the concept (e.g. from an earlier task and class discussion), asking them to find 'translations' is asking them to search for a pattern like 'translation' within the grid (see Pimm, 1995, pp 38- 45).

A main aim of the tasks in Chronaki's research was to encourage pupils to practice mathematical vocabulary by making use of 'formal' and 'informal' ways of expressing and communicating content. This meant adopting a *conscious* use of formal terminology (e.g. words such as translation, reflection, rotation) in the text of the worksheets so as to engage pupils in reading and problematising its nature. Pupils, of course, had difficulties in interpreting and using this 'formal' terminology. Although in lessons, 'formal' terminology was not imposed but introduced through examples and activities (and the worksheets were part of a series of lessons in which terminology was part of exploratory discussions), when pupils met a 'formal' term in written forms on worksheets they often faced problems in remembering its meaning and significance. Mastering links between 'informal' and 'formal' terms was not an easy task, and the handling of such situations meant that teachers' role and intervention was significant. It is therefore, of particular interest to understand how teachers assisted their pupils' mastery of formal vocabulary. Two extracts of teachers' work in such a direction is discussed below.

Extract 1: Building mathematical vocabulary on pupil's metaphors

Max: [Max colours a translation on the art pattern]
Teacher: What is the distance between the shapes in this one? [points to a translation].
Max: this far ... [points to the pattern]
Teacher: What is the distance between this one and this one? [points to two shapes in the translation].
Max: the same
Teacher: What about the distances here?
Max: the same.
Teacher: Now, if you imagine this shape being put over this shape... Where does that point go to?
Max: it is ...... how it has traveled
Teacher: That is the period of translation [emphasising]. How far each point has traveled...
Max: I can see how far it travels....
Teacher: If you imagine this shape—and I pick it up and move it over that shape
// Where is this point going to?
Max: there
Teacher: How far is that?

Max: [measures] .... 3.7 cm
Teacher: What about this point? [another point of the same shape].
Max: [measures again] ...3.7 cm
Teacher: And this one? How far?
Max: [measures for a third time] ...3.7 cm
Teacher: How far? The distance—each point moves to its new position is called period of translation.
Teacher: So, // the period of translation is 3.7 cm.
Max: [writes down the answer].

Extract 2: Unpacking formal terms
Teacher: Now, how can I measure that angle?
Paul: [no response]
Teacher: Don't you know? [pretends surprise]
Can you show me the dotted line that has rotated?
Paul: [Paul points on the line]
Teacher: Right... So, where has it turned from?
Paul: [Paul shows the point]
Teacher: And where has it turned to?
Paul: [Paul points again]
Teacher: So, you start with that line. And it's turned over to that one.../// Now. You need to put your protractor, so that it's flat along that line.... the line you start with... Yes?
Paul: [listening]
Teacher: And the centre of your protractor is on the pivot point
On the pivot.... Yes?
Paul: [Paul is listening]
Teacher: And you start from here.. So, you look at the nought
Paul: [tries to put the protractor so that to read the measuring]
Teacher: Not, the 180!
Look at the nought [emphasizing]And go around – on the outside scale. Do you see it goes around to 45? Okay?

The above extracts refer to teacher-pupil(s) classroom conversations over some specific mathematical content whilst using tasks in worksheets. The task in the first extract is about finding and colouring 'translations' within an art pattern, the task in the second one is about constructing a 'rotation' of a 45 degrees angle.

In the first extract, the teacher observes Max's colouring of translations and soon (line 2) focuses his discussion with Max over the distance between adjacent points in the translation he coloured. His aim is for Max to develop some understanding about the distance regularity amongst shapes who belong in the same translation and at the same time to use a new word, the 'period of translation' as a formal way of describing this regularity. The teacher does not jump in an explicit imposition of the 'formal' term, but he rather builds on Max's gradual understanding of the concept (i.e the distance regularity) and on Max's informal and metaphoric expression of what he sees on the picture (i.e. the regularity in shapes movement is described as travelling). Both the concept and the word (i.e. period of translation) are new for Max. From line 4 to line 9, the teacher works with Max in developing mainly the new concept, whilst from line 10 till line 22 the teacher concentrates in an interplay between concept development and introduction of 'formal' vocabulary as a way of naming this 'new concept'. For example, in line 10 the teacher says: 'That is the period of translation (emphasising). How far each point has traveled.'

