# YOUNG CHILDREN'S MATHEMATICAL IMAGERY: METHODOLGICAL ISSUES

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**Abstract**
*Imagery is a mental process, hidden to the researcher. This paper discusses the problems of accessing this hidden world, particularly when the research is being undertaken with young children. A pilot study is described in which these problems were experienced, resulting in the development of a grounded theory approach. Some early results from the first case study are used to illustrate three ways in which imagery has emerged using this method.*

## Introduction

As a teacher of special needs children, I had tried tasks based on visual imagery. These tasks had helped some, but not all, of the children to tackle mathematics that they were finding difficult. My current research is concerned with finding why these tasks helped some children and not others.

My early research questions asked whether children generated mathematical imagery when working with numbers and, if they did, how they used imagery in their mathematics. For me the most important question was this second idea 'how do children *use* imagery to help them work mathematically?' I also wanted to investigate whether children of differing attainments had different kinds of imagery and whether they used imagery in different ways.

## The nature of imagery

Psychological researchers have defined imagery as a mental representation based on an original sensory perception. Horowitz, for example says, *'Any thought representation that has a sensory quality we call an image.'* (Horowitz, 1978, p.3) and Kosslyn that images are *'internal representations that "stand in for" (re-present) the corresponding objects'* (Kosslyn, 1994, p.3) Images do not have to be exactly the same as the original perception – they can be elaborated to suit a particular purpose.

Kosslyn proposed a theory of image generation. He described a process where an image is generated and then has to be maintained while the imager inspects the image to see if it is suitable for the task ahead. If inspection shows that the image is suitable, the image can be transformed during 'use'. If the image is not suitable then another image is generated, maintained and inspected.

Intons-Peterson and McDaniel describe image generation or retrieval as a *'knowledge-weighted process'* (Intons-Peterson and McDaniel, 1991, p. 63). Generation of an image involves retrieval of components from long term memory. When images are being inspected, knowledge is required to assess their usefulness and appropriateness. Knowledge allows the imager to combine and manipulate different images. Knowledge is therefore used in selection, arrangement and manipulation of images.

**A hidden world - the problems of researching mathematical imagery**

Both Kosslyn and Horowitz describe the 'privacy' of imagery (Horowitz, 1978, p.5, Kosslyn, 1994, p.2) and make the point that it is difficult to access. Kosslyn says *'Mental images – like all mental events – are notoriously difficult to put on public display'* (1994, p2). He discusses the scepticism of scientists towards research in imagery. He points out that scientists do accept some unseen phenomena, for example electrons, because these cause measurable effects and leave tracks of their existence. Until the 1970s scientists asserted that psychologists *'could not even point to the tracks or the spoor left behind'* (p.2) of mental events called imagery.

This situation has now changed. Kosslyn describes how experimental work done in the 1960s and 1970s by Paivio, Shepherd and Cooper and others showed that the effect of imagery could be seen in empirical results. It is now generally accepted by psychologists that mental events happen and that some of these events are of a sensory nature and can therefore be called images. I know from my own experience that mental events, based on past sensory inputs, occur in my mind. The difficulty is that imagery is essentially a personal event. I can use various methods to communicate the nature of the event. I can use words to describe it or pencil and paper to draw it for example. However, this does not necessarily mean that the communication accurately describes the actual mental event. The very act of bringing a mental event into the open can cause changes.

Mental events are not necessarily conscious events. In raising them to the conscious level, they change. Additional features are added, either to interest the listener or to help the listener understand the communication. In attempting to gain access to hidden mental events, this progressive, changing situation needs to be acknowledged. Any new images formed are part of the whole but they may not be accurate.

When interviewing children it would be easy for the interviewer not only to lead the children to generate further images but to influence those new images as well. A question such as 'is there a line between the numbers?' could easily produce the answer 'yes' from children, who in my experience, strive to please a teacher. On the other hand the line may have been there but the child may not have thought it important to report or may not have been aware of it until asked. This is a thorny problem, do the questions of the interviewer trigger existing imagery or do they trigger creation of imagery.

**Towards a method**

If images are hidden events, then how can these images be brought into view? Some researchers working in this field design tasks to ask about imagery directly. Psychologists like Paivio (1971) and Kosslyn (1994) asked their subjects to form an image and then describe it. Thomas and

Mulligan (1995, p. 10) asked children *'to close their eyes and to imagine the numbers from 1 to 100 and . . . to draw what they saw in their minds'*.

From January to September 1998 I undertook a pilot study in a local primary school to establish a method. My original intention had been to work with a large sample, conducting a single interview with each child. The early pilot study work showed that I could not rely on data obtained in a single interview. I was using imaging tasks such as that used by Thomas and Mulligan – asking the children to close their eyes and 'see' the picture while they worked on mathematics. The children seemed unsure of what was required and in their efforts to please me produced some elaborate and non-mathematical images. When the tasks were repeated, stability was lacking and I felt that I could not be sure my research was valid.