In the second extract, the teacher works with Paul on a task that asks for the construction of a rotation according to a certain angle. Paul needs to measure the exact angle of the rotation. By reading this extract we realise that Paul does not know how to measure accurately the angle of his rotation. Paul has made a rotation, by turning the shape on the white paper about a line, but has not been able to measure its angle in order to specify the 45 rotation asked by the task. The teacher takes Paul, between lines 4 and 11, over a reproduction of how he rotated his shape. Then he shows him how to use the protractor for measuring the angle in this particular rotation. During the part of the rotation's reproduction one can notice that whilst the teacher initially uses the formal word 'rotated' (in line 4) he subsequently replaces it with the more informal terms of 'turned from', 'turned to' and 'turned over'. He realises that Paul needs to cope with learning also how to use the protractor so that to construct an accurate rotation which is an extra demand to what the task requires. It seems that, as a result of this, the teacher shifts from using the formal word 'rotate' and unpacks it to its informal forms of use. Appreciating that Paul should not be engaged with a coordination of learning both the use of 'formal' terminology and the use of a measuring instrument he makes room for him to concentrate in one of these.

Conclusion

Designing materials aiming at constructing routes for practicing geometry language does not guarantee that their 'reading' by the pupils who use them will be at all times unproblematic. Rather, such worksheets can be seen as a basis for work on mathematics in the sense that they entail tasks that aim towards some mathematising activity. This can be conceived as a 'mathematising space' where the pupils interact unassisted with the 'geometry text' (both with its embedded wording and pictures) in the worksheets as they try to read, interpret and act by drawing, writing or playing a game with another pupil. This unassisted interaction aims to guide pupils, according to their capacity and prior experiences, to some type of 'mathematising' since as they read and act, they are engaged in mathematical focusing (in the sense that instead of observing and commenting on the broad aspects of an art pattern they are asked to concentrate on specific mathematical regularity). But, the actual 'space for mathematising' that offers potentiality for bridging what a pupil already knows with what is coming as new, either as a form of observing or naming a nee concept, is the space that is being constructed between the teacher and pupils as part of lesson discussions. For example, in one of the extracts above, one can see that whilst Max could see the 'distance regularity' between shapes he did so through teacher's interaction. Within that 'space' the formal naming for describing the distance regularity is an interwoven part.

Examples of classroom use also show that pupils need to work not only with the 'new words' (e.g. period of translation, rotation) introduced in lessons as part of the materials, but also with the associated 'new concepts' (e.g. distance regularity) and the 'new ways' of grasping concepts (e.g. how to observe a picture, how to use the protractor). This also means that the teacher needs to be prepared to cope with all these simultaneously, to prioritise which one should be emphasised according to pupil's needs and potential. In both extracts above, one can see that the teacher realises and respects the potential and the powerfulness of informal language and metaphors. Informal language that the pupil uses whilst observing the picture is used as a basis to built 'formal' terminology (as in extract 1). Yet informal language also becomes a resource for the teacher when a pupil's need is realised (as in extract 2) and the formal term is replaced and unpacked to familiar language so as to create room for the pupil to cope with some extra demands in the task.

References

Adler, J. (1998), Lights and Limits: recontextualising Lave and Wenger to theorise knowledge of teaching and of learning school mathematics. In A. Watson (Ed), *Situated Cognition and the Learning of Mathematics*. University of Oxford: Centre for Mathematics Education Research.

Chronaki, A. (1997), *Teaching Mathematics through Art Based Activities*. Unpublished PhD Thesis. University of Bath.

Ernest, P. (1998), Mathematical Knowledge and Context. In A. Watson (Ed), *Situated Cognition and the Learning of Mathematics*. University of Oxford: Centre for Mathematics Education Research.

Lave, J. and Wenger, E. (1991), *Situated Learning: legitimate peripheral participation*. Cambridge University Press. Cambridge.

Love, E. (1995), The Functions of Visualisation in Learning Geometry. In R. Sutherland and J. Mason (Eds), *Exploring Mental Imagery with Computers in Mathematics Education*. Berlin: Springer.

McLeay, H. O'Driscoll-Tole, K and Jones, K. (1998) Using Imagery to Solve Spatial Problems. *Proceedings of the British Society for Research into Learning Mathematics*. Leeds: BSRLM. 83-88.

O'Driscoll-Tole, K. (1998), *Identifying Spatial Skills that Underpin the 5-14 Mathematics Curriculum in Scotland*. Presentation to the 1998 conference of the Scottish Educational Research Association held at the University of Dundee.

Pimm, D. (1995), *Symbols and Meanings in School Mathematics*. London: Routledge.

BSRLM Geometry Working Group

The BSRLM geometry working group focuses on the teaching and learning of geometrical ideas in its widest sense. The aim of the group is to share perspectives on a range of research questions which could become the basis for further collaborative work. Suggestions of topics for discussion are always welcome. The group is open to all.

Contact: Keith Jones, University of Southampton, Research and Graduate School of Education, Highfield, Southampton, SO17 1BJ, UK.

e-mail: dkj@southampton.ac.uk

tel: +44 (0)1703 592449

fax: +44 (0)1703 593556

http://www.soton.ac.uk/~gary/crime.html