During the latter stages of the pilot study I concentrated on working with the child on mathematical tasks (indirect tasks). I did not ask directly about imagery but used interpretative analysis of the data to see if I could detect whether imagery had been used. This meant that I spent far longer with each child and the research became a case study approach. Stability could be established over a variety of tasks and I felt more confident that I could obtain validity.

This approach required a rewording of my original research questions to match the qualitative nature of the research. My research questions at present are:

- Is there any evidence that the children in the study have mathematical imagery?
- How do the children use that imagery when working mathematically?
- What are the differences in the kinds of imagery and the way it is used by different children?
- Do these differences relate to differences in mathematical attainment?

The first question can be answered by analysis of individual case studies: the final two questions by a comparison of case studies.

**Method**

My research developed into an exploration of a child's ideas about the number system and how they use numbers in operations. Clinical interviewing is interspersed with periods of naturalistic observation. These can be described as indirect methods, in that the children are not asked 'What image do you see?'. By analysing their actions and language, I shall hope to find *'tracks or the spoor left behind'* (Kosslyn 1994, p.2) of mental events. Most of my data will be gathered using the method described above but I have included one direct task - Thomas and Mulligan's (1995) *'close your eyes and imagine numbers from 0 to 100'* task. Analysis of data is an integral part of the method rather than a stage following the fieldwork. Figure 1 shows how the fieldwork and analysis are integrated using a repeated cycle of data collection, transcription, analysis and planning (cycle shown by thick arrows). Towards the end of the case study, the data and analysis is reviewed. Tasks are designed to recheck the ideas that have emerged. When no new ideas seem to be emerging, the case study is concluded. In this way I build a case study of a child's ideas about the number system and arithmetical.

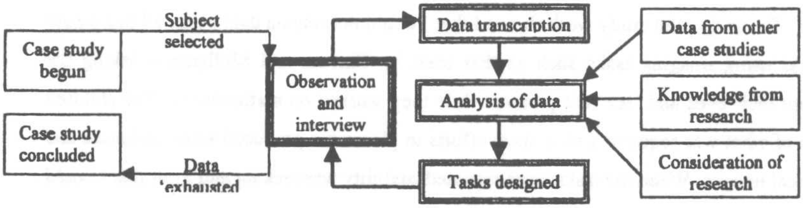


**Figure 1: method cycle for one case study**

This method has been developed from grounded theory. Glaser and Strauss described grounded theory research as *'an ever developing entity, not a perfected* product' (in Woods, 1996, p.67). Measor and Woods (1991, p.60) say *'theory is grounded in the material that is unearthed not specified in advance of the fieldwork'.* When considering the nature of imagery and learning, it would seem likely that imagery is deeply held and generated in fleeting moments when needed. By using a grounded theory method, I hope to be able to unearth the imagery, and so develop a 'theory of each child's imagery'. Woods (1985) expresses concern that, in concentrating on method and slavish adherence to technique, there is a danger of losing the creativity that is one of the strengths of interpretative research. I need to retain the creativity required to elicit the children's ideas. I see my research as *'a mixture of, on the one hand, dedication to the task, scrupulous attention to detail and method, and knowledge, and on the other, the ability to 'let go' of this rigorous application . . . experimenting with new combinations and patterns'* (Woods 1985, p.70)

**Early results**

Early analysis shows that imagery has emerged in 3 ways, which I have classified as 'direct imagery', 'overt imagery' and 'hidden imagery'. One example of each is given below from the case study of a 9 year-old average attainer.

**'Direct imagery'** emerges from responses to Thomas and Mulligan's imagery task. When asked to imagine numbers from 1 to 100, Aaron described a 'carousel' (my label). Each number was brought to the front in turn, inspected and then moved round to the left to make way for the next number.

**'Overt imagery'** emerges from responses to mathematical tasks. When asked to add fractions, Aaron described images of parts of circles to represent the fractions and then transforming the shapes when adding. Figure 2 shows Aaron's addition of ¾ and ¾.

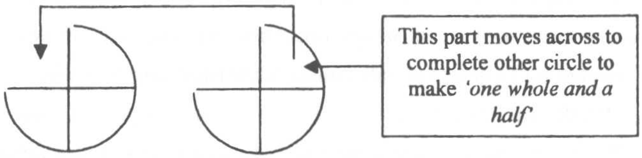


**Figure 2: Aaron's image for addition of fractions**

**'Hidden imagery'** emerges from analysis of mathematical tasks. Aaron has developed a complex method of calculating division. First he 'splits' the tens and units. Then he divides the units by the divisor. Then he splits the tens into separate tens. Then he divides each ten by the divisor. He remembers how many groups he has made and how many remainders there are. He then combines the remainders to make more groups and finally combines all the groups. Figure 3 gives a diagrammatic representation of his method using the example 44 divided by 3.

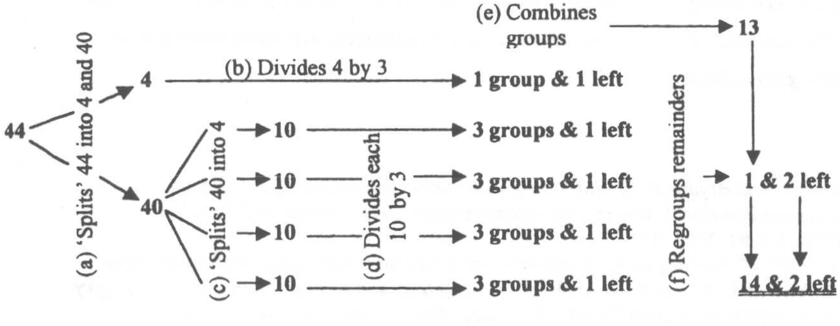


**Figure 3: Aaron's division**

**Discussion**

'Direct imagery' and 'overt imagery' are easily identified. Both examples above could be described as dynamic imagery, using the categories used by Presmeg (1986). The example of 'hidden imagery' is more difficult to justify. Exactly how Aaron arranged the various groups and remainders is not known. Aaron was only able to tell me that he *'had groups and remainders'*.

All three images match Kosslyn's theory of image generation. In the first example the 'carousel' is generated and maintained while each number is inspected in turn. Transformation of the image, moving the next number to front, takes place between each inspection. In the second example Aaron generates the images for ¾ and ¾. He maintains them while he transforms the image with an action. In the third example, Aaron seems to generate the image in sequence until he eventually reaches the result. There are two inspection points – the first where he totals the groups, the second where he combines the remainders (steps 5 and 6 in figure 3). The method had limits. When dividing 88 by 4, for example, he maintained the image until step 5 but during the inspection of the remainders he could no longer maintain the image. This is not surprising when one considers that he is attempting to remember the 18 groups already formed, while regrouping 8 remainders.

Pirie and Kieran (1994) describe the use of imagery as part of a recursive process called the growth of understanding. This theory describes how learners gain an increasingly complex structure of knowledge. Using previous knowledge, they learn new imagery and then use it in increasingly complex ways. Representation then moves away from imagery and ideas become formal and generalised. Looking at Aaron's imagery in terms of this theory, we can see that Aaron is able to use the images successfully. The division image in particular is highly complex.

It is necessary too to look at Aaron's mathematics and ask whether he has moved beyond imagery. Aaron's understanding of the division process itself has progressed to generalised understanding. Correa, Nunes and Bryant (1998, p. 322) say of division, *'the concept of division involves understanding the relationships between three values represented by the dividend, the divisor and the quotient'*. The very fact that Aaron was able to create his computation method infers that he understood these relationships.

**References**

Correa, J., Nunes, T. and Bryant, P. (1998) Young Children's Understanding of Division: The Relationship Between Division Terms in a Noncomputational Task. *Journal of Educational Psychology.* 1998. Vol.**90**, No.2, 321-329

Horowitz, M.J. (1978 2nd edition) *Image Formation and Cognition.* New York: Verlag-Springer

Intons-Peterson, M.J. and McDaniel, M.A. (1991) 'Symmetries and Assymmtries between Imagery and Perception'. Cornoldi, C. and McDaniel, M.A. (eds.) *Imagery and Cognition.* New York: Springer-Verlag

Kosslyn, S.M. (1994) *Image and Brain* Cambridge, Ma: The MIT Press

Measor, L. and Woods, P. (1991) Breakthroughs and blockages in ethnographic research. Walford, G. (ed.) *Doing Educational Research.* London: Routledge

Paivio, A. (1971) *Imagery and Verbal Processes.* New York: Holt, Rinehart and Winston, Inc.

Pirie, S. and Kieren, T. (1994) Growth in mathematical understanding: how can we characterise it and how can we represent it? *Educational Studies in Mathematics* **26**: 165-190, 1994. Kluwer Academic Publishers

Thomas, N. and Mulligan, J. (1995) Dynamic Imagery in Children's Representations of Number. *Mathematics Education Research Journal.* 1995, Vol. **7**, No. 2, 89-100

Woods, P. (1985) Ethnography and theory construction in educational research. Burgess, R. (ed.) *Field Study Methods in Educational Research.* Lewes: The Falmer Press

Woods, P. (1996) Researching the Art of Teaching: Ethnography for Educational Use. London: Routledge